

Steel Beam Analysis

- Steel Codes: ASD vs. LRFD
- Analysis Methods



Steel Beams by LRFD

Yield Stress Values

- A36 Carbon Steel $F_y = 36$ ksi
- A992 High Strength $F_y = 50$ ksi

Elastic Analysis for Bending

- * Plastic Behavior (zone 1)
 - $M_n = M_p = F_y Z_x < 1.5 M_y$
 - Braced against LTB ($L_b < L_p$)

- Inelastic Buckling "Decreased" (zone 2)

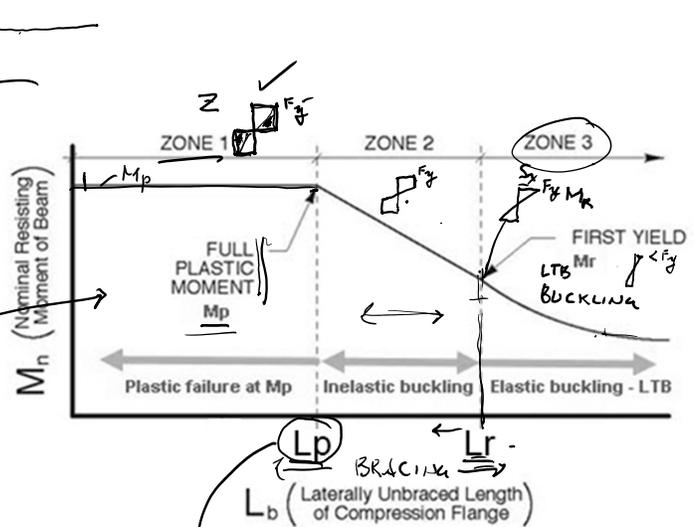
$$M_n = C_b [M_p - (M_p - M_r) \frac{(L_b - L_p)}{(L_r - L_p)}] < M_p$$

- $L_p < L_b < L_r$

- Elastic Buckling "Decreased Further" (zone 3)

$$M_{cr} = C_b * \frac{\pi^2 E I_y}{L_b^2} \sqrt{G^2 J + (\pi^2 E / L_b)^2 I_y C_w}$$

- $L_b > L_r$

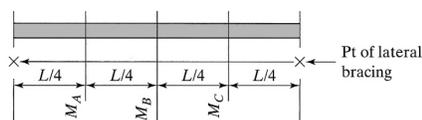


$$L_p = 1.76 r_y \sqrt{E / F_y}$$

$$M_p = F_y Z_x$$

$$M_r = 0.7 F_y S_x$$

C_b is LTB modification factor



$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C}$$

Steel Beams by LRFD

Analysis for Bending

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- Plastic Behavior (zone 1)

$$M_n = M_p = F_y Z_x < 1.5 M_y$$

- Braced against LTB ($L_b < L_p$)

- Inelastic Buckling "Decreased" (zone 2)

$$M_n = C_b (M_p - (M_p - M_r) [(L_b - L_p) / (L_r - L_p)]) < M_p$$

- $L_p < L_b < L_r$

- Elastic Buckling "Decreased Further" (zone 3)

$$M_{cr} = C_b * \pi / L_b \sqrt{(E * I_y * G * J + (\pi * E / L_b)^2 * I_y C_w)}$$

