

Steel Beam Analysis

- Steel Codes: ASD vs. LRFD
- Analysis Methods



Steel Beams by LRFD

Yield Stress Values

- A36 Carbon Steel $F_y = 36$ ksi
- A992 High Strength $F_y = 50$ ksi

Elastic Analysis for Bending

• Plastic Behavior (zone 1)

$$M_n = M_p = F_y Z' < 1.5 M_y$$

- Braced against LTB ($L_b < L_p$)

• Inelastic Buckling "Decreased" (zone 2)

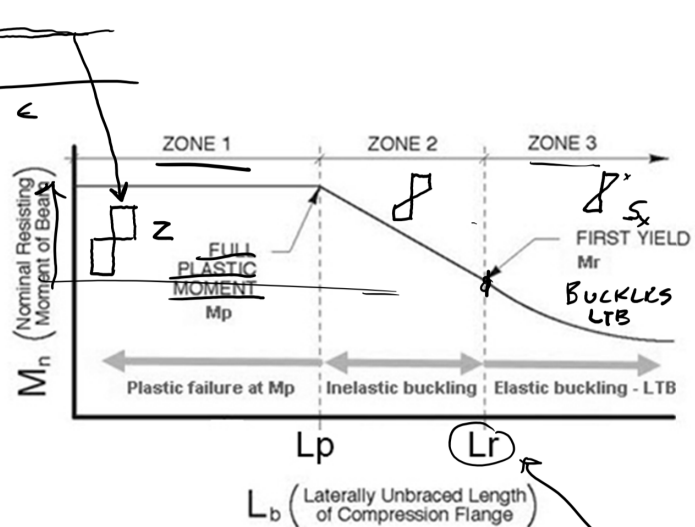
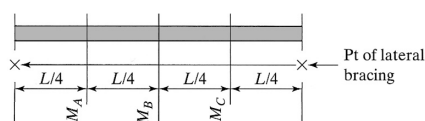
$$M_n = C_b [M_p - (M_p - M_r) \frac{(L_b - L_p)}{(L_r - L_p)}] < M_p$$

- $L_p < L_b < L_r$

• Elastic Buckling "Decreased Further" (zone 3)

$$M_{cr} = C_b \frac{\pi^2 EI_y}{L_b^2} \sqrt{E I_y G J + (\pi^2 E I_y / L_b^2)^2 I_y C_w}$$

- $L_b > L_r$



$$L_p = 1.76 r_y \sqrt{E/F_y}$$

$$M_p = F_y Z_x$$

$$M_r = 0.7 F_y S_x$$

C_b is LTB modification factor

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C}$$

Steel Beams by LRFD

Analysis for Bending

AISC 16th ed.

- Plastic Behavior (zone 1)

$$M_n = M_p = F_y Z_x < 1.5 M_y$$

- Braced against LTB ($L_b < L_p$)

- Inelastic Buckling "Decreased" (zone 2)

$$M_n = C_b (M_p - (M_p - M_r) [(L_b - L_p) / (L_r - L_p)]) < M_p$$

- $L_p < L_b < L_r$

- Elastic Buckling "Decreased Further" (zone 3)

$$M_{cr} = C_b * \pi / L_b \sqrt{(E * I_y * G * J + (\pi^2 E / L_b)^2 * I_y C_w)}$$

- $L_b > L_r$

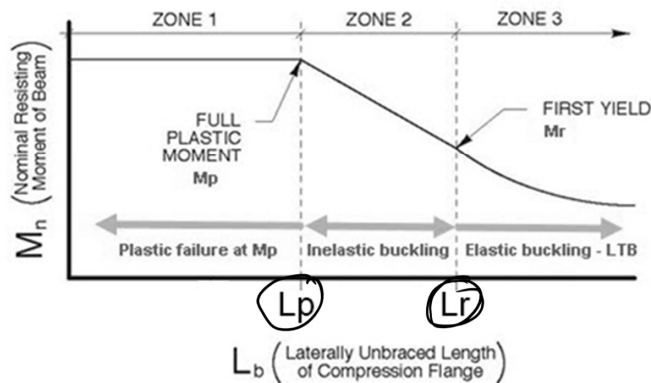


Table 3-2 (continued)

