ARCHITECTURE 324
Structures II

Recitation 08

Sections 04&05

Instructor
Peter von Buelow

GSI
Alireza Fazel

March 14, 2025

Office Hours

→ Office Hours

→ Day: Fridays, 12:00 PM - 1:00 PM

→ Location Options:

- In-person meetings: [2223B]
- Virtual meetings via Zoom

Please make sure to sign up at least 24 hours in advance to allow for proper scheduling via this link:

https://docs.google.com/forms/d/e/1FAIpQLSdOb4gAc6SoCdsMAZP4zKrn3ecPyGt6dwVahVcOD3EqXGG-oA/viewform?usp=dialog

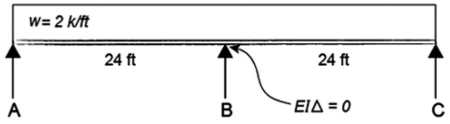
If the slots are fully booked or if you have a time conflict, please email me directly to find an alternative time (arfazel@umich.edu)

Contents

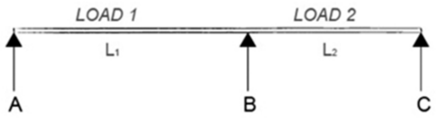
Three Moment Theorem

Continuous Beam Analysis

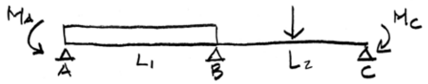
Deflection Method
- Two continuous, symmetric spans
- Symmetric Load



Slope Method
- Two continuous spans
- Non-symmetric loads and spans



3-Moment Theorem
- Any number of continuous spans
- Non-Symmetric Load and Spans



$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

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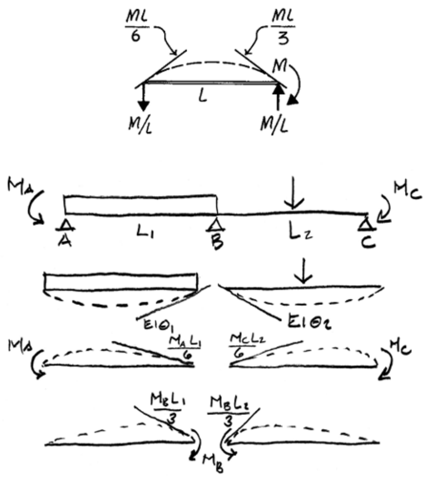
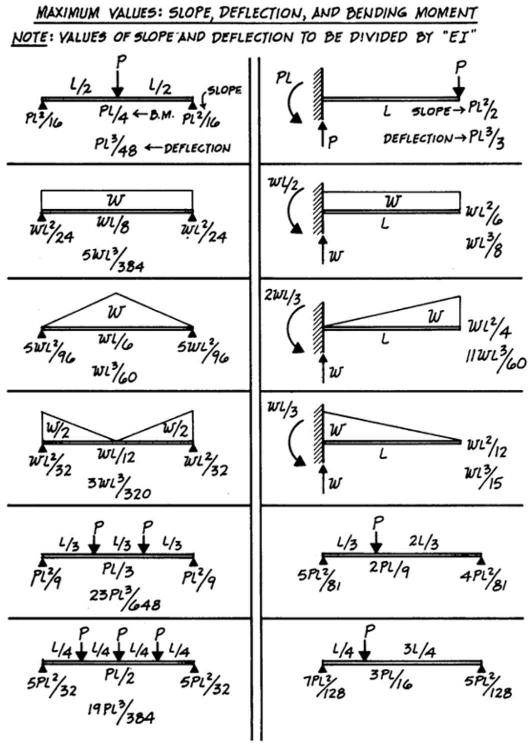
3-Moment Theorem

- Any number of continuous spans
- Non-Symmetric Load and Spans



$$+EI\Theta_1 - \frac{M_A L_1}{6} - \frac{M_B L_1}{3} = -EI\Theta_2 + \frac{M_C L_2}{6} + \frac{M_B L_2}{3}$$

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$



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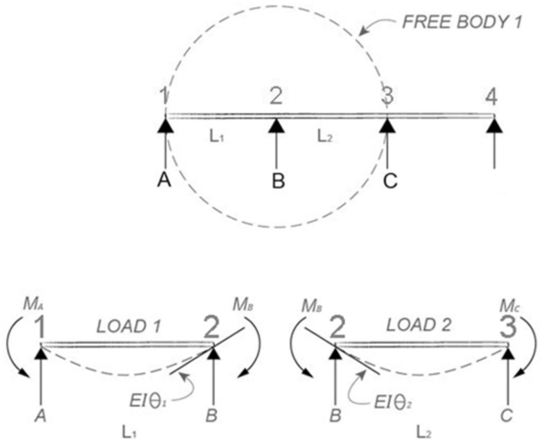
Three Moment Theorem

Three-Moment Theorem

- Any number of spans
- Symmetric or non-symmetric



Procedure:

1. Draw a free body diagram of the first two spans.
2. Label the spans L1 and L2 and the supports (or free end) A, B and C as show.
3. Use the Three-Moment equation to solve for each unknown moment, either as a value or as an equation.

$$M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]$$

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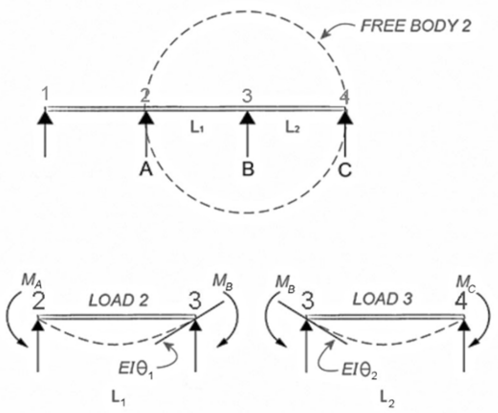
Three-Moment Theorem

Procedure (continued):

4. Move one span further and repeat the procedure.
5. In a 3 span beam, the mid-moment from step 3 above (B), can now be solved using the two equations from step 4 and 3 together, by writing 2 equations with 2 unknowns.
6. Repeat as needed, always moving one span to the right and writing a new set of moment equations.
7. Solve 2 simultaneous equations for 3 spans, or 3 equations for more than 3 spans, to get the interior moments.
8. Once all interior moments are known, solve for reactions using free body diagrams of individual spans.
9. Draw shear and moment diagrams as usual. This will also serve as a check for the moment values.



