ARCH 324
STRUCTURE II

FACULTY: Prof. Peter Von Bulow

GSI: Faezeh Choobkar

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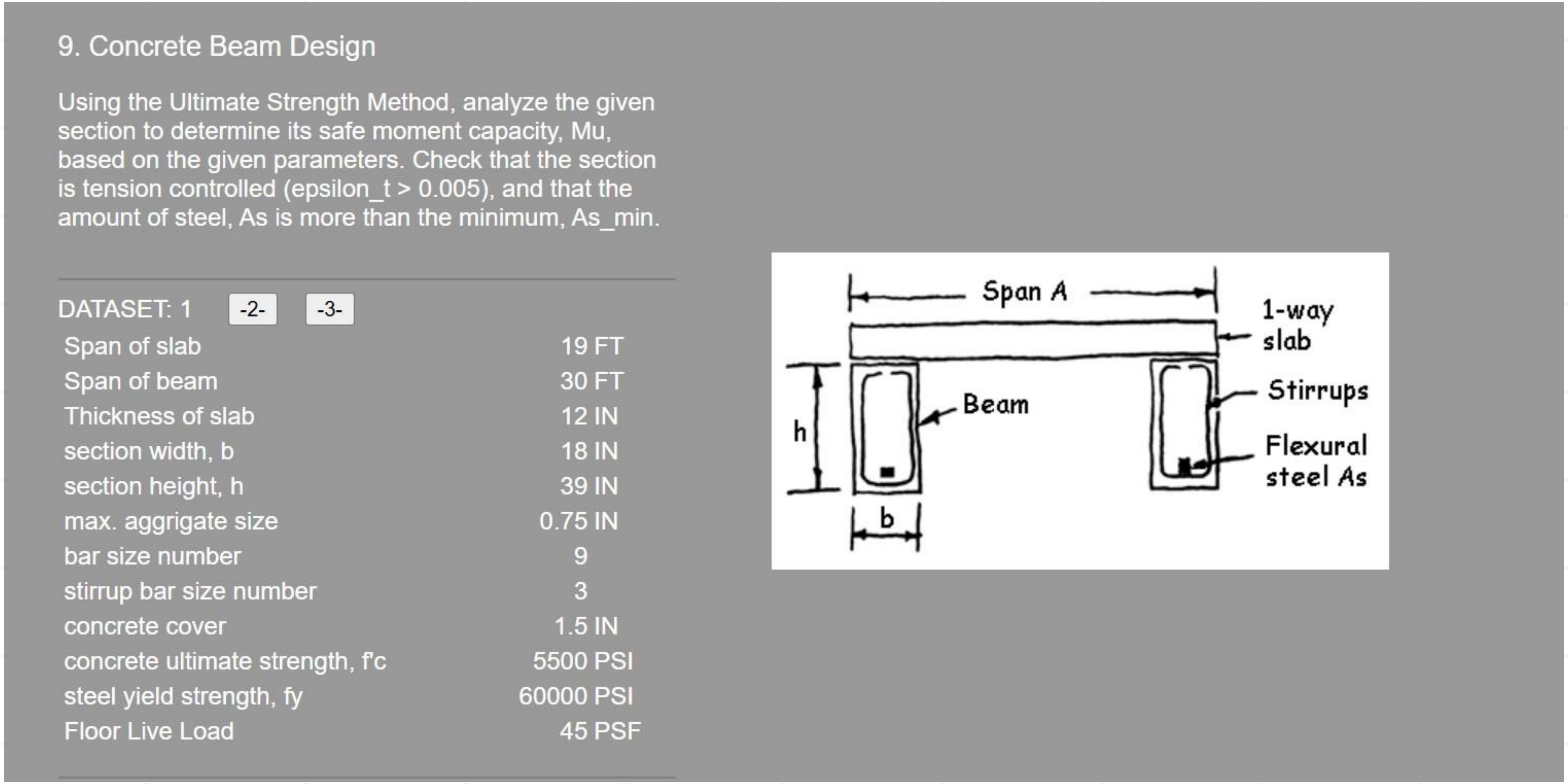
Recitation