$F_y = 50$ ksi ASD LRFD W-Shapes Selection by Z_x

Z_x

Shape	Z_x in. ³	ASD		LRFD		$\phi_b M_n$		$\phi_b M_r$		$\phi_b B F$		L_p ft	L_r ft	I_x in. ⁴	V_n / Ω_v		$\phi_v V_n$	
		kip-ft	kip-ft	kip-ft	kip-ft	kip-ft	kip-ft	kip-ft	kip-ft	kip-ft	kip-ft							
W21x55	126	314	473	192	289	10.8	16.3	6.11	17.4	1140	156	234						
W14x74	126	314	473	196	294	5.31	8.05	8.76	31.0	795	128	192						
W18x60	123	307	461	189	284	9.62	14.4	5.93	18.2	984	151	227						
W12x79	119	297	446	187	281	3.78	5.67	10.8	39.9	662	117	175						
W14x68	115	287	431	180	270	5.19	7.81	8.69	29.3	722	116	174						
W10x88	113	282	424	172	259	2.62	3.94	9.29	51.2	534	131	196						
W18x55	112	279	420	172	258	9.15	13.8	5.90	17.6	890	141	212						
W21x50	110	274	413	165	248	12.1	18.3	4.59	13.6	984	158	237						
W12x72	108	269	405	170	256	3.69	5.56	10.7	37.5	597	106	159						
W21x48 ⁽¹⁾	107	265	398	162	244	9.89	14.8	6.09	16.5	959	144	216						
W16x57	105	262	394	161	242	7.98	12.0	5.65	18.3	758	141	212						
W14x61	102	254	383	161	242	4.93	7.48	8.65	27.5	640	104	156						
W18x50	101	252	379	155	233	8.76	13.2	5.83	16.9	800	128	192						
W10x77	97.6	244	366	150	225	2.60	3.90	9.18	45.3	455	112	169						
W12x65 ⁽¹⁾	96.8	237	356	154	231	3.58	5.39	11.9	35.1	533	94.4	142						
W21x44	95.4	238	358	143	214	11.1	16.8	4.45	13.0	843	145	217						
W18x50	92.0	230	345	141	213	7.69	11.4	5.62	17.2	659	124	186						
W18x46	90.7	226	340	138	207	9.63	14.6	4.56	13.7	712	130	195						
W14x53	87.1	217	327	136	204	5.22	7.93	6.78	22.3	541	103	154						
W12x58	86.4	216	324	136	205	3.82	5.69	8.87	29.8	475	87.8	132						
W10x68	85.3	213	320	132	199	2.56	3.85	9.15	40.6	394	97.8	147						
W16x45	82.3	205	309	127	191	7.12	10.8	5.55	16.5	586	111	167						
W18x40	78.4	196	294	119	180	8.94	13.2	4.49	13.1	612	113	169						
W14x48	78.4	196	294	123	184	5.09	7.67	6.75	21.1	484	93.8	141						
W12x53	77.9	194	292	123	185	3.65	5.50	8.76	28.2	425	83.5	125						
W10x60	74.6	186	280	116	175	2.54	3.82	9.08	36.6	341	85.7	129						
W16x40	73.0	182	274	113	170	6.67	10.0	5.55	15.9	518	97.6	146						
W12x50	71.9	179	270	112	169	3.97	5.98	6.92	23.8	391	90.3	135						
W8x67	70.1	175	263	105	159	1.75	2.59	7.49	47.6	272	103	154						
W14x43	69.6	174	261	109	164	4.88	7.28	6.68	20.0	428	83.6	125						
W10x54	66.6	166	250	105	158	2.48	3.75	9.04	33.6	303	74.7	112						

⁽¹⁾Shape exceeds compact limit for flexure with $F_y = 50$ ksi; tabulated values have been adjusted accordingly.