$$M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]$$

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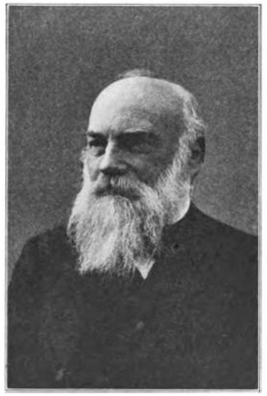
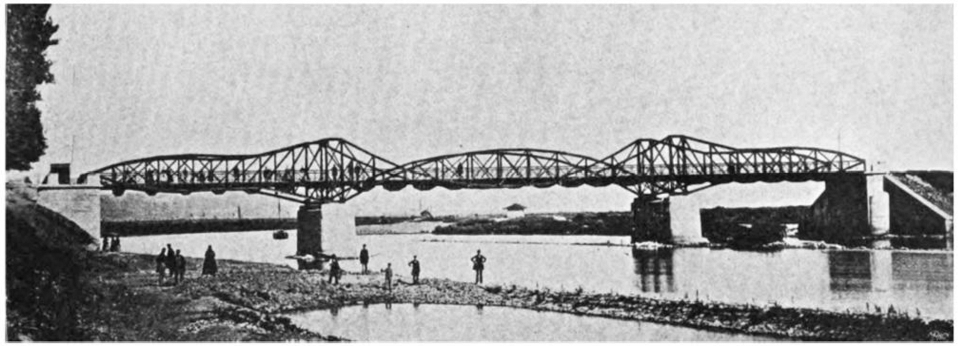
Gerber Beams

Gottfried Heinrich Gerber
(1832-1912)

Developed a cantilever bridge spanning system used in many bridges worldwide. The system became know as the “Gerber Beam” and uses cantilever segments to support a simple span.

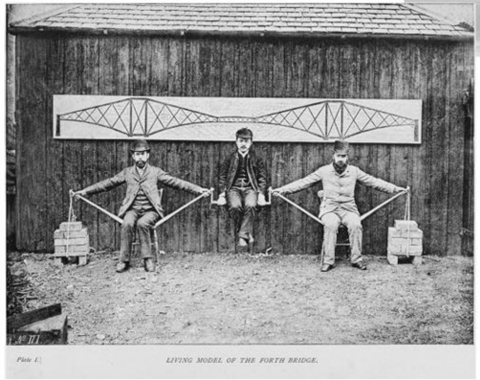


Haßfurter Brücke, 1864. Span of 38 m over the Main River.