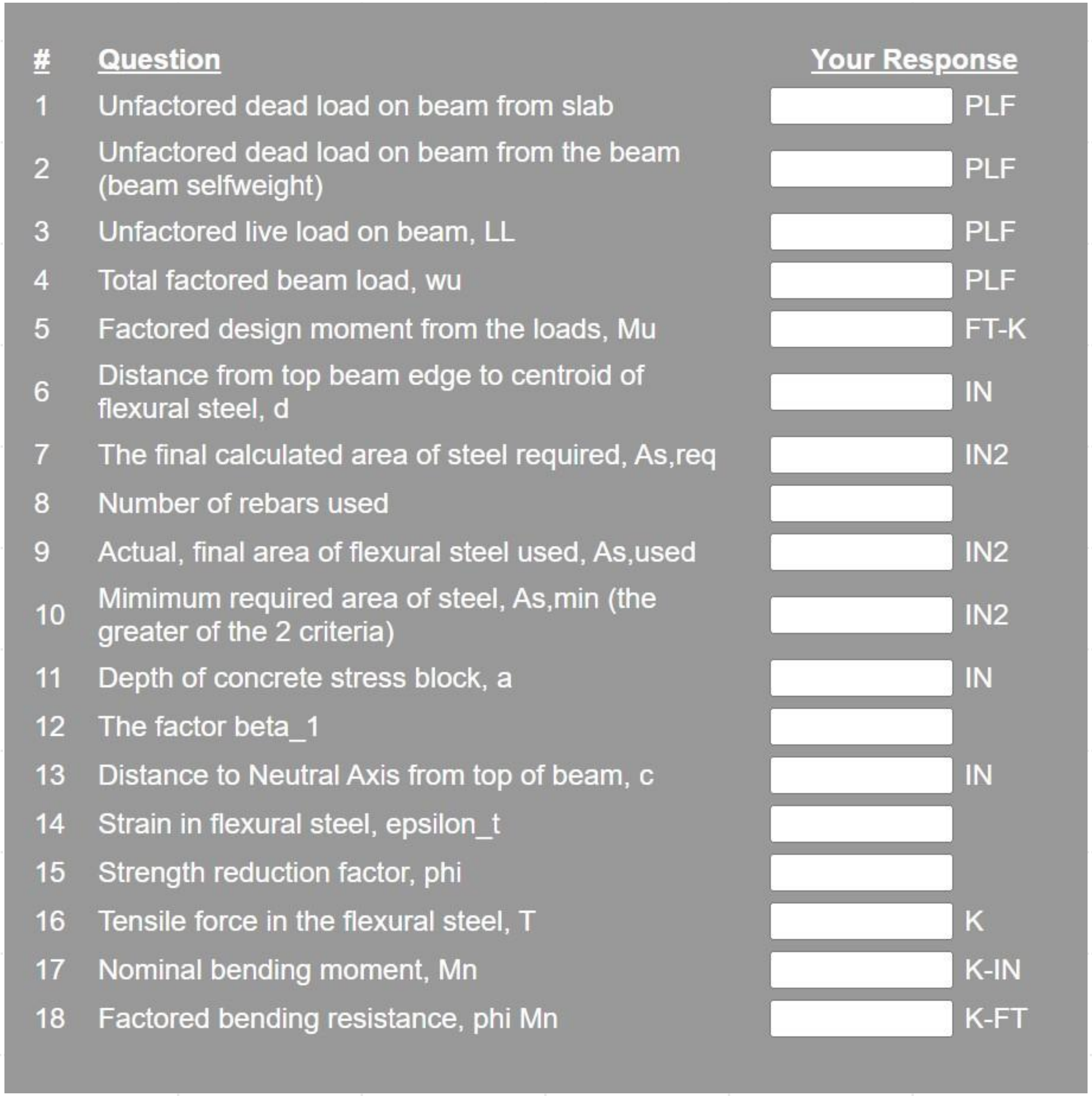


9. Concrete Beam Design

Using the Ultimate Strength Method, analyze the given section to determine its safe moment capacity, Mu, based on the given parameters. Check that the section is tension controlled (epsilon_t > 0.005), and that the amount of steel, As is more than the minimum, As_min.