$\Omega_b = 1.67$ $\phi_b = 0.90$
 $\Omega_v = 1.50$ $\phi_v = 1.00$

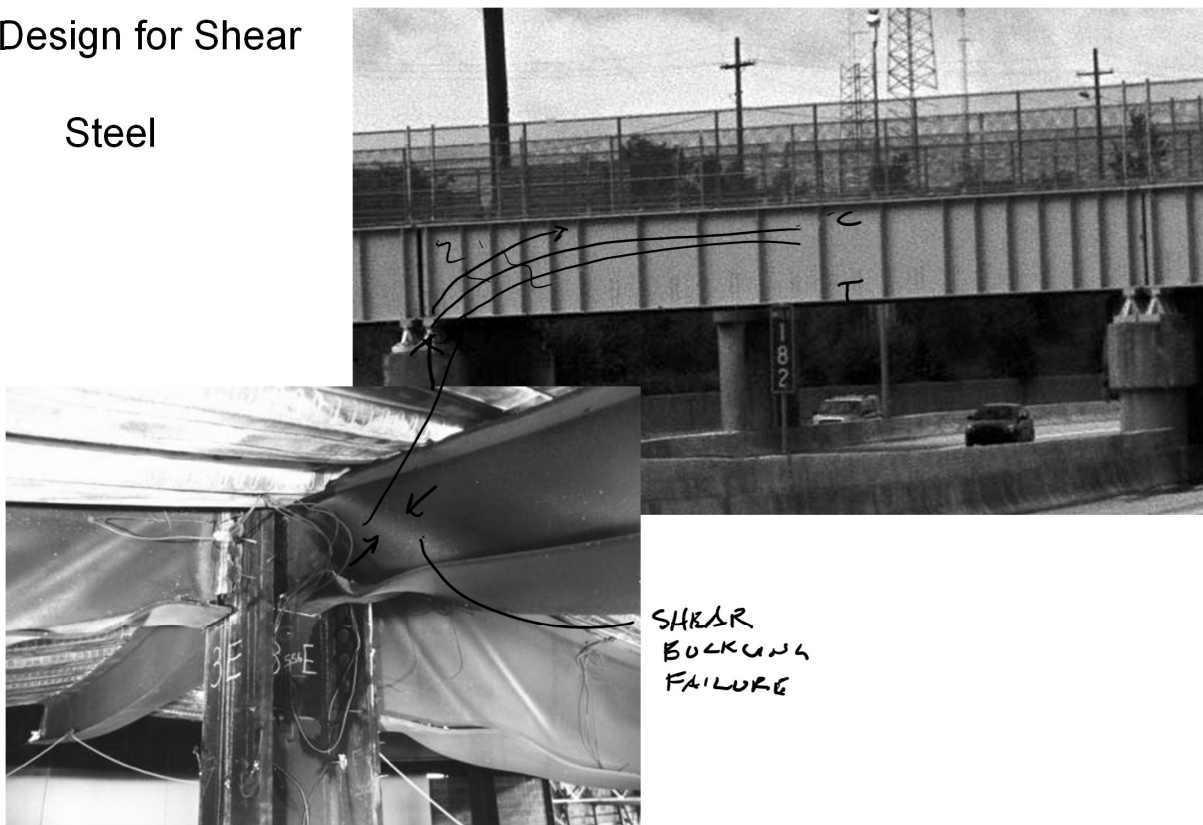
University of Michigan, TCAUP

Structures II

Slide 3 of 19

Design for Shear

Steel



University of Michigan, TCAUP

Structures II

Slide 4 of 19

Design for Shear

Shear stress in steel sections is approximated by averaging the stress in the web:

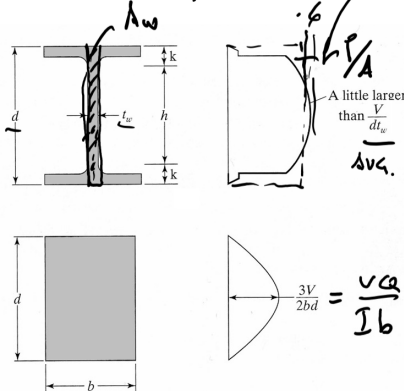
$$F_v = V / A_w$$

$$A_w = d * t_w$$

To adjust the stress a reduction factor of 0.6 is applied to F_y

$$F_v = 0.6 F_y$$

so, $V_n = 0.6 F_y A_w$ (Zone 1)



The equations for the 3 stress zones:
(ϕ in all cases = 1.0)

Zone 1:

WEB YIELDING (Most beam sections fall into this category)

if $\frac{h}{t_w} \leq 2.45 \sqrt{E/F_y} = 59$ (for 50 ksi steel) ✓

then: $V_n = 0.6 F_y A_w > V_u$

Zone 2:

INELASTIC WEB BUCKLING

if $2.45 \sqrt{E/F_y} < \frac{h}{t_w} \leq 3.07 \sqrt{E/F_y} = 74$ (for 50 ksi steel)

then: $V_n = 0.6 F_y A_w (2.45 \sqrt{E/F_y}) / \frac{h}{t_w}$

Zone 3:

ELASTIC WEB BUCKLING

if $3.07 \sqrt{E/F_y} < \frac{h}{t_w} \leq 260$

then: $V_n = A_w \left[\frac{4.25 E}{\left(\frac{h}{t_w} \right)^2} \right]$

Procedure - Analysis of Steel Beams – for Zone 1 $L_b < L_p$

Pass/Fail

Given: yield stress, steel section, loading, bracing (L_b)