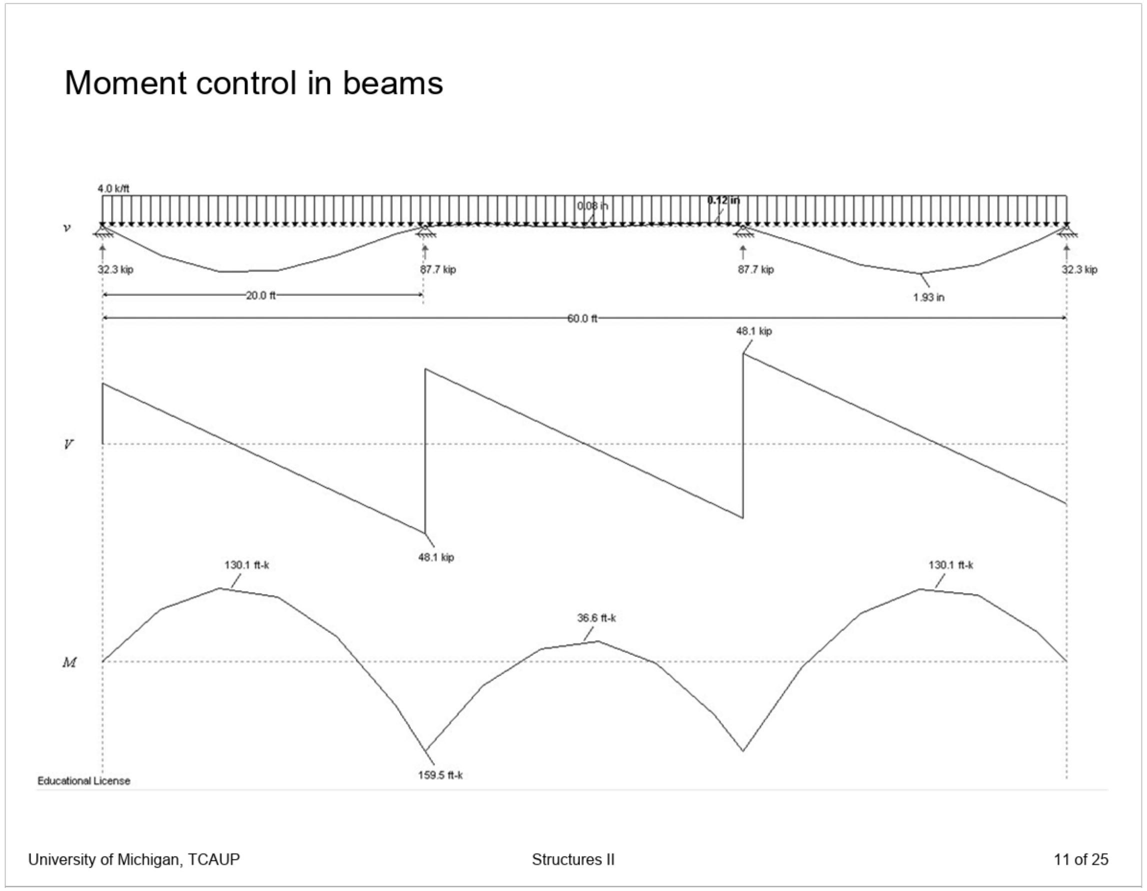
Examples of the Gerber system

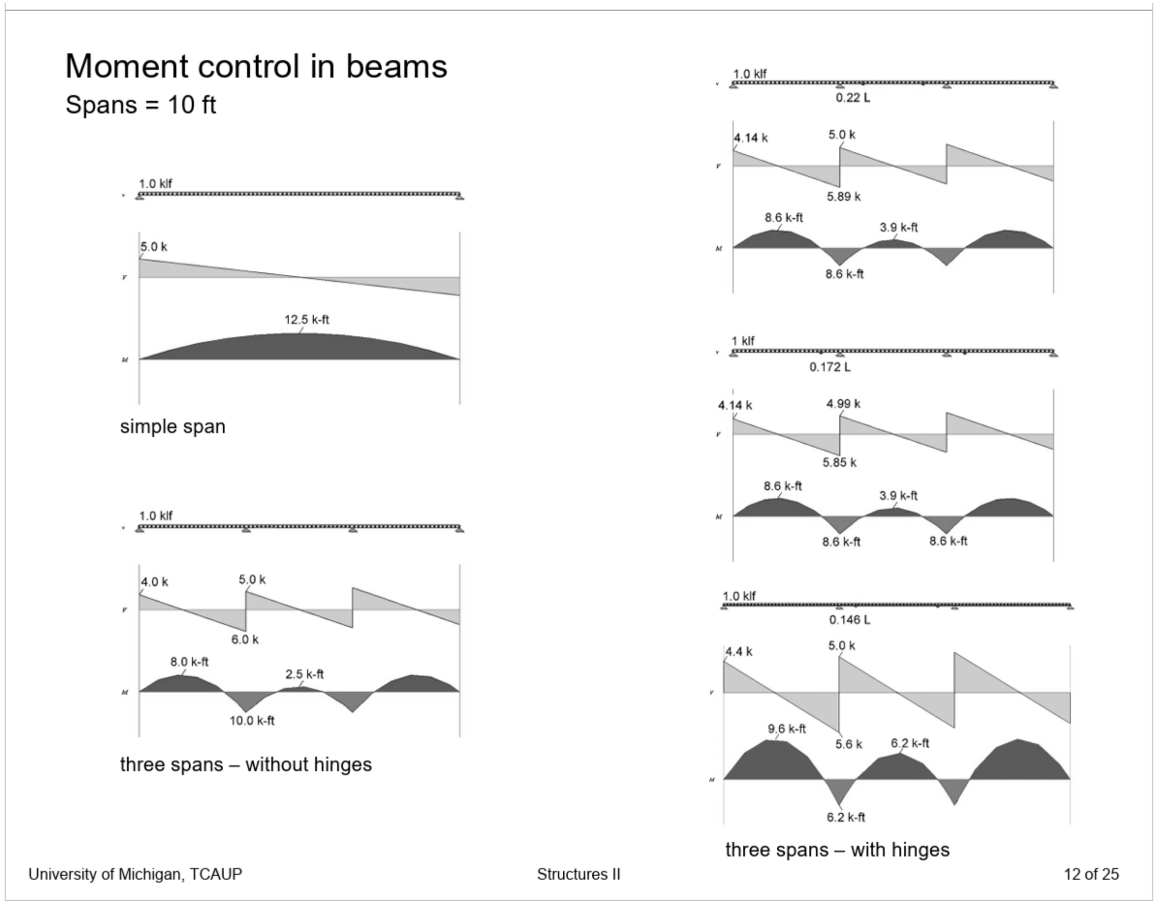
Firth of Fourth Bridge, 1890
- total length 8094 ft.
- central span 1700 ft.
- Design Fowler & Baker
- Construction 1882 - 1889



Static modeling of the Firth of Forth Bridge by Fowler & Baker

Gerber Beams





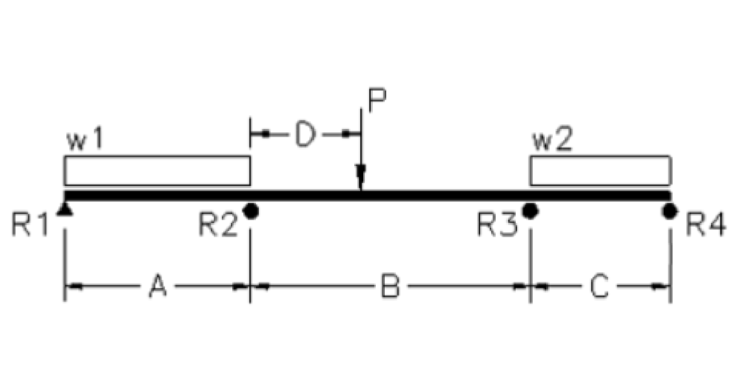
Problem Set 07

7. Three Moment Theorem

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

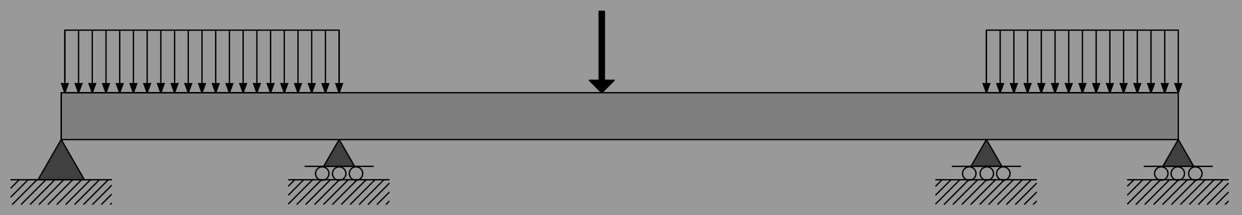
DATASET: 1 -2- -3-

Span A	17 FT
Span B	30 FT
Span C	10 FT
Uniform load on span A, w1	5 KLF
Uniform load on span C, w2	4 KLF
Point load on span b, P	44 K
Distance to point load P from R2, D	10 FT



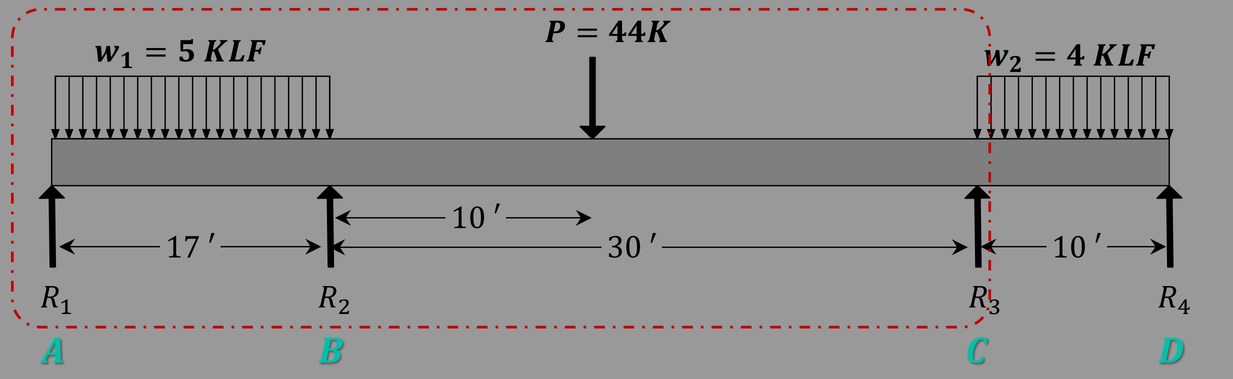
Problem Set 07

#Q1: Moment at support R1, M1 (- if tension on top)
#Q2: EI Theta on left side of R2
#Q3: EI Theta on right side of R2



7. Three Moment Theorem

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.