- $L_b > L_r$

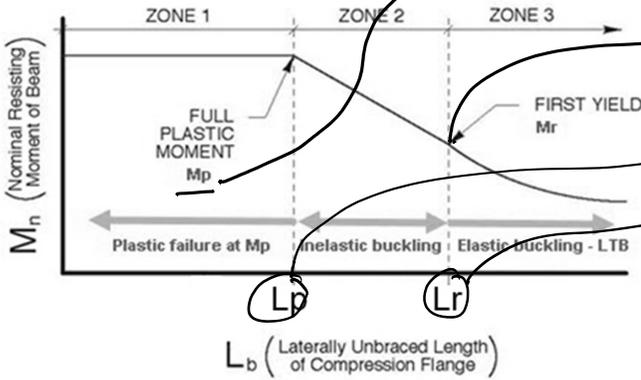


Table 3-2 (continued)
W-Shapes
Selection by Z_x

$F_y = 50$ ksi

LRFD

Z_x

Shape	Z_x in. ³	M_{px}/Ω_b		$\phi_b M_{px}$		M_{rx}/Ω_b		$\phi_b M_{rx}$		BF/Ω_b kips	$\phi_b BF$ kips	L_p ft	L_r ft	I_x in. ⁴	V_{nx}/Ω_v		$\phi_v V_{nx}$ kips
		kip-ft		kip-ft		kip-ft		kip-ft									
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD								
W21x55	126	314	473	182	289	10.8	16.3	6.11	7.4	1140	156	234					
W14x74	126	314	473	186	294	5.31	8.05	8.76	11.0	795	128	192					
W18x60	123	307	461	189	284	9.62	14.4	5.93	8.2	984	151	227					
W12x79	119	297	446	187	281	3.78	5.67	10.8	13.9	662	117	175					
W14x68	115	287	431	180	270	5.19	7.81	8.69	10.9	722	116	174					
W10x88	113	282	424	172	259	2.62	3.94	9.29	11.6	534	131	196					
W18x55	112	279	420	172	258	9.15	13.8	5.90	8.2	890	141	212					
W21x50	110	274	413	165	248	12.1	18.3	4.59	6.5	984	158	237					
W12x72	108	269	405	170	256	3.69	5.56	10.7	13.5	597	106	159					
W21x48 ⁽¹⁾	107	265	398	162	244	9.89	14.8	6.09	8.6	959	144	216					
W16x57	105	262	394	161	242	7.98	12.1	5.65	8.1	758	141	212					
W14x61	102	254	383	161	242	4.93	7.48	8.65	10.9	640	104	156					
W18x50	101	252	379	155	233	6.2	9.2	5.33	7.6	800	128	192					
W10x77	97.6	244	366	150	225	2.60	3.90	9.18	11.6	455	112	169					
W12x65 ⁽¹⁾	96.8	237	356	154	231	3.58	5.39	11.9	15.1	533	94.4	142					
W21x44	95.4	230	350	143	214	11.1	16.8	4.45	6.4	843	145	217					
W18x50	92.0	230	345	141	213	7.89	11.4	5.62	7.9	659	124	186					
W18x46	90.7	226	340	138	207	9.63	14.6	4.56	6.5	712	130	195					
W14x53	87.1	217	327	136	204	5.22	7.9	6.78	9.1	541	103	154					
W12x58	86.4	216	324	136	206	3.82	5.69	8.87	11.6	475	87.8	132					
W10x68	85.3	213	320	132	199	2.58	3.85	9.15	11.6	394	97.8	147					
W10x45	82.5	205	309	127	191	7.12	10.8	5.55	7.8	586	111	167					
W18x40	78.4	196	294	119	180	8.94	13.2	4.49	6.4	612	113	169					
W14x48	78.4	196	294	123	184	5.09	7.67	6.75	9.1	484	93.8	141					
W12x53	77.9	194	292	123	185	3.65	5.50	8.76	11.2	425	85.5	125					
W10x60	74.6	186	280	116	178	2.54	3.82	9.08	11.6	341	85.7	129					
W16x40	73.0	182	274	113	170	6.67	10.0	5.55	7.8	518	97.6	146					
W12x50	71.9	179	270	112	169	3.97	5.98	6.92	9.3	391	90.3	135					
W10x67	70.1	175	265	105	159	1.75	2.59	7.49	10.4	272	103	154					
W14x43	69.6	174	261	109	164	4.88	7.28	6.68	9.1	428	83.6	125					
W10x54	66.6	166	250	105	158	2.48	3.75	9.04	11.6	303	74.7	112					

ASD LRFD ⁽¹⁾Shape exceeds compact limit for flexure with $F_y = 50$ ksi; tabulated values have been adjusted accordingly.

$\Omega_b = 1.67$ $\phi_b = 0.90$
 $\Omega_y = 1.50$ $\phi_y = 1.00$

Design for Shear

Steel



Design for Shear

Shear stress in steel sections is approximated by averaging the stress in the web:

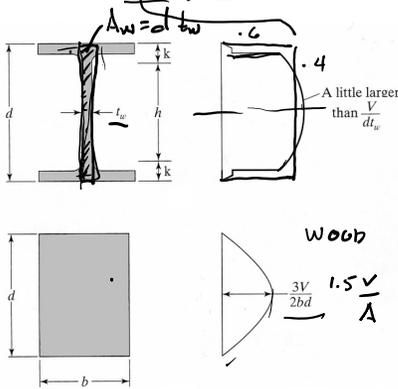
$$F_v = V / A_w$$

$$A_w = d * t_w$$

To adjust the stress a reduction factor of 0.6 is applied to F_y

$$F_v = 0.6 F_y$$

so, $V_n = 0.6 F_y A_w$ (Zone 1)



The equations for the 3 stress zones:
(ϕ in all cases = 1.0)

Zone 1:

WEB YIELDING (Most beam sections fall into this category)

if $\frac{h}{t_w} \leq 2.45 \sqrt{E/F_y} = 59$ (for 50 ksi steel) ✓

then: $V_n = 0.6 F_y A_w$ MOST COMMON

Zone 2:

INELASTIC WEB BUCKLING 74

if $2.45 \sqrt{E/F_y} < \frac{h}{t_w} \leq 3.07 \sqrt{E/F_y} = 74$ (for 50 ksi steel)

then: $V_n = 0.6 F_y A_w (2.45 \sqrt{E/F_y}) / \frac{h}{t_w}$

Zone 3:

