

Steel Beam Analysis

- Steel Codes: ASD vs. LRFD
- Analysis Methods



Steel Beams by LRFD

Yield Stress Values

- A36 Carbon Steel $F_y = 36$ ksi
- A992 High Strength $F_y = 50$ ksi

Elastic Analysis for Bending

• Plastic Behavior (zone 1)

$$M_n = M_p = F_y Z < 1.5 M_y$$

- Braced against LTB ($L_b < L_p$)

• Inelastic Buckling “Decreased” (zone 2)

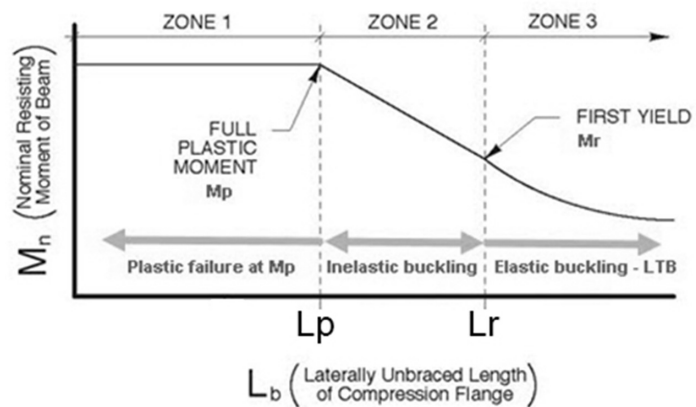
$$M_n = C_b (M_p - (M_p - M_r) [(L_b - L_p) / (L_r - L_p)]) < M_p$$

- $L_p < L_b < L_r$

• Elastic Buckling “Decreased Further” (zone 3)

$$M_{cr} = C_b * \pi / L_b \sqrt{(E * I_y * G * J + (\pi^2 E / L_b)^2 * I_y C_w)}$$

- $L_b > L_r$



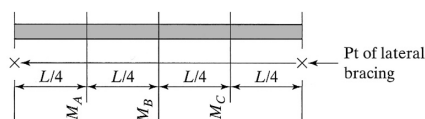
$$L_p = 1.76 r_y \sqrt{E / F_y}$$

$$M_p = F_y Z_x$$

$$M_r = 0.7 F_y S_x$$

C_b is LTB modification factor

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C}$$



Steel Beams by LRFD

Analysis for Bending

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- Plastic Behavior (zone 1)

$$M_n = M_p = F_y Z_x < 1.5 M_y$$

- Braced against LTB ($L_b < L_p$)

- Inelastic Buckling "Decreased" (zone 2)

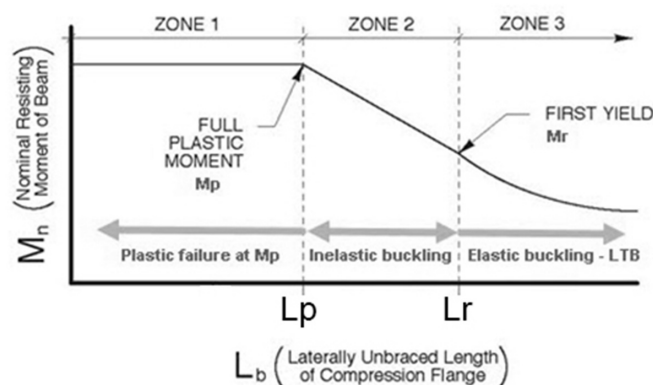
$$M_n = C_b(M_p - (M_p - M_r)[(L_b - L_p)/(L_r - L_p)]) < M_p$$

- $L_p < L_b < L_r$

- Elastic Buckling "Decreased Further" (zone 3)

$$M_{cr} = C_b * \pi / L_b \sqrt{(E * I_y * G * J + (\pi * E / L_b)^2 * I_y C_w)}$$