DATASET: 1 -2- -3-

Span A	17 FT
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Uniform load on span A, w1	5 KLF
Uniform load on span C, w2	4 KLF
Point load on span b, P	44 K
Distance to point load P from R2, D	10 FT

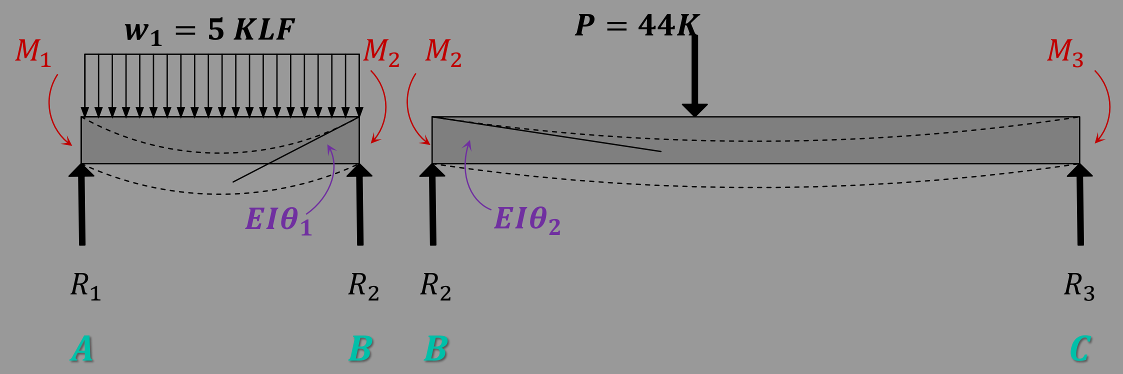
$$M_1 = 0\ (simple\ support)$$

on R_2

(Left) $$EI\theta_1 = \frac{WL_1^2}{24} = \frac{5(17)17^2}{24} = 1023.54$$

(right) $$EI\theta_2 = \frac{5PL_2^2}{81} = \frac{5(44)30^2}{81} = 2444.44$$

$$M_4 = 0\ (simple\ support)$$



Problem Set 07

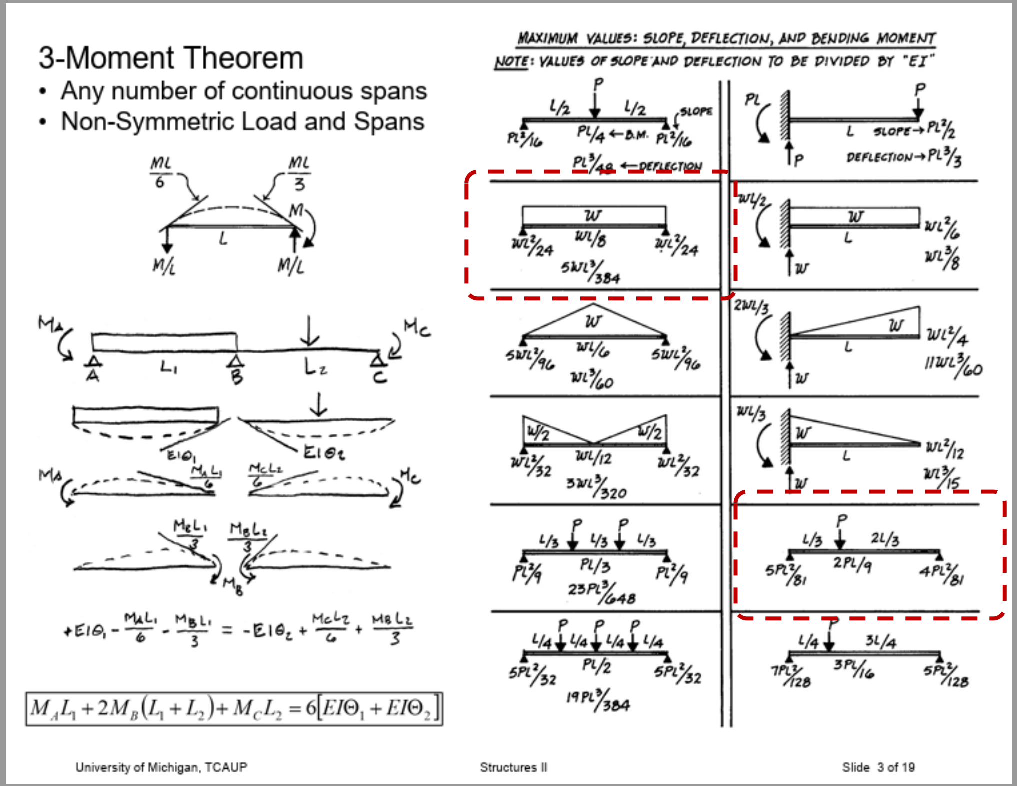
#Q1: Moment at support R1, M1
#Q2: EI Theta on left side of R2
#Q3: EI Theta on right side of R2
#Q4: Moment at support R4, M4

7. Three Moment Theorem

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

DATASET: 1 -2- -3-

Span A	17 FT
Span B	30 FT
Span C	10 FT
Uniform load on span A, w1	5 KLF
Uniform load on span C, w2	4 KLF
Point load on span b, P	44 K
Distance to point load P from R2, D	10 FT



