Peter von Buelow
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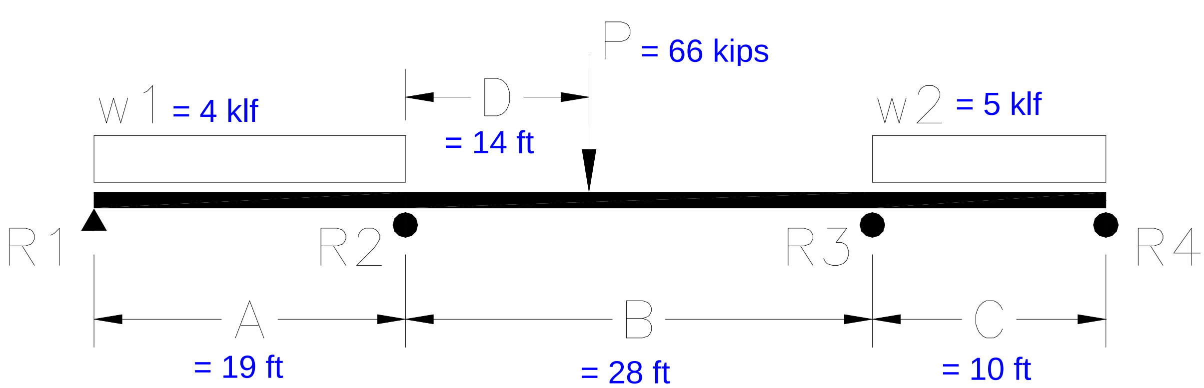
Continuous Beams with the Three Moment Theorem

Description
This project explores shear and bending moment diagrams for continuous beams.

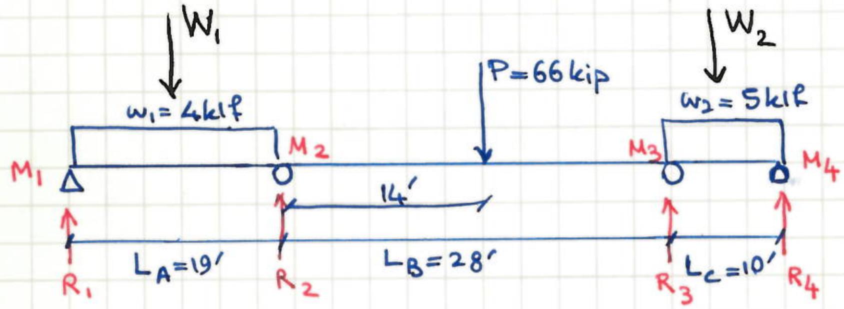
Goals
- to find reactions for statically indeterminate beams
- to construct shear diagrams for statically indeterminate beams
- to construct moment diagrams for statically indeterminate beams

Procedure
1. Use the loads and spans shown below, and the Three Moment Theorem to determine the moments at the inside reactions.
2. Determine any moments = 0.
3. Solve the three moment equation to find internal moments at R2 and R3.
4. Determine all support reactions.



Questions:

1. The moment at R1. (- if tension on top) 0 KIP-FT
2. EI Theta on left side of R2 1143.17
3. EI Theta on right side of R2 3234
4. The moment at R4. (- if tension on top) 0 KIP-FT
5. EI Theta on left side of R3 3234
6. EI Theta on right side of R3 208.33
7. The moment at R2. (- if tension on top) KIP-FT -222.905
8. The moment at R3. (- if tension on top) KIP-FT -189.64
9. The reaction at R1. (- if downward) 26.268 KIPS
10. The reaction at R2. (- if downward) 83.92 KIPS
11. The reaction at R3. (- if downward) 75.775 KIPS
12. The reaction at R4. (- if downward) 6.036 KIPS

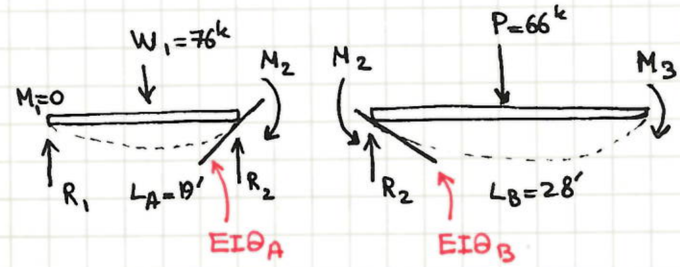


$W_1 = \omega_1 L_A = 4(k/ft) \times 19(ft) = 76$ kip

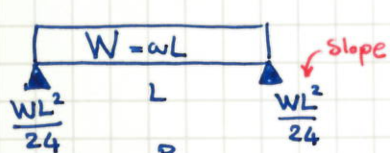
$W_2 = \omega_2 L_C = 5(k/ft) \times 10(ft) = 50$ kip