ELASTIC WEB BUCKLING 75

if $3.07 \sqrt{E/F_y} < \frac{h}{t_w} \leq 260$ MAX

then: $V_n = A_w \left[\frac{4.25 E}{\left(\frac{h}{t_w}\right)^2} \right]$

Procedure - Analysis of Steel Beams – for Zone 1 $L_b < L_p$

Pass/Fail

Given: yield stress, steel section, loading, bracing (L_b), Z_x

Find: pass/fail of section

1. Calculate the factored design load w_u

LRFD $w_u = 1.2 W_{DL} + 1.6 W_{LL}$

2. Determine the design moment (M_u)
 M_u will be the maximum beam moment using the factored loads

3. Insure that $L_b < L_p$ (zone 1)

$$L_p = 1.76 r_y \sqrt{E/F_y}$$

4. Determine the nominal moment, M_n

Zoned $M_n = F_y Z_x$ (look up Z_x for section)

5. Factor the nominal moment

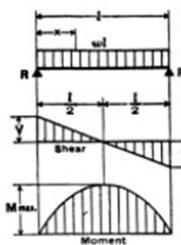
$\phi M_n = 0.90 M_n$

6. Check that $M_u < \phi M_n$ PASS

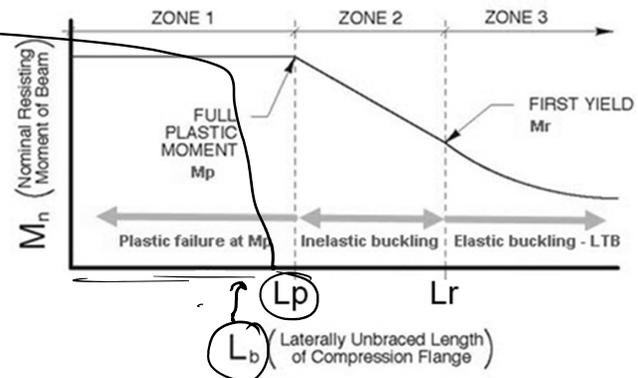
7. Check shear ✓

8. Check deflection ✓

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load	$= wl$
$R = V$	$= \frac{wl}{2}$
V_x	$= w \left(\frac{l}{2} - x \right)$
$M_{max.} \text{ (at center)}$	$= \frac{wl^2}{8}$
M_x	$= \frac{wx}{2} (l-x)$
$\Delta_{max.} \text{ (at center)}$	$= \frac{5wl^4}{384EI}$
Δ_x	$= \frac{wx}{24EI} (l^3 - 2lx^2 + x^3)$



Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

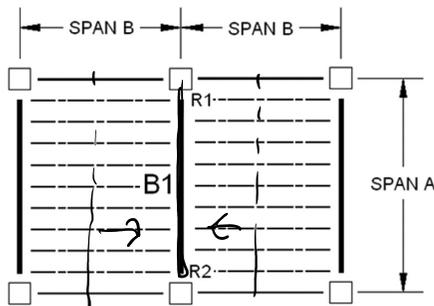
Given: yield stress, steel section, loading, braced 24" o.c.

Find: pass/fail of section

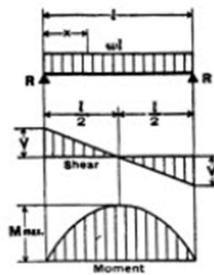
1. Calculate the factored design load w_u

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1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load	$= wl$
$R = V$	$= \frac{wl}{2}$
V_x	$= w(\frac{l}{2} - x)$
M max. (at center)	$= \frac{wl^2}{8}$
M_x	$= \frac{wx}{2}(l-x)$
Δ max. (at center)	$= \frac{5wl^4}{384EI}$
Δ_x	$= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

Handwritten calculations for the beam analysis:

$D = 1 \text{ KLF} + \text{BEAM}$ $L = 3 \text{ KLF}$

$W21 \times 44$
 A 992 STEEL
 $F_y = 50 \text{ ksi}$

FROM TABLE 1-1 AISC $Z_x = 95.4 \text{ in}^3$

$w_u = 1.2(1 + 0.044) + 1.6(3) = 6.05 \text{ KLF}$

$M_u = \frac{w_u l^2}{8} = \frac{6.05 \text{ KLF} \times 21'^2}{8} = 333.5 \text{ K-}'$

Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

3. Insure that $L_b < L_p$ (zone 1)

24"

$$L_p = 1.76 r_y \sqrt{E/F_y}$$

$$L_p = 1.76 (1.26) \sqrt{29000/50}$$

$$L_p = 53.4 \text{ in.} > 24 \text{ in.} \text{ ok}$$

so ZONE 1.