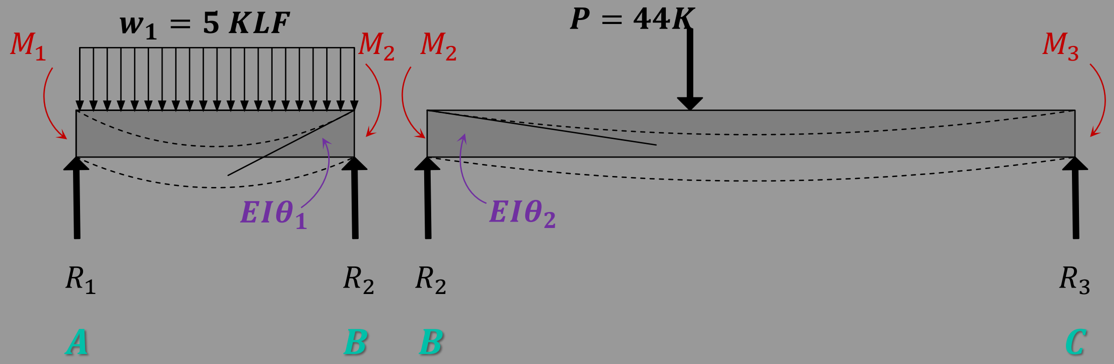
$$M_1 = 0 \ (simple\ support)$$

on R_2

(Left) $$EI\theta_1 = \frac{WL_1^2}{24} = \frac{5(17)17^2}{24} = 1023.54$$

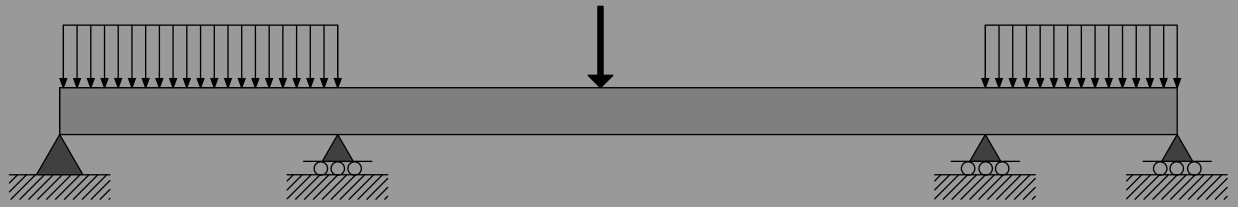
(right) $$EI\theta_2 = \frac{5PL_2^2}{81} = \frac{5(44)30^2}{81} = 2444.44$$

$$M_4 = 0 \ (simple\ support)$$



Problem Set 07

#Q5: EI Theta on left side of R3
#Q6: EI Theta on right side of R3

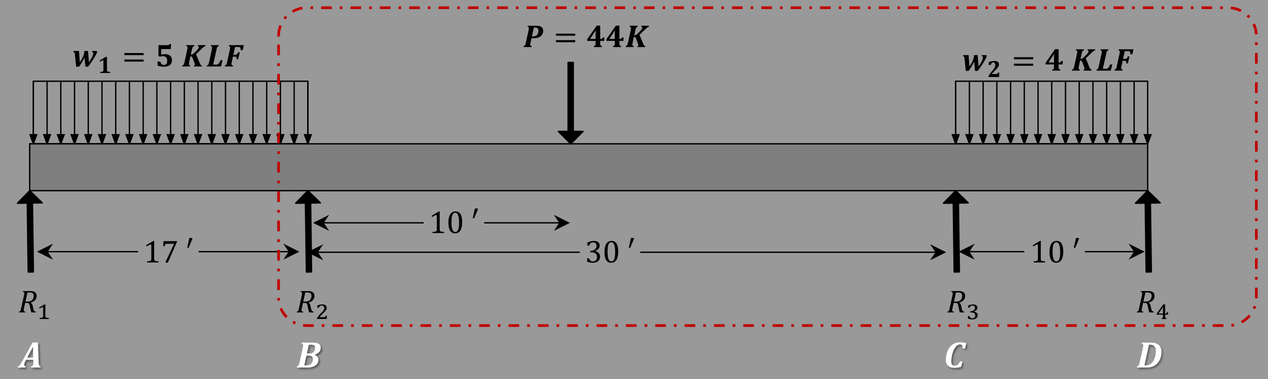


7. Three Moment Theorem

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

DATASET: 1 -2- -3-

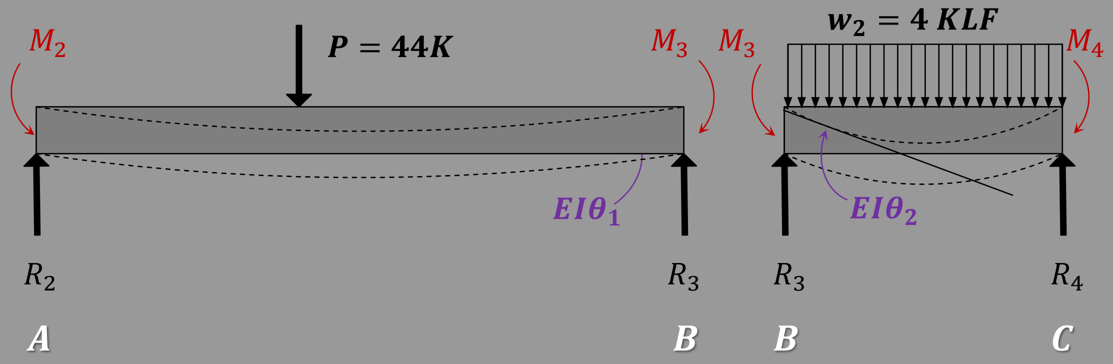
Span A	17 FT
Span B	30 FT
Span C	10 FT
Uniform load on span A, w1	5 KLF
Uniform load on span C, w2	4 KLF
Point load on span b, P	44 K
Distance to point load P from R2, D	10 FT



on R_3

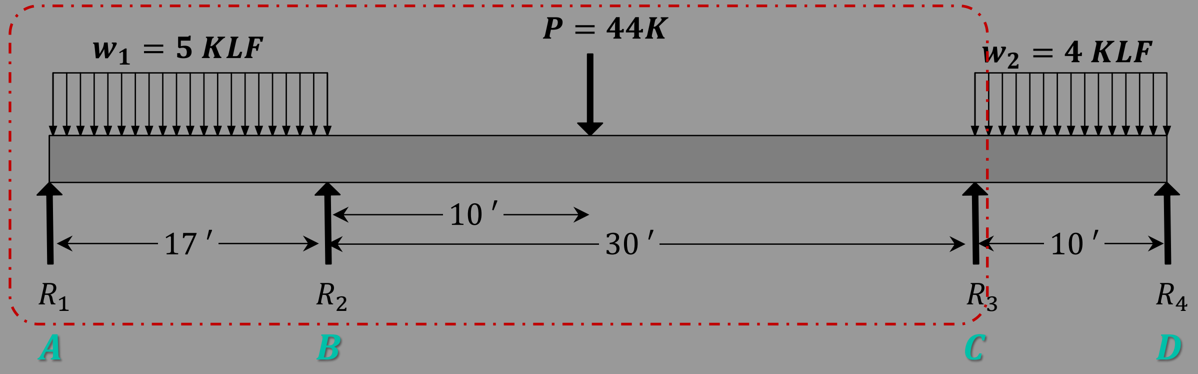
(Left) $$EI\theta_1 = \frac{4PL_2^2}{81} = \frac{5(44)30^2}{81} = 1955.55$$