Find: pass/fail of section

1. Calculate the factored design load w_u

$$w_u = 1.2 w_{DL} + 1.6 w_{LL}$$

2. Determine the design moment M_u .

Design M_u will be the maximum beam moment using the factored loads

3. Insure that $(L_b) < L_p$ (zone 1)

$$L_p = 1.76 (r_y) \sqrt{E/F_y}$$

SECTION

4. Determine the nominal moment, M_n

Zone 1 $M_n = F_y Z_x$ (look up Z_x for section)

5. Factor the nominal moment

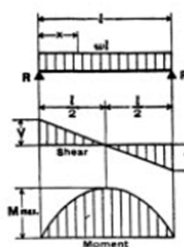
$$\phi M_n = 0.90 M_n$$

6. Check that $M_u < \phi M_n$

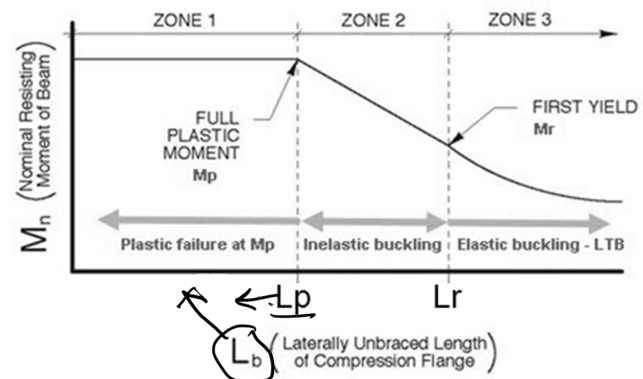
7. Check shear

8. Check deflection

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load	$= w l$
$R = V$	$= \frac{w l}{2}$
V_x	$= w \left(\frac{l}{2} - x \right)$
M max. (at center)	$= \frac{w l^2}{8}$
M_x	$= \frac{w x}{2} (l - x)$
Δ max. (at center)	$= \frac{5 w l^4}{384 E I}$
Δ_x	$= \frac{w x}{24 E I} (l^3 - 2 l x^2 + x^3)$



Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

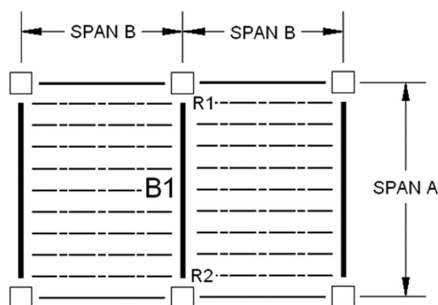
Given: yield stress, steel section, loading, braced 24" o.c.

Find: pass/fail of section

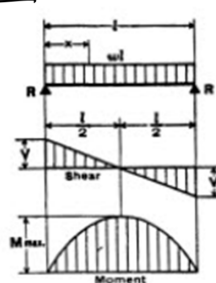
1. Calculate the factored design load w_u

$$w_u = 1.2W_{DL} + 1.6W_{LL}$$

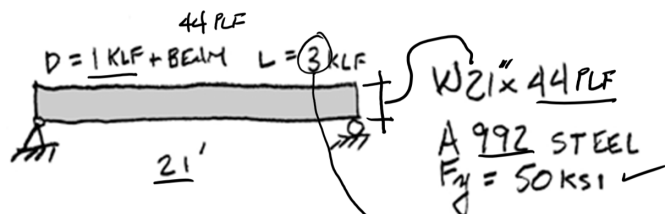
2. Determine the design moment M_u . M_u will be the maximum beam moment using the factored loads.



1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



$$\begin{aligned} \text{Total Equiv. Uniform Load} &= wl \\ R = V &= \frac{wl}{2} \\ V_x &= w\left(\frac{l}{2} - x\right) \\ M_{\text{max. (at center)}} &= \frac{wl^2}{8} \\ M_x &= \frac{wx}{2}(l-x) \\ \Delta_{\text{max. (at center)}} &= \frac{5wl^4}{384EI} \\ \Delta_x &= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3) \end{aligned}$$

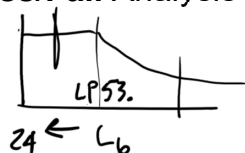


FROM TABLE 1-1 AISC $Z_x = 95.4 \text{ in}^3$

$$w_u = 1.2(1 + 0.044) + 1.6(3) = 6.05 \text{ KLF}$$

$$M_u = \frac{w_u l^2}{8} = \frac{6.05 \text{ KLF} \times 21'^2}{8} = 333.5 \text{ K-ft}$$

Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$



3. Insure that $L_b < L_p$ (zone 1)

$$L_p = 1.76 r_y \sqrt{E/F_y}$$

$$L_p = 1.76 (1.26) \sqrt{29000/50}$$

$$L_p = 53.4 \text{ in.} > 24 \text{ in.} \text{ ok}$$

4. Determine the nominal moment, M_n

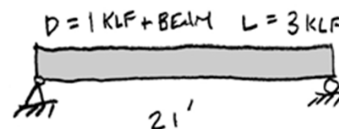
$$M_n = M_p = F_y Z_x \text{ (for zone 1)}$$

(look up Z_x for section)

5. Factor the nominal moment

$$\phi M_n = 0.90 M_n$$

6. Check that $M_u < \phi M_n$



W21x44
A 992 STEEL
 $F_y = 50 \text{ ksi}$

FROM TABLE 1-1 AISC $Z_x = 95.4 \text{ in}^3$

$$M_n = F_y Z_x = 50 \text{ ksi} \times 95.4 \text{ in}^3 = 4770 \text{ K-in.}$$

$$M_n = 4770 \text{ K-in.} / 12 = 397.5 \text{ K-ft}$$

$$\phi M_n = 0.9 (397.5) = 357.7 \text{ K-ft}$$

DESIGN

$$M_u = 333.5 \text{ K-ft} < 357.7 \text{ K-ft} = \phi M_n$$

\therefore PASS STRENGTH

Slide 10 of 19

Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

7. Check shear (zone 1)

FROM AISC TABLE 1-1

$$\frac{h}{t_w} = 53.6 < 59 \text{ (zone 1)}$$

Zone 1:

WEB YIELDING (Most beam sections fall into this category)

$$\text{if } \frac{h}{t_w} \leq 2.45 \sqrt{E/F_y} = 59 \text{ (for 50 ksi steel)}$$

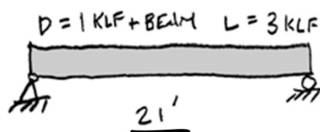
$$\text{then: } V_n = 0.6 F_y A_w$$

CHECK SHEAR:

$$V_u = \frac{w_u \ell}{2} = \frac{6.05(21)}{2} = 63.5 \text{ K}$$

FROM AISC TABLE 1-1

$$\frac{h}{t_w} = 53.6 < 59 \text{ (zone 1)}$$



W21x44
A 992 STEEL
 $F_y = 50 \text{ KSI}$

FROM TABLE 1-1 AISC $Z_x = 95.4 \text{ in}^3$

$$w_u = 1.2(1 + 0.044) + 1.6(3) = 6.05 \text{ KLF}$$

$$V = \frac{w_u \ell}{2} = 63.5 \text{ K}$$

$$V_n = 0.6 F_y A_w = 0.6(50)(20.7 \times 0.35) = 217.35 \text{ K}$$

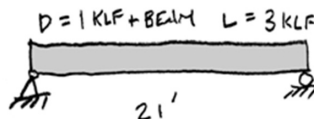
$$\phi V_n = 1.0(217.35) = 217.35 \text{ K STRENGTH}$$

$$V_u = 63.5 \text{ K} < 217.3 \text{ K} = \phi V_n \quad \checkmark$$

Therefore, pass.

Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

8. Check deflection



W21x44
A 992 STEEL
 $F_y = 50 \text{ KSI}$

FROM TABLE 1-1 AISC $Z_x = 95.4 \text{ in}^3$

$$w_u = 1.2(1 + 0.044) + 1.6(3) = 6.05 \text{ KLF}$$

$$\Delta_{\text{MAX}} = \frac{5 w_u \ell^4}{384 EI} = \frac{5(3000) 21^4 (1728)}{384(29000000)(843)}$$

$$= 0.535 \text{ inches}$$

$$\frac{\ell}{360} = \frac{21(12)}{360} = 0.7 \text{ inches}$$

$$\Delta_{\text{ACTUAL}} = 0.535 \text{ inches} < 0.7 \text{ inches} = \Delta_{\text{ALLOWABLE}} \quad \checkmark$$