- $L_b > L_r$



Shape	Z_x in. ³	M_{px}/Ω_b		$\phi_b M_{px}$		M_{rx}/Ω_b		$\phi_b M_{rx}$		BF/Ω_b		$\phi_b BF$		L_p ft	L_r ft	I_x in. ⁴	V_{nx}/Ω_v		$\phi_v V_{nx}$	
		kip-ft	LRFD	kip-ft	LRFD	kip-ft	LRFD	ASD	LRFD	ASD	LRFD	kip-ft	LRFD				kip-ft	LRFD		
W21x55	126	314	473	192	289	10.8	16.3	6.11	17.4	1140	156	234								
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W12x53	77.9	194	292	123	185	3.65	5.50	8.76	28.2	425	83.5	125								
W10x60	74.6	186	280	116	175	2.54	3.82	9.08	36.6	341	85.7	129								
W16x40	73.0	182	274	113	170	6.67	10.0	5.55	15.9	518	97.6	146								
W12x50	71.9	179	270	112	169	3.97	5.98	6.92	23.8	391	90.3	135								
W8x67	70.1	175	263	105	159	1.75	2.59	7.49	47.6	272	103	154								
W14x43	69.6	174	261	109	164	4.88	7.28	6.68	20.0	428	83.6	125								
W10x54	66.6	166	250	105	158	2.48	3.75	9.04	33.6	303	74.7	112								

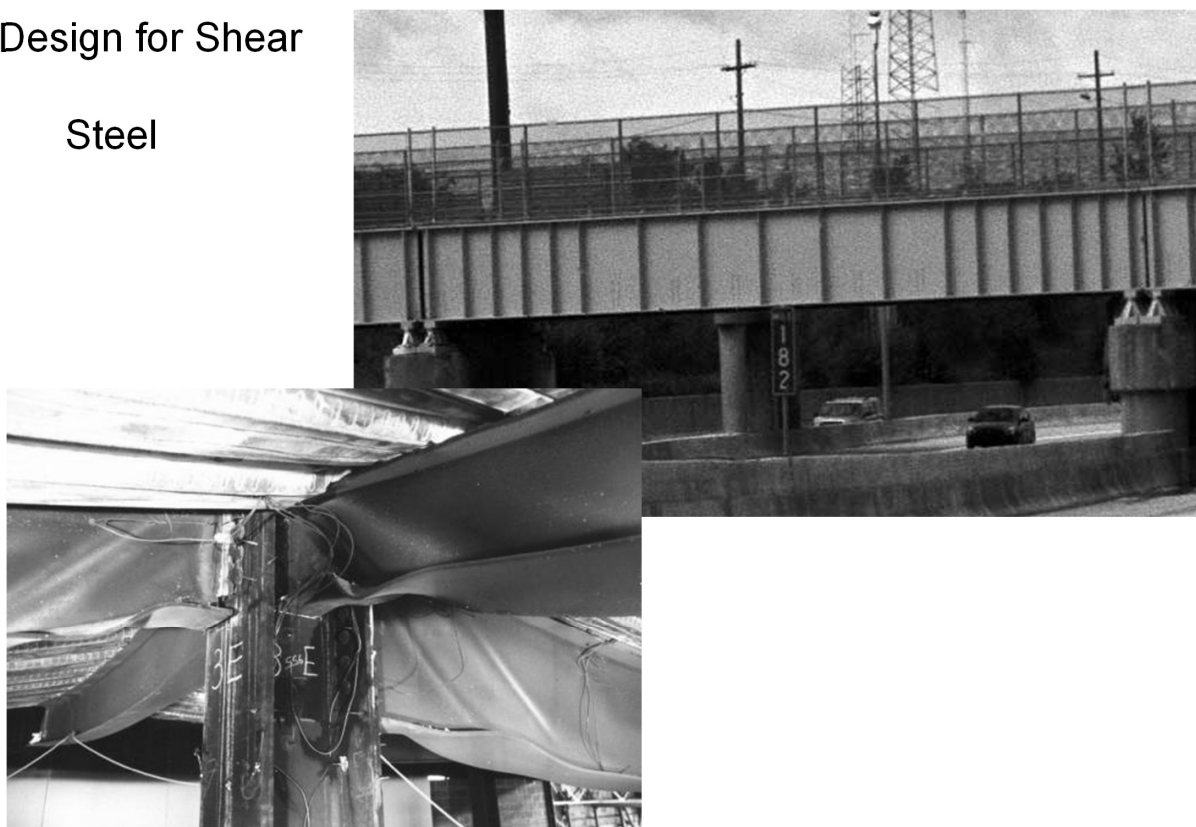
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Structures II