(right) $$EI\theta_2 = \frac{WL_3^2}{24} = \frac{4(10)10^2}{24} = 166.66$$



Problem Set 07

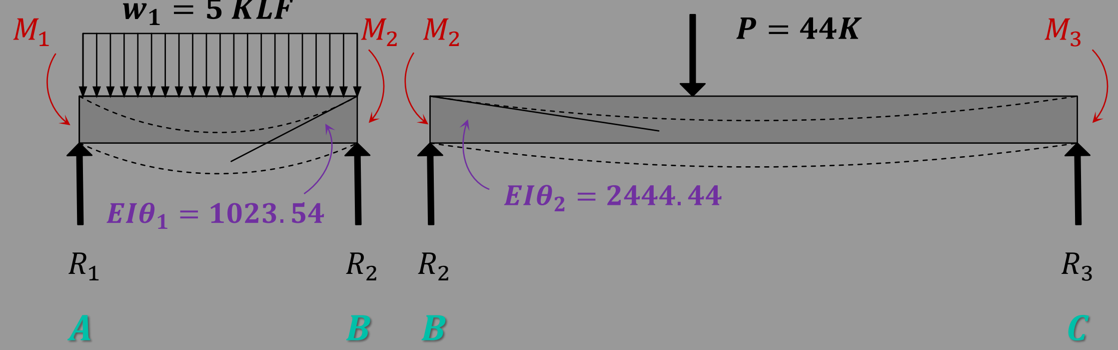
#Q7: Moment at support R2, M2 (- if tension on top)
#Q8: Moment at support R3, M3 (- if tension on top)



$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6\ (EI\theta_1 + EI\theta_2)$$

$$0(17) + 2M_2(17 + 30) + M_3 30 = 6\ (1023.54 + 2444.44)$$

$$94M_2 + 30M_3 = 20807.88 \quad Eq.1$$



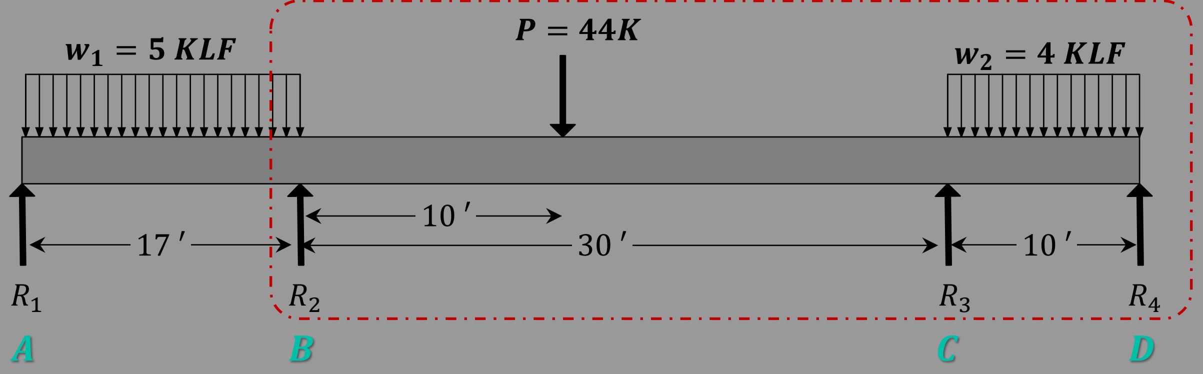
Problem Set 07

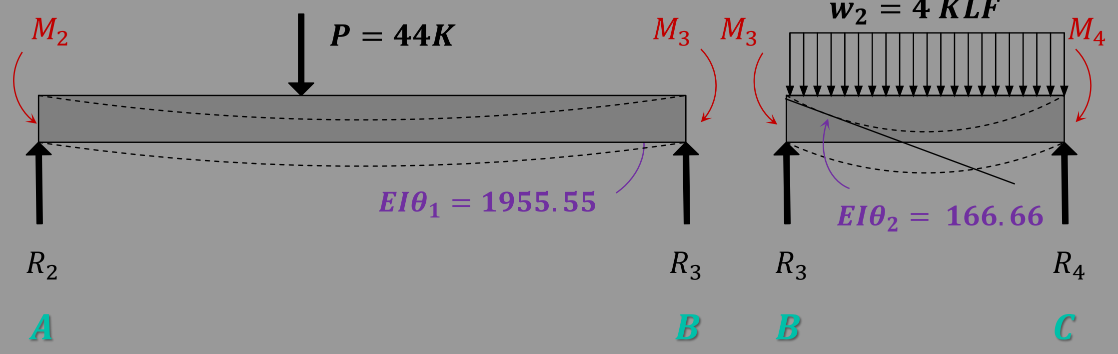
#Q7: Moment at support R2, M2 (- if tension on top)
#Q8: Moment at support R3, M3 (- if tension on top)

$$\boldsymbol{M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6\,(EI\theta_1 + EI\theta_2)}$$

$$M_2(30) + 2M_3(30 + 10) + 0(10) = 6\,(1955.55 + 166.66)$$

$$\boldsymbol{30M_2 + 80M_3 = 12733.26} \quad \boldsymbol{Eq. 2}$$





Problem Set 07

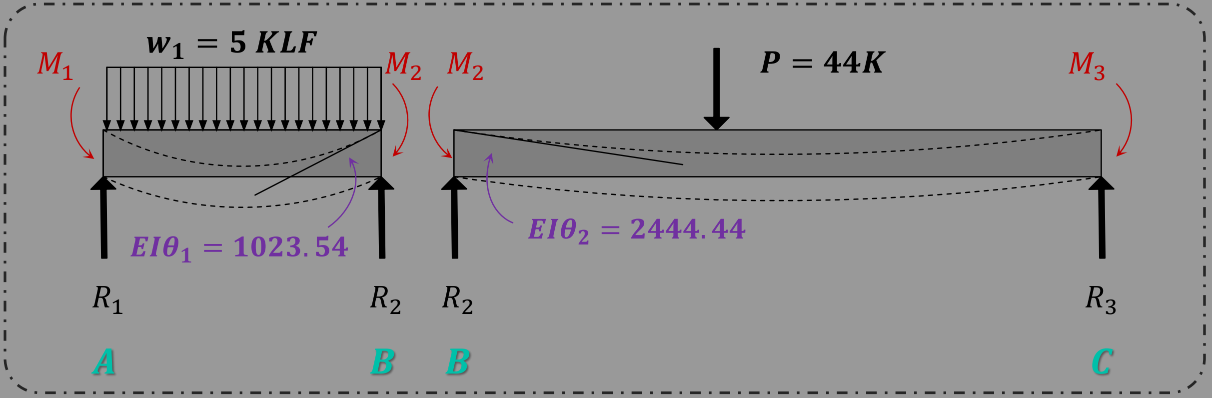
#Q7: Moment at support R2, M2 (- if tension on top)
#Q8: Moment at support R3, M3 (- if tension on top)