4. Determine the nominal moment, M_n
 $M_n = M_p = F_y Z_x$ (for zone 1)
 (look up Z_x for section)

5. Factor the nominal moment
 $\phi M_n = 0.90 M_n$

6. Check that $M_u < \phi M_n$

Handwritten calculations for the beam analysis:

$D = 1 \text{ KLF} + \text{BEAM}$ $L = 3 \text{ KLF}$

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$M_n = F_y Z_x = 50 \text{ ksi} \times 95.4 \text{ in}^3 = 4770 \text{ K-}'$

$M_n = 4770 \text{ K-}' / 12 = 397.5 \text{ K-}'$

$\phi M_n = 0.9 (397.5) = 357.7 \text{ K-}'$

DESIGN LOAD $M_u = 333.5 \text{ K-}' < 357.7 \text{ K-}' = \phi M_n$

\therefore PASS ✓

Steel Beams by LRFD

Analysis for Bending

AISC 16th ed.

- Plastic Behavior (zone 1)

$$M_n = M_p = F_y Z_x$$

- $L_p = 4.45 \text{ ft} = 53.4 \text{ in.} > 24 \text{ in. o.c.}$
- $\phi_b M_{px} = 358 \text{ k-ft} > M_u = 333.5 \text{ k-ft}$

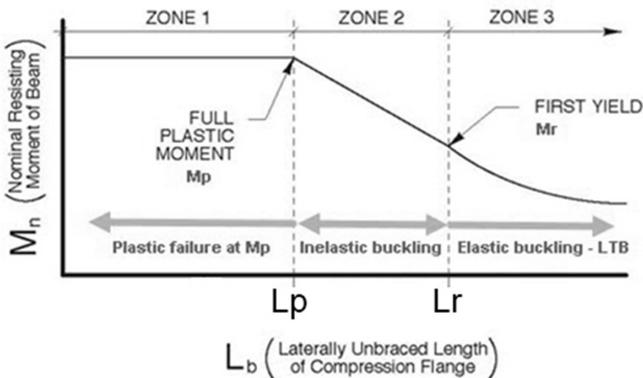


Table 3-2 (continued)
 $F_y = 50 \text{ ksi}$
W-Shapes
 Selection by Z_x

Z_{max}
 M_p
 M_r

Shape	Z_x in. ³	M_{px}/Ω_b		M_{rx}/Ω_b		BF/Ω_b		$\phi_b BF$	L_r ft	I_x in. ⁴	V_{ux}/Ω_v		$\phi_v V_{ux}$	
		kip-ft	kip-ft	kip-ft	kip-ft	ASD	LRFD				ASD	LRFD		
W21x55	126	314	473	192	289	10.8	16.3	6.11	17.4	1140	156	234		
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W10x54	66.6	166	250	105	158	2.48	3.75	9.04	33.6	303	74.7	112		

ASD LRFD ⁽¹⁾Shape exceeds compact limit for flexure with $F_y = 50 \text{ ksi}$; tabulated values have been adjusted accordingly.

$\Omega_b = 1.67$ $\phi_b = 0.90$
 $\Omega_v = 1.50$ $\phi_v = 1.00$

Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

7. Check shear for W21x44

CHECK SHEAR:

Zone 1:

WEB YIELDING (Most beam sections fall into this category)

FROM AISC TABLE 1-1

$$\frac{20.7}{1.35} \text{ if } \frac{h}{t_w} \leq 2.45 \sqrt{E/F_y} = 59 \text{ (for 50 ksi steel)}$$

$$\frac{h}{t_w} = 53.6 < 59 \text{ (zone 1)}$$

then: $V_n = 0.6 F_y A_w$

Table 1-1 (continued)
W-Shapes
 Dimensions

Shape	Area, A in. ²	Depth, D in.	Web		Flange		Distance				Work-able Gage				
			Thickness, t _w in.	t _w ² in.	Width, b _f in.	Thickness, t _f in.	k	k _{des}	k _{set}	T					
W21x93	27.3	21.6	2 1/8	0.580	9/16	5/16	8.42	8 3/8	0.930	1 5/16	1.43	1 3/8	1 3/8	18 3/8	5 1/2
>83°	24.4	21.4	2 1/8	0.515	1/2	1/4	8.36	8 3/8	0.835	1 3/16	1.34	1 1/2	7/8		
>73°	21.5	21.2	2 1/8	0.455	7/16	1/4	8.30	8 3/4	0.740	3/4	1.24	1 7/16	7/8		
>68°	20.0	21.1	2 1/8	0.430	7/16	1/4	8.27	8 3/4	0.685	1 1/16	1.19	1 3/8	7/8		
>62°	18.3	21.0	2 1/8	0.400	3/8	3/16	8.24	8 3/4	0.615	5/8	1.12	1 5/16	1 3/16		
>55°	16.2	20.8	2 0 3/4	0.375	3/8	3/16	8.22	8 3/4	0.522	1/2	1.02	1 3/16	1 3/16		
>48° ⁽¹⁾	14.1	20.6	2 0 3/8	0.350	3/8	3/16	8.14	8 3/8	0.430	7/16	0.930	1 1/8	1 3/16		
W21x57 ⁽²⁾	16.7	21.1	2 1/8	0.405	3/8	3/16	6.56	6 1/2	0.650	5/8	1.15	1 5/16	1 3/16	18 3/8	3 1/2
>50°	14.7	20.8	2 0 7/8	0.380	3/8	3/16	6.53	6 1/2	0.535	9/16	1.04	1 1/4	1 3/16		
>44°	13.0	20.7	2 0 3/4	0.350	3/8	3/16	6.50	6 1/2	0.450	7/16	0.950	1 1/8	1 3/16		