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Design for Shear

Steel



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Structures II

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Design for Shear

Shear stress in steel sections is approximated by averaging the stress in the web:

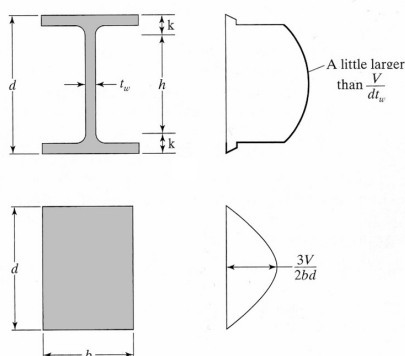
$$F_v = V / A_w$$

$$A_w = d * t_w$$

To adjust the stress a reduction factor of 0.6 is applied to F_y

$$F_v = 0.6 F_y$$

$$\text{so, } V_n = 0.6 F_y A_w \quad (\text{Zone 1})$$



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The equations for the 3 stress zones:
(ϕ in all cases = 1.0)

Zone 1:

WEB YIELDING (Most beam sections fall into this category)

$$\text{if } \frac{h}{t_w} \leq 2.45 \sqrt{E/F_y} = 59 \text{ (for 50 ksi steel)}$$

$$\text{then: } V_n = 0.6 F_y A_w$$

Zone 2:

INELASTIC WEB BUCKLING

$$\text{if } 2.45 \sqrt{E/F_y} < \frac{h}{t_w} \leq 3.07 \sqrt{E/F_y} = 74 \text{ (for 50 ksi steel)}$$

$$\text{then: } V_n = 0.6 F_y A_w (2.45 \sqrt{E/F_y}) / \frac{h}{t_w}$$

Zone 3:

ELASTIC WEB BUCKLING

$$\text{if } 3.07 \sqrt{E/F_y} < \frac{h}{t_w} \leq 260$$

$$\text{then: } V_n = A_w \left[\frac{4.25 E}{\left(\frac{h}{t_w} \right)^2} \right]$$

Structures II

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Procedure - Analysis of Steel Beams – for Zone 1 $L_b < L_p$

Pass/Fail

Given: yield stress, steel section, loading, bracing (L_b)

Find: pass/fail of section

1. Calculate the factored design load w_u
 $w_u = 1.2W_{DL} + 1.6W_{LL}$

2. Determine the design moment M_u .
 M_u will be the maximum beam moment using the factored loads

3. Insure that $L_b < L_p$ (zone 1)
 $L_p = 1.76 r_y \sqrt{E/F_y}$

4. Determine the nominal moment, M_n
 $M_n = F_y Z_x$ (look up Z_x for section)

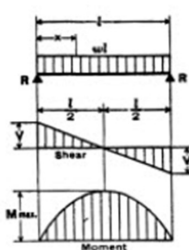
5. Factor the nominal moment
 $\phi M_n = 0.90 M_n$

6. Check that $M_u < \phi M_n$

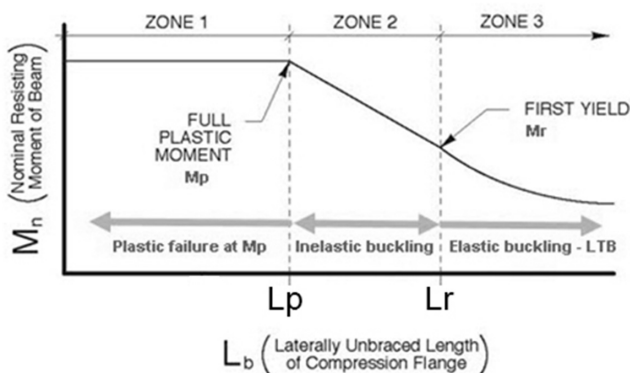
7. Check shear

8. Check deflection

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load	$= wl$
$R = V$	$= \frac{wl}{2}$
V_x	$= w \left(\frac{l}{2} - x \right)$
M max. (at center)	$= \frac{wl^2}{8}$
M_x	$= \frac{wx}{2} (l - x)$
Δ max. (at center)	$= \frac{5wl^4}{384EI}$
Δ_x	$= \frac{wx}{24EI} (l^3 - 2lx^2 + x^3)$