TABLE 1604.3 DEFLECTION LIMITS^{a, b, c, h, i}

CONSTRUCTION	$\frac{L}{360}$	$\frac{S \text{ or } W}{180}$	$\frac{D + L^d, g}{240}$
Roof members: ^e			
Supporting plaster or stucco ceiling	//360	//360	//240
Supporting nonplaster ceiling	//240	//240	//180
Not supporting ceiling	//180	//180	//120
Floor members	//360	—	//240
Exterior walls:			
With plaster or stucco finishes	—	//360	—
With other brittle finishes	—	//240	—
With flexible finishes	—	//120	—
Interior partitions: ^b			
With plaster or stucco finishes	//360	—	—
With other brittle finishes	//240	—	—
With flexible finishes	//120	—	—
Farm buildings	—	—	//180
Greenhouses	—	—	//120

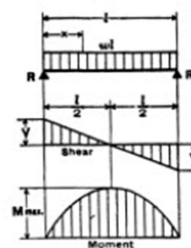
Procedure - Analysis of Steel Beam - Capacity

Given: yield stress, steel section, bracing

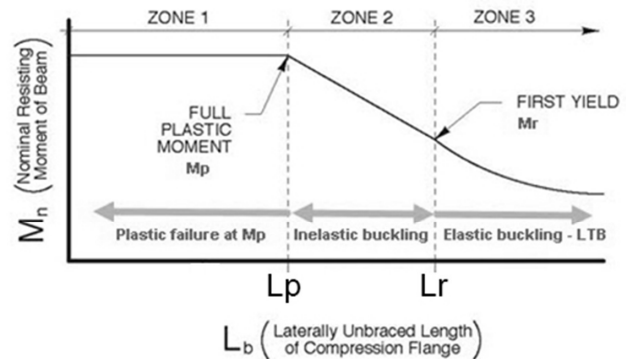
Find: moment or load capacity

1. Determine the unbraced length of the compression flange (L_b).
2. Find the L_p and L_r values from the AISC Z_x Table 3-2
3. Compare L_b to L_p and L_r and determine which equation for M_n or M_{cr} to be used.
4. Determine the beam load equation for maximum moment in the beam.
 $M_u = \phi_b M_n$
5. Calculate load based on maximum moment. $M_u = \phi_b M_n$
 \downarrow SOLVE $\frac{w l^2}{8}$

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



$$\begin{aligned} \text{Total Equiv. Uniform Load} &= w l \\ R = V &= \frac{w l}{2} \\ V_x &= w \left(\frac{l}{2} - x \right) \\ M_{\text{max. (at center)}} &= \frac{w l^2}{8} \\ M_x &= \frac{w x}{2} (l - x) \\ \Delta_{\text{max. (at center)}} &= \frac{5 w l^4}{384 E I} \\ \Delta x &= \frac{w x}{24 E I} (l^3 - 2 l x^2 + x^3) \end{aligned}$$



Example – Analysis of Steel Beam - Capacity

Given:

$F_y = 50$ ksi, Fully Braced 20 ft span

Section: W21x44

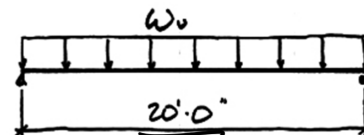
Find:

applied live load capacity, w_{LL} in KLF

$$\begin{aligned} w_u &= 1.2 w_{DL} + 1.6 w_{LL} \\ w_{DL} &= \text{beam} + \text{floor} = 44 \text{ plf} + 1500 \text{ plf} \quad DL \end{aligned}$$

1. Find the Plastic Modulus (Z_x) for the given section from the AISC table 1-1
2. Check that $L_b < L_p$ (fully braced – ok)
3. Determine $M_n = M_p = F_y Z_x$
4. Set $M_u = \phi_b M_n$
 $\phi_b = 0.90$

GIVEN: $F_y = 50$ ksi
W21x44
FULLY BRACED



For a W21x44 FROM TABLE
 $Z_x = 95.4 \text{ in}^3$

$$\begin{aligned} \text{ZONE 1} \\ M_n = F_y Z_x &= 50 \text{ ksi} \times 95.4 = 4,770 \text{ k-in} \\ \phi_b M_n &= 0.9 \times 4,770 \text{ k-in} \\ M_u &= 4,293 \text{ k-in} = 357.75 \text{ k-ft} \end{aligned}$$

$$\frac{w l^2}{8}$$

Steel Beams by LRFD

Analysis for Bending

AISC 16th ed.