Table 1-1 (continued)
W-Shapes
 Properties

Nominal wt. lb/ft	Compact Section Criteria			Axis X-X			Axis Y-Y				Torsional Properties			
	b _f /2t _f	h/t _w	l _c /t _w	I	S	r	Z	I	S	r	Z	r _{ts}	h _o	J
93	4.53	32.3	2070	192	8.70	221	92.9	22.1	1.84	34.7	2.24	20.7	6.03	9940
83	5.00	36.4	1830	171	8.67	196	81.4	19.5	1.83	30.5	2.21	20.6	4.34	8630
73	5.60	41.2	1600	151	8.64	172	70.6	17.0	1.81	26.6	2.19	20.5	3.02	7410
68	6.04	43.6	1480	140	8.60	160	64.7	15.7	1.80	24.4	2.17	20.4	2.45	6760
62	6.70	46.9	1330	127	8.54	144	57.5	14.0	1.77	21.7	2.15	20.4	1.83	5960
55	7.87	50.0	1140	110	8.40	126	48.4	11.8	1.73	18.4	2.11	20.3	1.24	4980
48	9.47	53.6	959	93.0	8.24	107	38.7	9.52	1.66	14.9	2.05	20.2	0.803	3950
57	5.04	46.3	1170	111	8.36	129	30.6	9.35	1.35	14.8	1.68	20.5	1.77	3190
50	6.10	49.4	984	94.5	8.18	110	24.9	7.64	1.30	12.2	1.64	20.3	1.14	2570
44	7.21	53.6	843	81.6	8.06	95.4	20.7	6.37	1.26	10.2	1.60	20.3	0.770	2110

Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

7. Check shear (zone 1)

FROM AISC TABLE 1-1

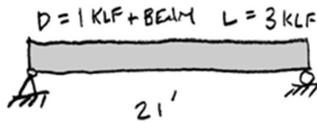
$$\frac{h}{t_w} = 53.6 < 59 \text{ (zone 1)}$$

Zone 1:

WEB YIELDING (Most beam sections fall into this category)

$$\text{if } \frac{h}{t_w} \leq 2.45 \sqrt{E/F_y} = 59 \text{ (for 50 ksi steel)}$$

$$\text{then: } V_n = 0.6 F_y A_w$$



W21x44
A992 STEEL
 $F_y = 50 \text{ KSI}$

FROM TABLE 1-1 AISC $Z_x = 95.4 \text{ in}^3$

$$w_u = 1.2(1 + 0.044) + 1.6(3) = 6.05 \text{ KLF}$$

CHECK SHEAR:

$$V_u = \frac{w_u L}{2} = \frac{6.05(21)}{2} = 63.5 \text{ K}$$

FROM AISC TABLE 1-1

$$\frac{h}{t_w} = 53.6 < 59 \text{ (zone 1)}$$

STRENGTH

$$V_n = 0.6 F_y A_w = 0.6(50)(20.7 \times 0.35)$$

$$V_n = 217.35 \text{ K}$$

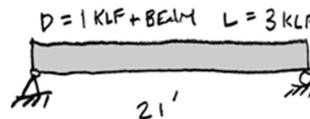
$$\phi V_n = 1.0(217.35) = 217.35 \text{ K}$$

$$V_u = 63.5 \text{ K} < 217.3 \text{ K} = \phi V_n \checkmark$$

Therefore, pass.

Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

8. Check deflection



W21x44
A992 STEEL
 $F_y = 50 \text{ KSI}$

FROM TABLE 1-1 AISC $Z_x = 95.4 \text{ in}^3$

$$w_u = 1.2(1 + 0.044) + 1.6(3) = 6.05 \text{ KLF}$$

$$\Delta_{\text{MAX}} = \frac{5 w_u L^4}{384 EI} = \frac{5(3000) 21^4 (1728)}{384(29000000)(843)}$$

$$= 0.535''$$

$$\frac{l}{360} = \frac{21(12)}{360} = 0.7''$$

$$\Delta_{\text{ACTUAL}} = 0.535'' < 0.7'' = \Delta_{\text{ALLOWABLE}} \checkmark$$

TABLE 1604.3 DEFLECTION LIMITS^{a, b, c, h, i}

CONSTRUCTION	L	S or W	$D + L^{d, g}$
Roof members: ^e			
Supporting plaster or stucco ceiling	//360	//360	//240
Supporting nonplaster ceiling	//240	//240	//180
Not supporting ceiling	//180	//180	//120
Floor members	//360	—	//240
Exterior walls:			
With plaster or stucco finishes	—	//360	—
With other brittle finishes	—	//240	—
With flexible finishes	—	//120	—
Interior partitions: ^b			
With plaster or stucco finishes	//360	—	—
With other brittle finishes	//240	—	—
With flexible finishes	//120	—	—
Farm buildings	—	—	//180
Greenhouses	—	—	//120