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Structures II

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Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

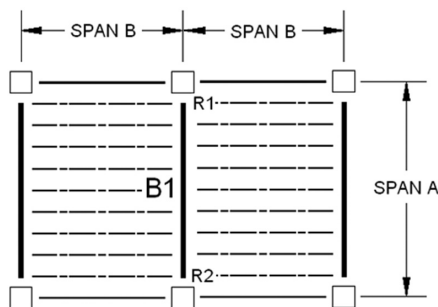
Given: yield stress, steel section, loading, braced 24" o.c.

Find: pass/fail of section

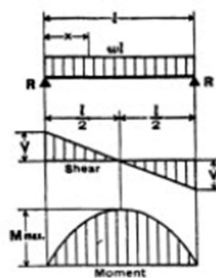
1. Calculate the factored design load w_u

$$w_u = 1.2w_{DL} + 1.6w_{LL}$$

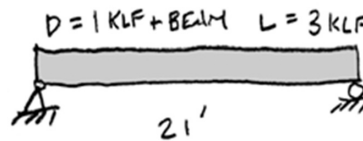
2. Determine the design moment M_u . M_u will be the maximum beam moment using the factored loads.



1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



$$\begin{aligned} \text{Total Equiv. Uniform Load} &= w \\ R = V &= \frac{wl}{2} \\ V_x &= w\left(\frac{l}{2} - x\right) \\ M_{\text{max. (at center)}} &= \frac{wl^2}{8} \\ M_x &= \frac{wx}{2}(l-x) \\ \Delta_{\text{max. (at center)}} &= \frac{5wl^4}{384EI} \\ \Delta_x &= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3) \end{aligned}$$



W21x44
A992 STEEL
 $F_y = 50 \text{ ksi}$

FROM TABLE 1-1 AISC $Z_x = 95.4 \text{ in}^3$

$$w_u = 1.2(1 + 0.044) + 1.6(3) = 6.05 \text{ KLF}$$

$$M_u = \frac{w_u l^2}{8} = \frac{6.05 \text{ KLF} \times 21'^2}{8} = 333.5 \text{ K-ft}$$

Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

3. Insure that $L_b < L_p$ (zone 1)

$$L_p = 1.76 r_y \sqrt{E/F_y}$$

$$L_p = 1.76 (1.26) \sqrt{29000/50}$$

$$L_p = 53.4 \text{ in.} > 24 \text{ in. ok}$$

4. Determine the nominal moment, M_n

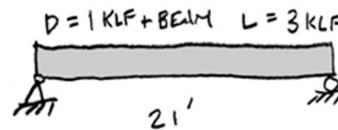
$$M_n = M_p = F_y Z_x \quad (\text{for zone 1})$$

(look up Z_x for section)

5. Factor the nominal moment

$$\phi M_n = 0.90 M_n$$

6. Check that $M_u < \phi M_n$



W21x44
A992 STEEL
 $F_y = 50 \text{ ksi}$

FROM TABLE 1-1 AISC $Z_x = 95.4 \text{ in}^3$

$$M_n = F_y Z_x = 50 \text{ ksi} \times 95.4 \text{ in}^3 = 4770 \text{ K-in.}$$

$$M_n = 4770 \text{ K-in.} / 12 = 397.5 \text{ K-ft}$$

$$\phi M_n = 0.9 (397.5) = 357.7 \text{ K-ft}$$

$$M_u = 333.5 \text{ K-ft} < 357.7 \text{ K-ft} = \phi M_n$$

$\therefore$ PASS

Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

7. Check shear (zone 1)

FROM AISC TABLE 1-1

$$\frac{h}{t_w} = 53.6 < 59 \text{ (zone 1)}$$

Zone 1:

WEB YIELDING (Most beam sections fall into this category)

$$\text{if } \frac{h}{t_w} \leq 2.45 \sqrt{E/F_y} = 59 \text{ (for 50 ksi steel)}$$