- Plastic Behavior (zone 1)

$$M_n = M_p = F_y Z$$

- $L_p = 4.45 \text{ ft} = 53.4 \text{ in.}$
- $\phi_b M_{px} = 358 \text{ k-ft} = M_u$

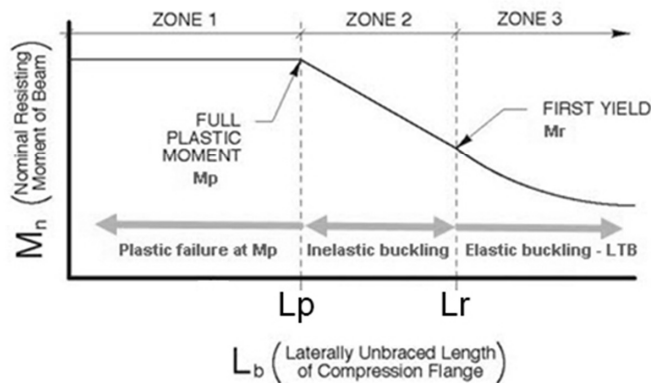


Table 3-2 (continued)																		
W-Shapes																		
Selection by Z_x																		
$F_y = 50 \text{ ksi}$																		
Shape	Z_x in. ³	M_{px}/Ω_b		$\phi_b M_{px}$		M_{rx}/Ω_b		$\phi_b M_{rx}$		BF/Ω_b	$\phi_b BF$	L_p	L_r	I_x	V_{nx}/Ω_v		$\phi_v V_{nx}$	
		kips	LRFD	kips	LRFD	kips	LRFD	kips	LRFD	kips	LRFD				kips	LRFD		
W21x55	126	314	473	192	289	10.8	16.3	6.11	17.4	1140	156	234						
W14x74	126	314	473	196	294	5.31	8.05	8.76	31.0	795	128	192						
W18x60	123	307	461	189	284	9.62	14.4	5.93	18.2	984	151	227						
W12x79	119	297	446	187	281	3.78	5.67	10.8	39.9	662	117	175						
W14x68	115	287	431	180	270	5.19	7.81	8.69	29.3	722	116	174						
W10x88	113	282	424	172	259	2.62	3.94	9.29	51.2	534	131	196						
W18x55	112	279	420	172	258	9.15	13.8	5.90	17.6	890	141	212						
W21x50	110	274	413	165	248	12.1	18.3	4.59	13.6	984	158	237						
W12x72	108	269	405	170	256	3.69	5.56	10.7	37.5	597	106	159						
W21x48 ⁽¹⁾	107	265	398	162	244	9.89	14.8	6.09	16.5	959	144	216						
W16x57	105	262	394	161	242	7.98	12.0	5.65	18.3	758	141	212						
W14x61	102	254	383	161	242	4.93	7.48	8.65	27.5	640	104	156						
W18x50	101	252	379	155	233	8.76	13.2	5.83	16.9	800	128	192						
W10x77	97.6	244	366	150	225	2.60	3.90	9.18	45.3	455	112	169						
W12x65 ⁽¹⁾	96.8	237	356	154	231	3.58	5.39	11.9	35.1	533	94.4	142						
W21x44	95.4	238	358	143	214	11.1	16.8	4.45	13.0	843	145	217						
W16x50	92.0	230	345	141	213	7.69	11.4	5.62	17.2	659	124	186						
W18x46	90.7	226	340	138	207	9.63	14.6	4.56	13.7	712	130	195						
W14x53	87.1	217	327	136	204	5.22	7.93	6.78	22.3	541	103	154						
W12x58	86.4	216	324	136	205	3.82	5.69	8.87	29.8	475	87.8	132						
W10x68	85.3	213	320	132	199	2.58	3.85	9.15	40.6	394	97.8	147						
W16x45	82.3	205	309	127	191	7.12	10.8	5.55	16.5	586	111	167						
W18x40	78.4	196	294	119	180	8.94	13.2	4.49	13.1	612	113	169						
W14x48	78.4	196	294	123	184	5.09	7.67	6.75	21.1	484	93.8	141						
W12x53	77.9	194	292	123	185	3.65	5.50	8.76	28.2	425	83.5	125						
W10x60	74.6	186	280	116	175	2.54	3.82	9.08	36.6	341	85.7	129						
W16x40	73.0	182	274	113	170	6.67	10.0	5.55	15.9	518	97.6	146						
W12x50	71.9	179	270	112	169	3.97	5.98	6.92	23.8	391	90.3	135						
W8x67	70.1	175	263	105	159	1.75	2.59	7.49	47.6	272	103	154						
W14x43	69.6	174	261	109	164	4.88	7.28	6.68	20.0	428	83.6	125						
W10x54	66.6	166	250	105	158	2.48	3.75	9.04	33.6	303	74.7	112						
ASD	LRFD	⁽¹⁾ Shape exceeds compact limit for flexure with $F_y = 50 \text{ ksi}$; tabulated values have been adjusted accordingly.																
$\Omega_b = 1.67$	$\phi_b = 0.90$																	
$\Omega_v = 1.50$	$\phi_v = 1.00$																	