Procedure - Analysis of Steel Beam - Capacity

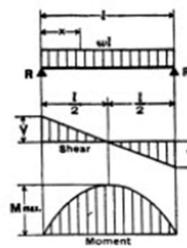
Given: yield stress, steel section, bracing

Find: moment or load capacity

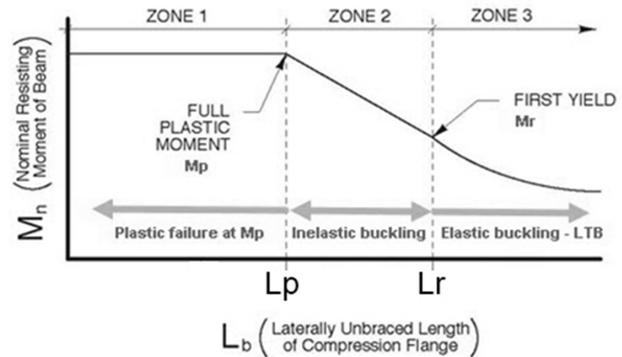
1. Determine the unbraced length of the compression flange (L_b).
2. Find the L_p and L_r values from the AISC Z_x Table 3-2
3. Compare L_b to L_p and L_r and determine which equation for M_n or M_{cr} to be used.
4. Determine the beam load equation for maximum moment in the beam.
5. Calculate load based on maximum moment. $M_u = \phi_b M_n$

Load STRIP width

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load	...	$= wl$
$R = V$...	$= \frac{wl}{2}$
V_x	...	$= w(\frac{l}{2} - x)$
M max. (at center)	...	$= \frac{wl^2}{8}$
M_x	...	$= \frac{wx}{2}(l-x)$
Δ max. (at center)	...	$= \frac{5wl^4}{384EI}$
Δx	...	$= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$



Example – Analysis of Steel Beam - Capacity

Given:

$F_y = 50$ ksi, Fully Braced 20 ft span

Section: W21x44

Find:

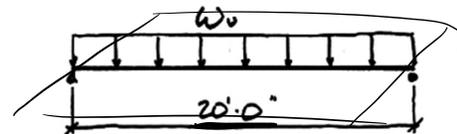
applied live load capacity (w_{LL}) in KLF

$$w_u = 1.2w_{DL} + 1.6w_{LL}$$

$$w_{DL} = \frac{\text{beam} + \text{floor}}{44} = 44\text{plf} + 1500\text{plf}$$

1. Find the Plastic Modulus (Z_x) for the given section from the AISC table 1-1
2. Check that $L_b < L_p$ (fully braced – ok)
3. Determine $M_n = M_p = F_y Z_x$
4. Set $M_u = \phi_b M_n$

GIVEN: $F_y = 50$ ksi
W21x44
FULLY BRACED



FOR A W21x44 FROM TABLE
 $Z_x = 95.4 \text{ in}^3$

$$M_n = F_y Z_x = 50 \text{ ksi} \times 95.4 = 4,770 \text{ k-in}$$

$$M_u = \phi_b M_n = 0.9 \times 4,770 \text{ k-in}$$

$$M_u = 4,293 \text{ k-in} = 357.75 \text{ k-ft}$$

STRENGTH

Steel Beams by LRFD

Analysis for Bending

AISC 16th ed.

- Plastic Behavior (zone 1)

$$M_n = M_p = F_y Z$$

- $L_p = 4.45 \text{ ft} = 53.4 \text{ in.}$
- $\phi_b M_{px} = 358 \text{ k-ft} = M_u$