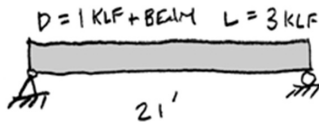
$$\text{then: } V_n = 0.6 F_y A_w$$

CHECK SHEAR:

$$V_u = \frac{w_u \ell}{2} = \frac{6.05(21)}{2} = 63.5 \text{ K}$$

FROM AISC TABLE 1-1

$$\frac{h}{t_w} = 53.6 < 59 \text{ (zone 1)}$$



W21x44
A992 STEEL
 $F_y = 50 \text{ ksi}$

$$\text{FROM TABLE 1-1 AISC } Z_x = 95.4 \text{ in}^3$$

$$w_u = 1.2(1 + 0.044) + 1.6(3) = 6.05 \text{ KLF}$$

$$V_n = 0.6 F_y A_w = 0.6(50)(20.7 \times 0.35)$$

$$V_n = 217.35 \text{ K}$$

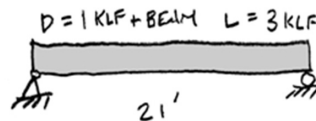
$$\phi V_n = 1.0(217.35) = 217.35 \text{ K}$$

$$V_u = 63.5 \text{ K} < 217.3 \text{ K} = \phi V_n$$

Therefore, pass.

Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

8. Check deflection



W21x44
A992 STEEL
 $F_y = 50 \text{ ksi}$

$$\text{FROM TABLE 1-1 AISC } Z_x = 95.4 \text{ in}^3$$

$$w_u = 1.2(1 + 0.044) + 1.6(3) = 6.05 \text{ KLF}$$

$$\Delta_{\text{MAX}} = \frac{5 w_u \ell^4}{384 EI} = \frac{5(3000)21^4(1728)}{384(29000000)(843)}$$

$$= 0.535 \text{''}$$

$$\frac{\ell}{360} = \frac{21(12)}{360} = 0.7 \text{''}$$

$$\Delta_{\text{ACTUAL}} = 0.535 \text{''} < 0.7 \text{''} = \Delta_{\text{ALLOWABLE}}$$

TABLE 1604.3 DEFLECTION LIMITS^{a, b, c, h, i}

CONSTRUCTION	L	S or W ^f	D + L ^{d, g}
Roof members: ^e			
Supporting plaster or stucco ceiling	//360	//360	//240
Supporting nonplaster ceiling	//240	//240	//180
Not supporting ceiling	//180	//180	//120
Floor members	//360	—	//240
Exterior walls:			
With plaster or stucco finishes	—	//360	—
With other brittle finishes	—	//240	—
With flexible finishes	—	//120	—
Interior partitions: ^b			
With plaster or stucco finishes	//360	—	—
With other brittle finishes	//240	—	—
With flexible finishes	//120	—	—
Farm buildings	—	—	//180
Greenhouses	—	—	//120

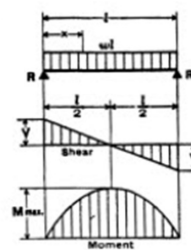
Procedure - Analysis of Steel Beam - Capacity

Given: yield stress, steel section, bracing

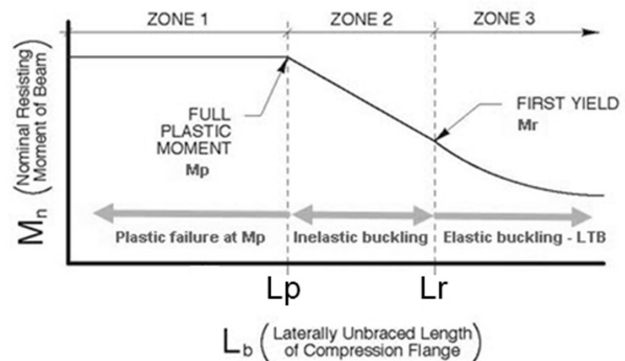
Find: moment or load capacity

1. Determine the unbraced length of the compression flange (L_b).
2. Find the L_p and L_r values from the AISC Z_x Table 3-2
3. Compare L_b to L_p and L_r and determine which equation for M_n or M_{cr} to be used.
4. Determine the beam load equation for maximum moment in the beam.
5. Calculate load based on maximum moment. $M_u = \phi_b M_n$