University of Michigan, TCAUP

Structures II

Slide 15 of 19

Example – Analysis of Steel Beam - Capacity

- Using the maximum moment equation, solve for the factored distributed loading, w_u

$$\phi M_u = M_u = \frac{w_u l^2}{8} \Rightarrow w_u = \frac{8 M_u}{l^2}$$

$$w_u = \frac{8 \times 357.75 \text{ k-ft}}{20 \text{ ft}^2}$$

$$w_u = 7.155 \text{ k/ft}$$

- The applied (unfactored) load $w = w_u / (\text{g factors})$
 $w_u = 1.2\text{WDL} + 1.6\text{WLL}$

$$w_u = 7.155 \text{ k/ft} = 1.2(0.044 + 1.5) + 1.6(w_{LL})$$

$$w_u = 1.853 + 1.6 w_{LL} = 7.155 \text{ k/ft}$$

$$w_{LL} = 3.31 \text{ k/ft}$$

University of Michigan, TCAUP

Structures II

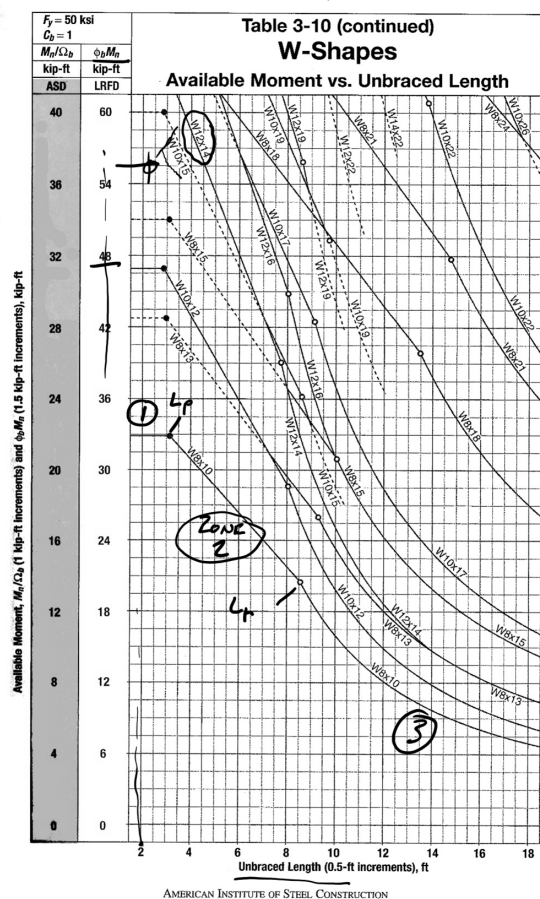
Slide 16 of 19

Steel Beams by LRFD

Moment Capacity with L_b Graphs

Analysis for Bending

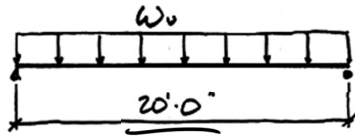
- Plastic Behavior (zone 1)
 - $M_n = M_p$
 - Braced against LTB ($L_b < L_p$)
- Inelastic Buckling "Decreased" (zone 2)
 - $M_n < M_p$
 - $L_p < L_b < L_r$
- Elastic Buckling "Decreased Further" (zone 3)
 - $M_n = M_{cr}$
 - $L_b > L_r$



Steel Beams by LRFD

Moment Capacity Graphs

GIVEN: $F_y = 50 \text{ ksi}$
 $W21 \times 44$
 FULLY BRACED



FOR A $W21 \times 44$ FROM TABLE
 $Z_x = 95.4 \text{ in}^3$

$$M_n = F_y Z_x = 50 \text{ ksi} \times 95.4 = 4,770 \text{ kip-in}$$

$$M_u = \phi_b M_n = 0.9 \times 4,770 \text{ kip-in}$$

$$M_u = 4,293 \text{ kip-in} = \underline{\underline{357.75 \text{ kft}}}$$