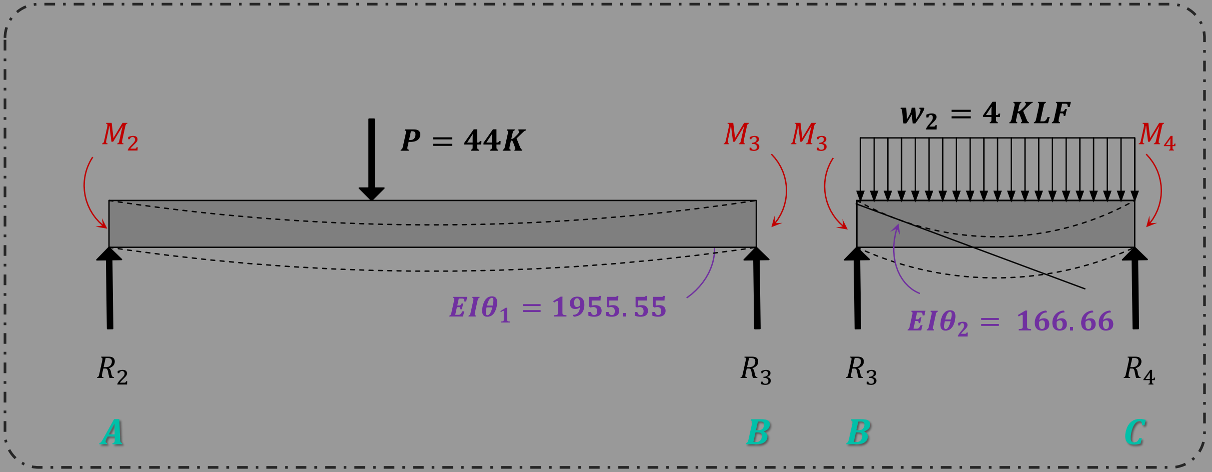
$$94M_2 + 30M_3 = 20807.88 \quad Eq.1$$

$$30M_2 + 80M_3 = 12733.26 \quad Eq.2$$

$\rightarrow M_3 = -86.50\ \text{K} - \text{FT}$

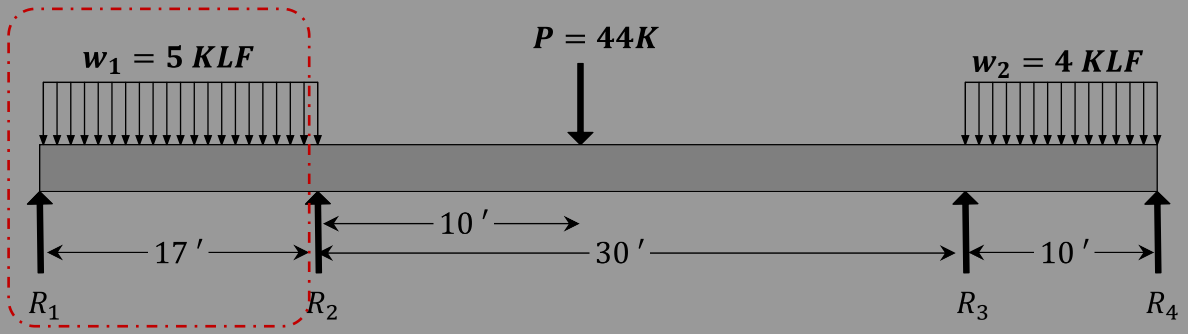
$\rightarrow M_2 = -193.74\ K - FT$





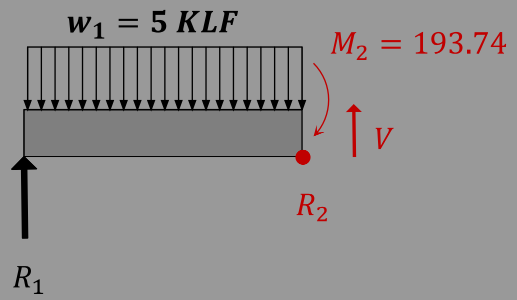
Problem Set 07

#Q9: Support reaction, R1 (- if downward)



$$\sum M_{R_2} = 0 \rightarrow R_1(17) - 5(17)\left(\frac{17}{2}\right) + 193.74 = 0$$

$$\rightarrow \boldsymbol{R_1 = 31.1\ K}$$

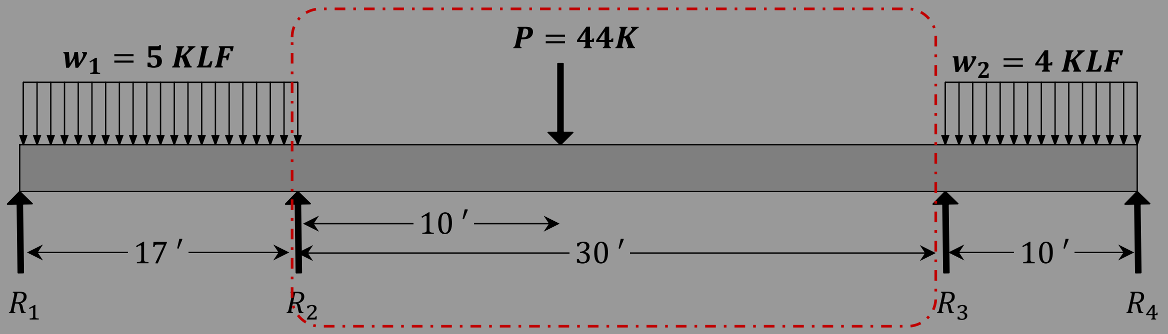


$$\sum F_V = 0 \rightarrow R_1 + V = 5(17)$$

$$\rightarrow \boldsymbol{V = 53.9\ K}$$

Problem Set 07

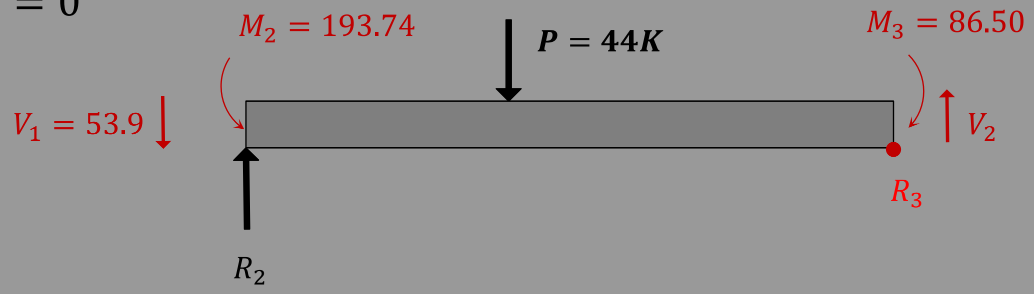
#Q10: Support reaction, R2 (- if downward)