DATASET: 1 -2- -3-

Span of slab	19 FT
Span of beam	30 FT
Thickness of slab	12 IN
section width, b	18 IN
section height, h	39 IN
max. aggrigate size	0.75 IN
bar size number	9
stirrup bar size number	3
concrete cover	1.5 IN
concrete ultimate strength, f'c	5500 PSI
steel yield strength, fy	60000 PSI
Floor Live Load	45 PSF



#	Question	Your Response	
1	Unfactored dead load on beam from slab		PLF
2	Unfactored dead load on beam from the beam (beam selfweight)		PLF
3	Unfactored live load on beam, LL		PLF
4	Total factored beam load, wu		PLF
5	Factored design moment from the loads, Mu		FT-K
6	Distance from top beam edge to centroid of flexural steel, d		IN
7	The final calculated area of steel required, As,req		IN2
8	Number of rebars used		
9	Actual, final area of flexural steel used, As,used		IN2
10	Mimimum required area of steel, As,min (the greater of the 2 criteria)		IN2
11	Depth of concrete stress block, a		IN
12	The factor beta_1		
13	Distance to Neutral Axis from top of beam, c		IN
14	Strain in flexural steel, epsilon_t		
15	Strength reduction factor, phi		
16	Tensile force in the flexural steel, T		K
17	Nominal bending moment, Mn		K-IN
18	Factored bending resistance, phi Mn		K-FT

reinforced Concrete density $= 150 \ \frac{lb}{ft^3}$

$1 \ ft^3 \longrightarrow 150 \ lb$

$19 \times 1 \times \frac{12}{12} \longrightarrow ?$ 2850 Plf : total weight of slab

load on beam $= \frac{2850}{2} = 1425$

Beam Self - weight :

$1 \ ft^3 \quad 150 \ lb$

$\left(\frac{39}{12} \times \frac{18}{12} \times 1 \right) \quad ? \longrightarrow 731{,}25$ Plf live load on beam:

$$\text{live load} \times \frac{\text{slab span}}{2} \times \text{per linear foot of beam}$$

$$45\,\text{psf} \times \frac{19}{2} \times 1 = 427{,}5$$

Total factored load on beam: (wu)

$$1.2D + 1.6L$$

$$1.2(1425 + 737.25) + 1.6(427{,}5)$$

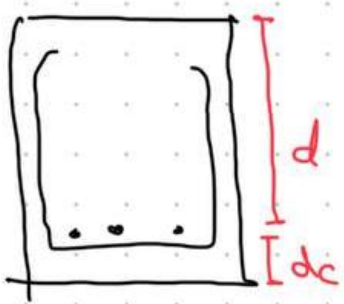
$$2587{,}5 + 684 = 3271{,}5$$

Factored Design Moment

This is a simple beam →

$$M_u = \frac{w\ell^2}{8} = \frac{3277{,}5\,(30)^2}{8} = 368043{,}75 \times {}^{1}\!/_{1000}$$