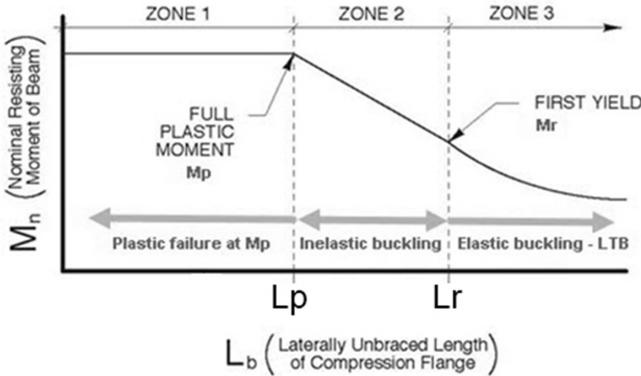


Table 3-2 (continued)
 $F_y = 50 \text{ ksi}$
W-Shapes
Selection by Z_x

Z_x

Shape	Z_x in. ³	M_p		M_r		BF/Ω_b		L_p ft	L_r ft	I_x in. ⁴	V_{nx}/Ω_v		$\phi_v V_{nx}$	
		kip-ft	kip-ft	kip-ft	kip-ft	ASD	LRFD				ASD	LRFD		
		ASD	LRFD	ASD	LRFD	ASD	LRFD				ASD	LRFD		
W21x55	126	314	473	182	289	10.8	16.3	6.11	17.4	1140	156	234		
W14x74	126	314	473	186	294	5.31	8.05	8.76	31.0	795	128	192		
W18x60	123	307	461	189	284	9.62	14.4	5.93	18.2	984	151	227		
W12x79	119	297	446	187	281	3.78	5.67	10.8	39.9	662	117	175		
W14x68	115	287	431	180	270	5.19	7.81	8.69	29.3	722	116	174		
W10x88	113	282	424	172	259	2.62	3.94	9.29	51.2	534	131	196		
W18x55	112	279	420	172	258	9.15	13.8	5.90	17.6	890	141	212		
W21x50	110	274	413	165	248	12.1	18.3	4.59	13.6	984	158	237		
W12x72	108	269	405	170	256	3.69	5.56	10.7	37.5	597	106	159		
W21x48 ⁽¹⁾	107	265	398	162	244	9.89	14.8	6.09	16.5	959	144	216		
W16x57	105	262	394	161	242	7.98	12.0	5.65	18.3	758	141	212		
W14x61	102	254	383	161	242	4.93	7.48	8.65	27.5	640	104	156		
W18x50	101	252	379	155	233	8.76	13.2	5.83	16.9	800	128	192		
W10x77	97.6	244	366	150	225	2.60	3.90	9.18	45.3	455	112	169		
W12x65 ⁽¹⁾	96.8	237	356	154	231	3.58	5.39	11.9	35.1	533	94.4	142		
W21x44	95.4	238	358	143	214	11.1	16.8	4.45	13.0	843	145	217		
W16x50	92.0	230	345	141	213	7.69	11.4	5.62	17.2	659	124	186		
W18x46	90.7	226	340	138	207	9.63	14.6	4.56	13.7	712	130	195		
W14x53	87.1	217	327	136	204	5.22	7.93	6.78	22.3	541	103	154		
W12x58	86.4	216	324	136	205	3.82	5.69	8.87	29.8	475	87.8	132		
W10x68	85.3	213	320	132	199	2.58	3.85	9.15	40.6	394	97.8	147		
W16x45	82.3	205	309	127	191	7.12	10.8	5.55	16.5	586	111	167		
W18x40	78.4	196	294	119	180	8.94	13.2	4.49	13.1	612	113	169		
W14x48	78.4	196	294	123	184	5.09	7.67	6.75	21.1	484	93.8	141		
W12x53	77.9	194	292	123	185	3.65	5.50	8.76	28.2	425	83.5	125		
W10x60	74.6	186	280	116	175	2.54	3.82	9.08	36.6	341	85.7	129		
W16x40	73.0	182	274	113	170	6.67	10.0	5.55	15.9	518	97.6	146		
W12x50	71.9	179	270	112	169	3.97	5.98	6.92	23.8	391	90.3	135		
W8x67	70.1	175	263	105	159	1.75	2.59	7.49	47.6	272	103	154		
W14x43	69.6	174	261	109	164	4.88	7.28	6.68	20.0	428	83.6	125		
W10x54	66.6	166	250	105	158	2.48	3.75	9.04	33.6	303	74.7	112		

Example – Analysis of Steel Beam - Capacity

6. Using the maximum moment equation, solve for the factored distributed loading, w_u

$$M_p = \frac{M_u}{\phi_b} = \frac{w_u l^2}{8} \Rightarrow w_u = \frac{8 M_u}{l^2}$$