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



$$\begin{aligned} \text{Total Equiv. Uniform Load} &= wl \\ R = V &= \frac{wl}{2} \\ V_x &= w\left(\frac{l}{2} - x\right) \\ M_{\text{max. (at center)}} &= \frac{wl^2}{8} \\ M_x &= \frac{wx}{2}(l - x) \\ \Delta_{\text{max. (at center)}} &= \frac{5wl^4}{384EI} \\ \Delta x &= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3) \end{aligned}$$



Example – Analysis of Steel Beam - Capacity

Given:

$F_y = 50$ ksi, Fully Braced 20 ft span

Section: W21x44

Find:

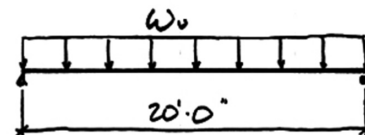
applied live load capacity, w_{LL} in KLF

$$w_u = 1.2w_{DL} + 1.6w_{LL}$$

$$w_{DL} = \text{beam} + \text{floor} = 44\text{plf} + 1500\text{plf}$$

1. Find the Plastic Modulus (Z_x) for the given section from the AISC table 1-1
2. Check that $L_b < L_p$ (fully braced – ok)
3. Determine $M_n = M_p = F_y Z_x$
4. Set $M_u = \phi_b M_n$
 $\phi_b = 0.90$

GIVEN: $F_y = 50$ ksi
W21x44
FULLY BRACED



For a W21x44 FROM TABLE
 $Z_x = 95.4 \text{ in}^3$

$$M_n = F_y Z_x = 50 \text{ ksi} \times 95.4 = 4,770 \text{ k-in}$$

$$M_u = \phi_b M_n = 0.9 \times 4,770 \text{ k-in}$$

$$M_u = 4,293 \text{ k-in} = 357.75 \text{ k-ft}$$

Steel Beams by LRFD

Analysis for Bending

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- Plastic Behavior (zone 1)

$$M_n = M_p = F_y Z$$

- $L_p = 4.45 \text{ ft} = 53.4 \text{ in.}$
- $\phi_b M_{px} = 358 \text{ k-ft} = M_u$

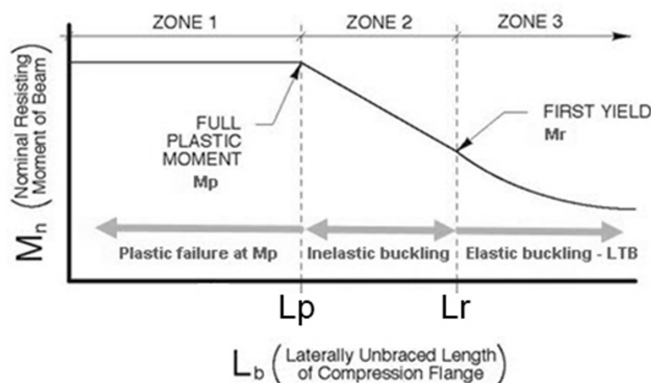


Table 3-2 (continued) W-Shapes Selection by Z_x													
$F_y = 50 \text{ ksi}$													
Shape	Z_x in. ³	M_p/Ω_b		$\phi_b M_{px}$		M_{rx}/Ω_b		$\phi_b M_{rx}$		BF/Ω_b		$\phi_b BF$	
		kip-ft		kip-ft		kip-ft		kip-ft		kips		kips	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
W21x55	126	314	473	192	289	10.8	16.3	6.11	17.4	1140	156	234	234
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W10x77	97.6	244	366	150	225	2.60	3.90	9.18	45.3	455	112	169	169
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W14x48	78.4	196	294	123	184	5.09	7.67	6.75	21.1	484	93.8	141	141
W12x53	77.9	194	292	123	185	3.65	5.50	8.76	28.2	425	83.5	125	125
W10x60	74.6	186	280	116	175	2.54	3.82	9.08	36.6	341	85.7	129	129
W16x40	73.0	182	274	113	170	6.67	10.0	5.55	15.9	518	97.6	146	146
W12x50	71.9	179	270	112	169	3.97	5.98	6.92	23.8	391	90.3	135	135
W8x67	70.1	175	263	105	159	1.75	2.59	7.49	47.6	272	103	154	154
W14x43	69.6	174	261	109	164	4.88	7.28	6.68	20.0	428	83.6	125	125
W10x54	66.6	166	250	105	158	2.48	3.75	9.04	33.6	303	74.7	112	112
ASD	LRFD	⁽¹⁾ Shape exceeds compact limit for flexure with $F_y = 50 \text{ ksi}$; tabulated values have been adjusted accordingly.											
$\Omega_b = 1.67$ $\Omega_y = 1.50$	$\phi_b = 0.90$ $\phi_y = 1.00$												