$$= 368{,}043$$



find d: distance from top to center of flexural steel

$d = h - d_c$

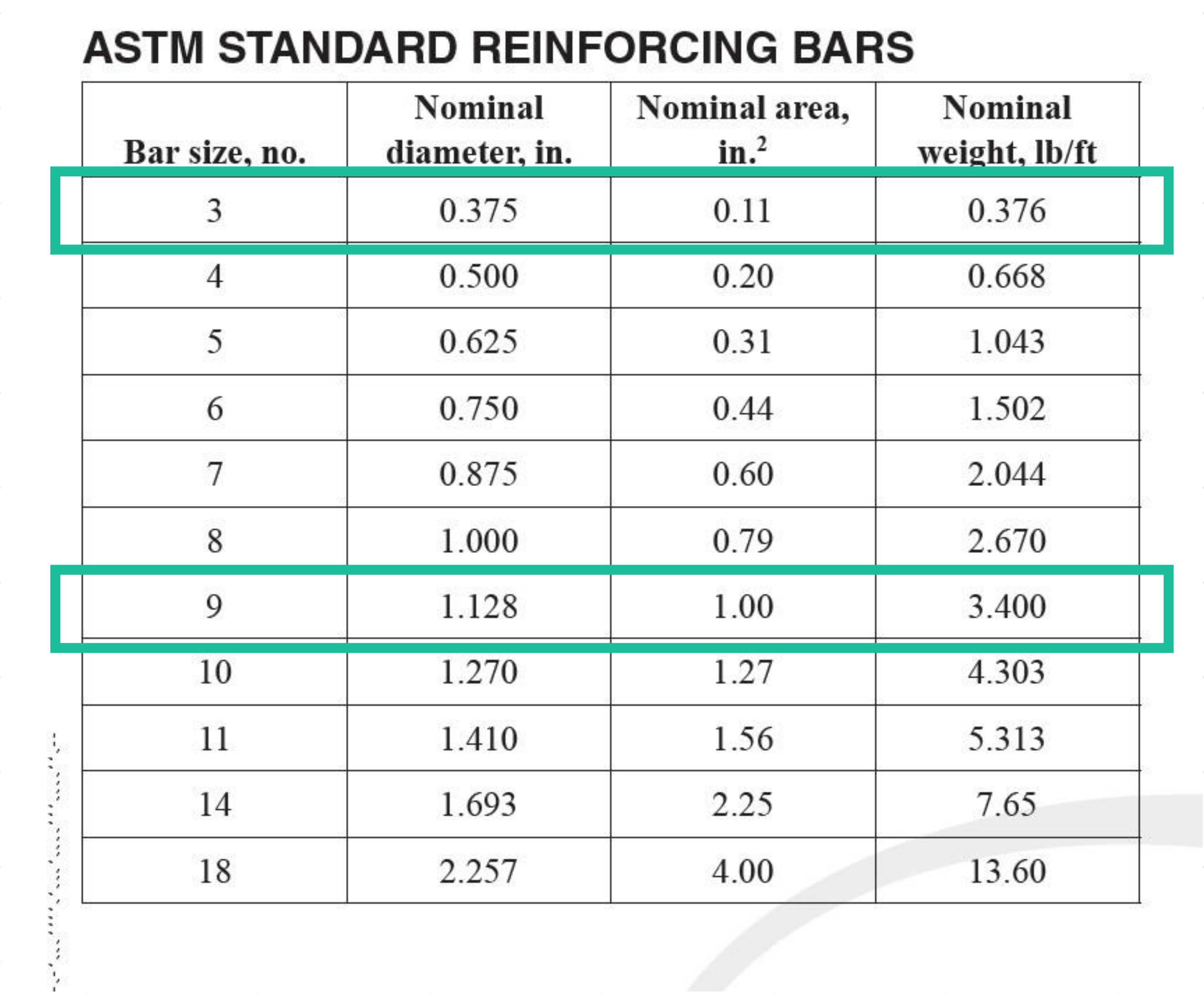
d_c : distance from bottom to center of flexural rebars

$$d_c = \text{Cover} + \underset{\#3}{\text{Stirrup}} + \frac{\text{Flex bar } \#9}{2}$$

$$= 1.5 + \boxed{0.375} + \frac{\boxed{1}}{2} = 2.375 \text{ in}$$

based on Table

$$d = 39 - 2.375 = 36.625 \text{ in}$$



ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.65
18	2.257	4.00	13.60

Calculate required As

estimate moment arm : $2 = jd = (0.9)(36.625)$

$= 32.9625$

unitConversion

$$1.\ A_s = \frac{M_u}{\phi \cdot f_y \boxed{(d-\frac{a}{2})}} = \frac{368.043 \times 10^3 \times 12}{0.9(60000)(32.9625)} = \underline{\underline{2.48}}$$

moment arm

$$2.\ a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.48(60000)}{0.85(5500)(18)} = \frac{148800}{84150} = 1.768 \text{ in}$$

moment arm: $d - \frac{a}{2} = 36.625 - \frac{1.768}{2} = 35.741$

$$\rightarrow 1.\ A_s = \frac{M_u}{\phi \cdot f_y (d - \frac{a}{2})} = \frac{4416516}{1930014} = \underline{\underline{2.28}}$$