$$w_u = \frac{8 \times 357.75 \text{ k-ft}}{20 \text{ ft}^2}$$

7. The applied (unfactored) load $w = w_u / (\gamma \text{ factors})$
 $w_{LL} = 1.2w_{DL} + 1.6w_{LL}$

$$w_u = 7.155 \text{ k/ft}$$

$$w_u = 7.155 \text{ k/ft} = 1.2(0.044 + 1.5) + 1.6(w_{LL})$$

$$w_u = 1.855 + 1.6 w_{LL} = 7.155 \text{ k/ft}$$

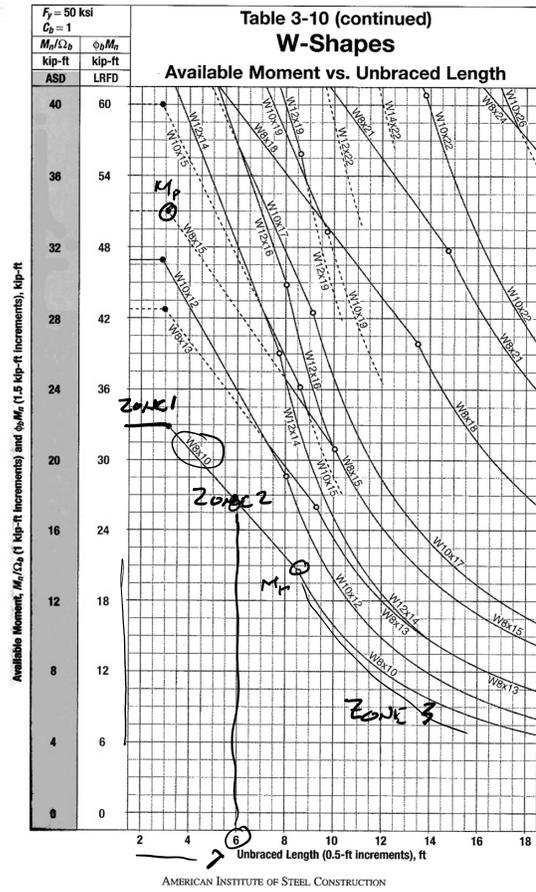
$$w_{LL} = 3.31 \text{ k/ft}$$

Steel Beams by LRFD

Moment Capacity with L_b Graphs

Analysis for Bending

- Plastic Behavior (zone 1)
 - $M_n = M_p$
 - Braced against LTB ($L_b < L_p$)
- Inelastic Buckling "Decreased" (zone 2)
 - $M_n < M_p$
 - $L_p < L_b < L_r$
- Elastic Buckling "Decreased Further" (zone 3)
 - $M_n = M_{cr}$
 - $L_b > L_r$



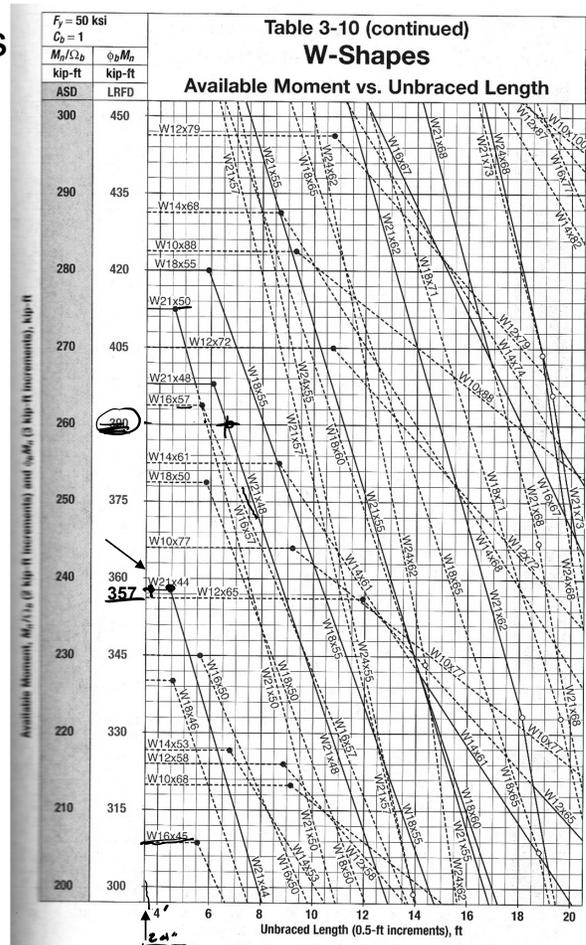
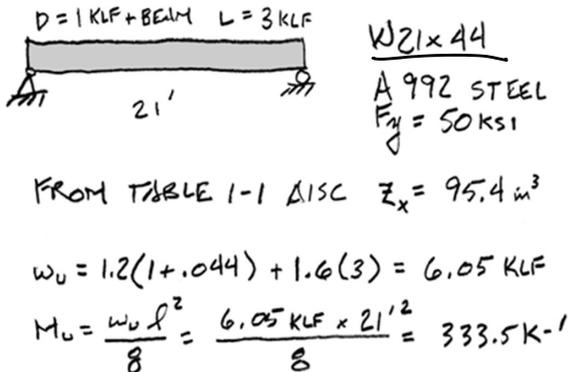
AISC 15th ed.

Pass/Fail Analysis of Steel Beams for Zone 1 $L_b < L_p$

Example:

Given: yield stress, steel section, loading, braced @ 24" o.c.

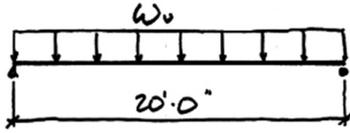
Find: pass/fail of section



Steel Beams by LRFD

Moment Capacity Graphs

GIVEN: $F_y = 50 \text{ ksi}$
 $W21 \times 44$
 FULLY BRACED



FOR A $W21 \times 44$ FROM TABLE
 $Z_x = 95.4 \text{ in}^3$

$$M_n = F_y Z_x = 50 \text{ ksi} \times 95.4 = 4,770 \text{ k-in}$$

$$M_u = \phi_b M_n = 0.9 \times 4,770 \text{ k-in}$$

$$M_u = 4,293 \text{ k-in} = \underline{\underline{357.75 \text{ k-ft}}}$$