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Structures II

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Example – Analysis of Steel Beam - Capacity

- Using the maximum moment equation, solve for the factored distributed loading, w_u

$$M_u = \frac{w_u l^2}{8} \Rightarrow w_u = \frac{8 M_u}{l^2}$$

$$w_u = \frac{8 \times 357.75 \text{ k-ft}}{20 \text{ ft}^2}$$

$$w_u = 7.155 \text{ k/ft}$$

- The applied (unfactored) load $w = w_u / (\text{g factors})$
 $w_u = 1.2w_{DL} + 1.6w_{LL}$

$$w_u = 7.155 \text{ k/ft} = 1.2(0.044 + 1.5) + 1.6(w_{LL})$$

$$w_u = 1.853 + 1.6 w_{LL} = 7.155 \text{ k/ft}$$

$$w_{LL} = 3.31 \text{ k/ft}$$

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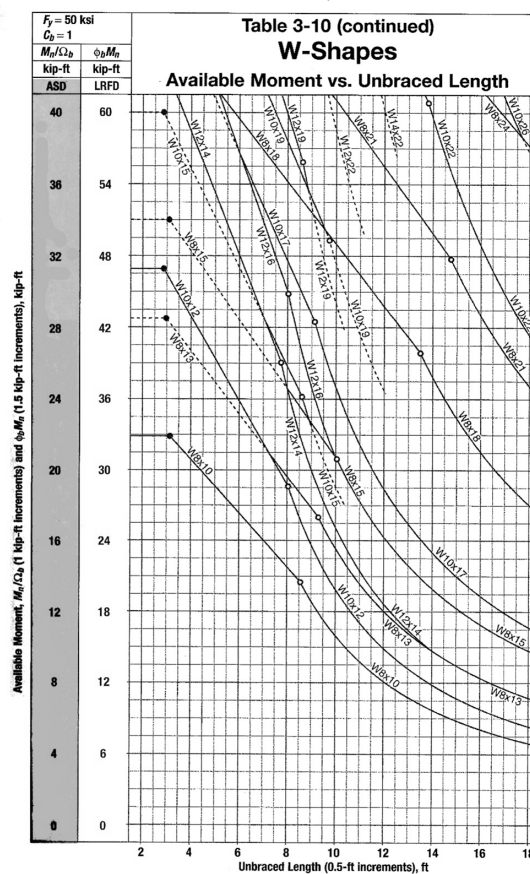
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Steel Beams by LRFD

Moment Capacity with L_b Graphs

Analysis for Bending

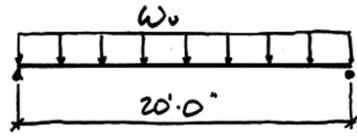
- Plastic Behavior (zone 1)
 $M_n = M_p$
 Braced against LTB ($L_b < L_p$)
- Inelastic Buckling "Decreased" (zone 2)
 $M_n < M_p$
 $L_p < L_b < L_r$
- Elastic Buckling "Decreased Further" (zone 3)
 $M_n = M_{cr}$
 $L_b > L_r$



Steel Beams by LRFD

Moment Capacity Graphs

GIVEN: $F_y = 50 \text{ ksi}$
 $W21 \times 44$
 FULLY BRACED



FOR A $W21 \times 44$ FROM TABLE
 $Z_x = 95.4 \text{ in}^3$

$$M_n = F_y Z_x = 50 \text{ ksi} \times 95.4 = 4,770 \text{ kip-in}$$

$$M_u = \phi_b M_n = 0.9 \times 4,770 \text{ kip-in}$$

$$M_u = 4,293 \text{ kip-in} = 357.75 \text{ kft}$$