Check accuracy $= \frac{2.48 - 2.28}{2.48} \simeq 8.06\,\%$ ✗ repeat until 2% accuracy

$$2.\ a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.28(60000)}{0.85(5500)(18)} = \frac{136800}{84150} = 1.62$$

$$1.\ A_s = \frac{4416516}{0.9(60000)(36.625 - \frac{1.62}{2})} = 2.28$$

Check accuracy = 0% ✓

required As = 2.28

Number of rebars:

Size is given = # 9 → Area of each rebar: 1.00

$$n = \frac{As}{Area} = \frac{2.28}{1} = 2.28 \rightarrow \text{round to next Num: } \underline{\underline{3}}$$

Actual As used: $3 \times 1.00 = 3\ in^2$

Minimum required steel

$$\text{Max of :}\ \left[\begin{array}{l} 1.\ \dfrac{3\sqrt{f_c'}(bd)}{f_y} = \dfrac{3\sqrt{5500}\,(18)(36.625)}{60000} = 2.44 \checkmark < 3 \checkmark \\[3em] 2.\ \dfrac{200(b)(d)}{f_y} = \dfrac{200(18)(36.625)}{60000} = 2.19 \end{array}\right.$$

depth of Conc Stress block, a :

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3(60000)}{0.85(5500)(18)} = \frac{180000}{84150} = 2.13$$

β_1 :

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) = 0.775 < 0.85$$

Distance from N.A from top of beam (C) :

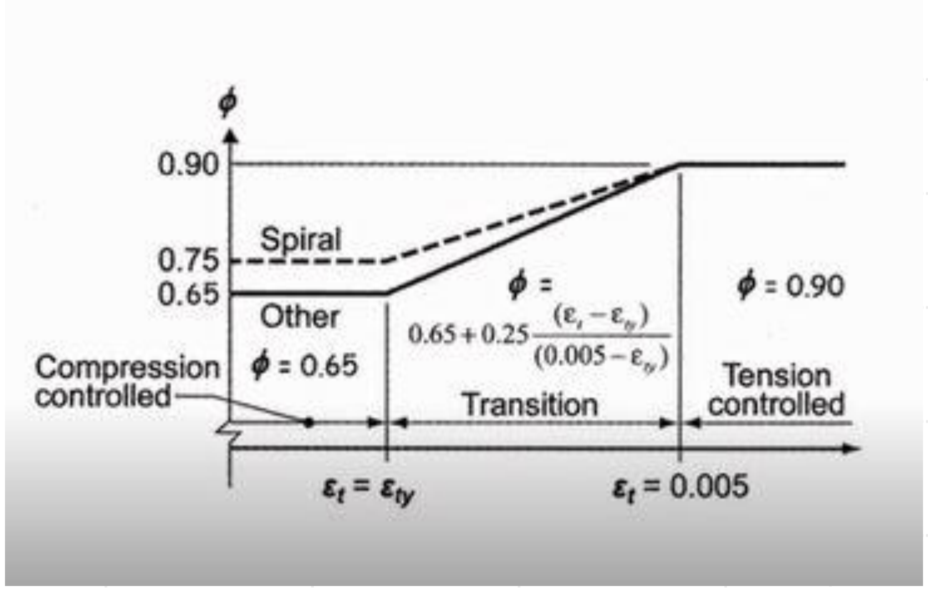
$$C = \frac{a}{\beta_1} = \frac{2.13}{0.775} = 2.7483$$

Strain in flexural steel :

$$\varepsilon_t = \frac{d-c}{c}(0.003) = 0.036 \geq 0.005$$ based on diagram

$\phi = 0.9$

Tension Controlled



Tensile Force in rebars (T):

$$T = A_s f_y = 3\,(60000) = 180000 \times \frac{1}{1000} = 180$$

Nominal bending moment, Mn:

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$= 3\,(60000)\left(36{,}625 - \frac{2.13}{2}\right) = 6400800 \times \frac{1}{1000} = \underset{\text{k-IN}}{6400.8}$$

$$\phi M_n : 0.9\,(6400.8) = 5760.72 \text{ k-IN} \times \frac{1}{12} = 480.06 \text{ k-ft}$$

- Thanks for your attention ☺