- Zero moments : $M_1 = M_4 = 0$ (1), (4)

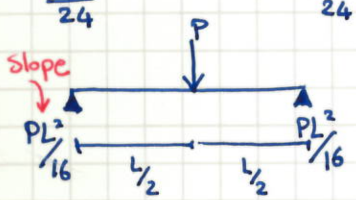
- find $EI\theta_A$ and $EI\theta_B$ on two sides of R_2 :



from slope table (Lecture slides):

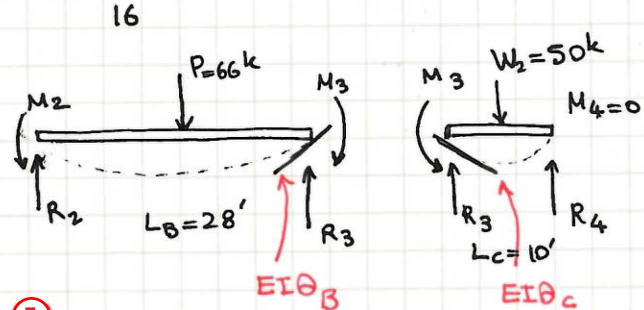


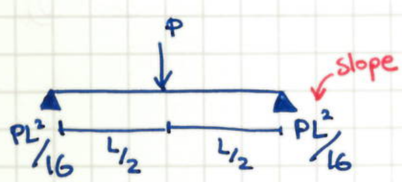
$$EI\theta_A = \frac{W_1 L_A^2}{24} = \frac{76 \times 19^2}{24} = 1143.17 \quad (2)$$



$$EI\theta_B = \frac{P L_B^2}{16} = \frac{66 \times 28^2}{16} = 3234 \quad (3)$$

- find $EI\theta_B$ and $EI\theta_C$ on two sides of R_3 :





$\rightarrow EI\theta_B = 3234$ (5)

↳ This will be a different value if P shifts from center.

$$\rightarrow EI\theta_C = \frac{W_2 L_C^2}{24} = \frac{50 \times 10^2}{24} = 208.33 \quad (6)$$

- find M_2 and M_3 :

from Lecture slides:

(I)
$$\cancel{M_1}^{0} L_A + 2M_2(L_A + L_B) + M_3 L_B = 6\left[EI\theta_A + EI\theta_B\right] \rightarrow \text{for Beams A and B}$$

$$M_2 L_B + 2M_3(L_B + L_C) + \cancel{M_4}^{0} L_C = 6\left[EI\theta_B + EI\theta_C\right] \rightarrow \text{for Beams B and C}$$
(II)

I $\rightarrow$ $2 \times M_2 \times (\underset{L_A}{19} + \underset{L_B}{28}) + M_3 \times \underset{L_B}{28} = 6 \times (\underset{②}{1143.17} + \underset{③}{3234})$

$\Rightarrow 94 M_2 + 28 M_3 = 26263 \rightarrow M_2 = \frac{1}{94}(26263 - 28 M_3)$ III

II $\rightarrow$ $\underset{L_B}{M_2 \times 28} + 2 \times M_3 (\underset{L_B}{28} + \underset{L_C}{10}) = 6 \times (\underset{④}{3234} + \underset{⑤}{208.33})$

$\Rightarrow 28 M_2 + 76 M_3 = 20654 \rightarrow M_2 = \frac{1}{28}(20654 - 76 M_3)$ IV

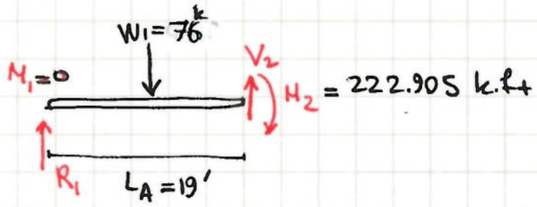
III = IV $\rightarrow$ $\frac{1}{94}(26263 - 28 M_3) = \frac{1}{28}(20654 - 76 M_3) \rightarrow M_3 = 189.64 \text{ k.ft}$

$\hookrightarrow -189.64$ ⑧

III $\rightarrow$ $M_2 = \frac{1}{94}(26263 - 28 \times \underset{M_3}{189.64}) = 222.905 \text{ k.ft}$

$\hookrightarrow -222.905$ ⑦

- find reactions $R_1 - R_4$:



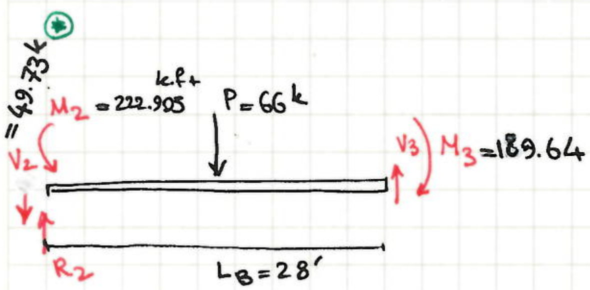
$\Sigma M @ R_2 = 0$

$M_2 + R_1(L_A) - W_1(L_A/2) = 0$

$222.905 + R_1 \times 19 - 76 \times 19/2 = 0 \longrightarrow R_1 = 26.268^k$ ⑨

$\Sigma f_y = 0$

$R_1 + V_2 - 76 = 0 \rightarrow V_2 = 49.73^k$ ✱



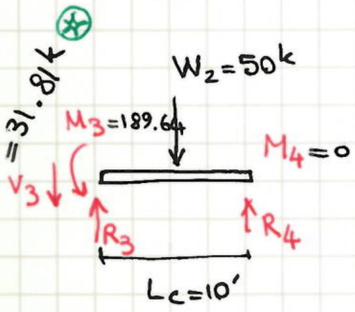
$\Sigma M @ R_3 = 0$

$M_3 + R_2(L_B) - P(L_B/2) - V_2(L_B) - M_2 = 0$

$189.64 + R_2 \times 28 - 66 \times 14 - 49.73 \times 28 - 222.905 = 0$

$R_2 = 83.92^k$ ⑩

$\Sigma f_y = 0$



$R_2+V_3-V_2-P=0$

$83.92+V_3-49.73-66=0 \longrightarrow V_3=31.81^k$

$\Sigma M @ R_3=0$

$W_2\left(L_c/2\right)-R_4(L_c)-M_3=0$

$50\times 5-R_4\times 10-189.64=0 \longrightarrow R_4=6.036^k$ (12)

$\Sigma F_y=0$

$R_3+R_4-V_3-W_2=0$

$R_3+6.036-31.81-50=0 \longrightarrow R_3=75.775^k$ (11)

Recitation 7: Three Moment Theorem

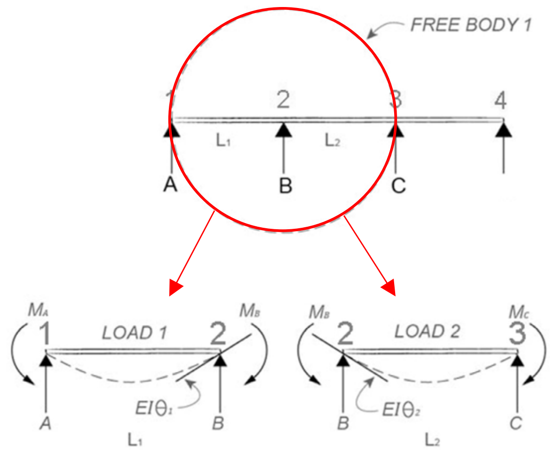
From Lecture Slides:

Three-Moment Theorem

- Any number of spans
- Symmetric or non-symmetric

Procedure:

1. Draw a free body diagram of the first two spans.
2. Label the spans L1 and L2 and the supports (or free end) A, B and C as show.
3. Use the Three-Moment equation to solve for each unknown moment, either as a value or as an equation.

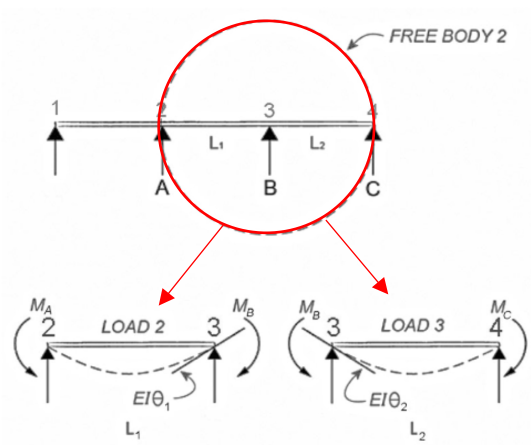


$$M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]$$

Three-Moment Theorem

Procedure (continued):

4. Move one span further and repeat the procedure.
5. In a 3 span beam, the mid-moment from step 3 above (B), can now be solved using the two equations from step 4 and 3 together, by writing 2 equations with 2 unknowns.
6. Repeat as needed, always moving one span to the right and writing a new set of moment equations.
7. Solve 2 simultaneous equations for 3 spans, or 3 equations for more than 3 spans, to get the interior moments.
8. Once all interior moments are known, solve for reactions using free body diagrams of individual spans.
9. Draw shear and moment diagrams as usual. This will also serve as a check for the moment values.



$$M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]$$

MAXIMUM VALUES: SLOPE, DEFLECTION, AND BENDING MOMENT

NOTE: VALUES OF SLOPE AND DEFLECTION TO BE DIVIDED BY "EI"