$$\sum M_{R_3} = 0 \ \rightarrow V_1(30) + R_2(30) - M_2 \ - 44(20) + M_3$$

$$\rightarrow -53.9(30) + R_2(30) - 193.74 - 44(20) + 86.50 = 0$$

$$\rightarrow R_2 \ = 86.81 \text{ K}$$



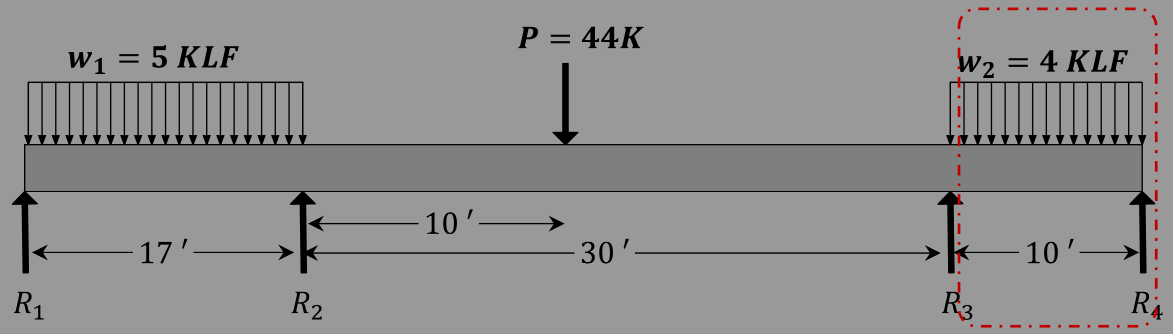
$$\sum F_V = 0 \ \rightarrow R_2 - V_1 - P + V_2 = 5(17)$$

$$86.81 - 53.9 \ - 44 + V_2 = 0$$

$$\rightarrow V_2 = 11.09 \text{ K}$$

Problem Set 07

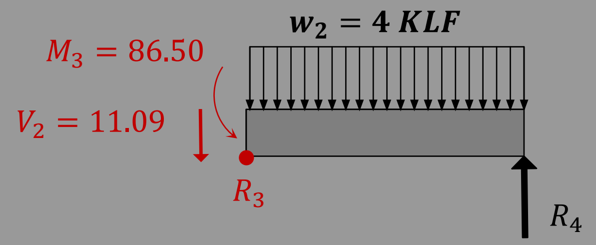
#Q11: Support reaction, R3 (- if downward)
#Q12: Support reaction, R4 (- if downward)



$$\sum M_{R_3} = 0 \rightarrow -M_3 + 4(10)\left(\frac{10}{2}\right) - R_4\,(10) = 0$$

$$-86.50 + 4(10)(5) - 10R_4 = 0$$

$$\rightarrow R_4 = 11.35\text{K}$$



$$\sum F_V = 0 \rightarrow R_3 - V_2 + R_4 + 4(10) = 0$$

$$R_3 - 11.09 - 40 + 11.35 = 0$$

$$\rightarrow R_3 = 39.74\text{ K}$$

Lab05

Structures II

Arch 324

Name 1 ____________________

Name 2 ____________________

Name 3 ____________________

Continuous Beams

Description

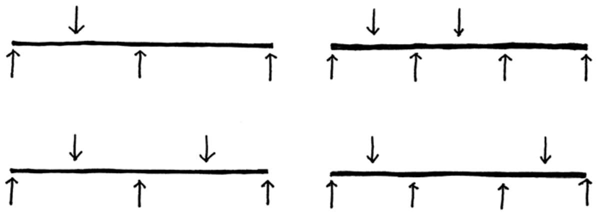
This project uses observation to understand the behavior of beams continuous over multiple supports.

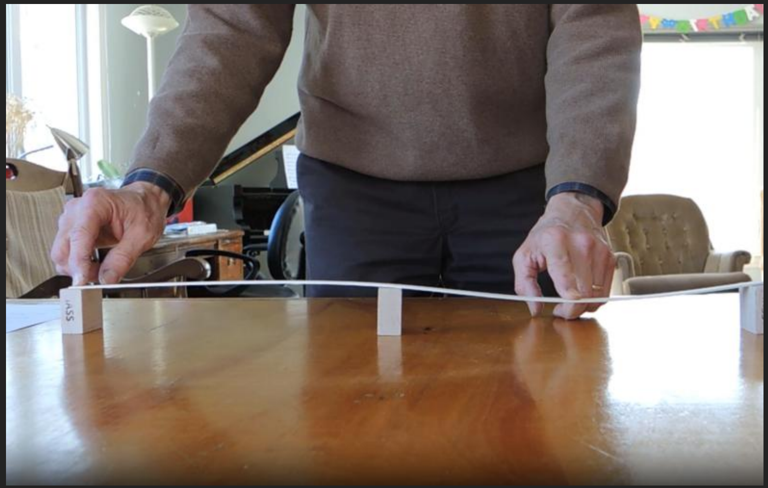
Goals

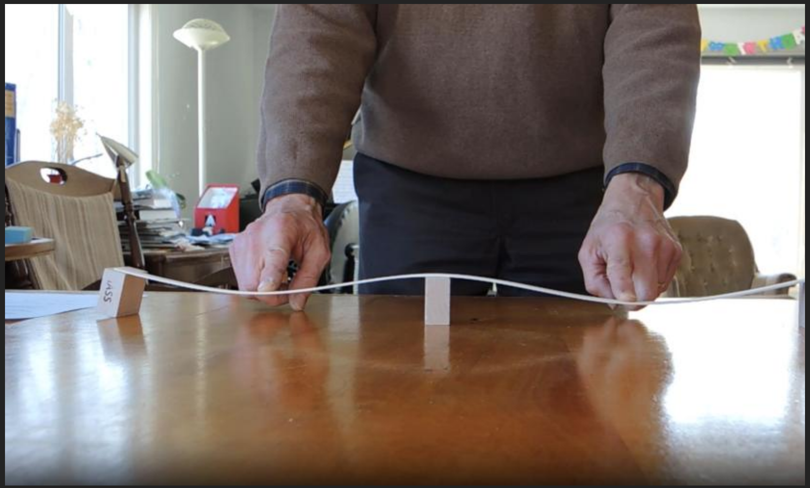
To observe the behavior of continuous beams under different loadings
To estimate locations of contraflexure and effective lengths
To determine areas of positive and negative moment based on curvature

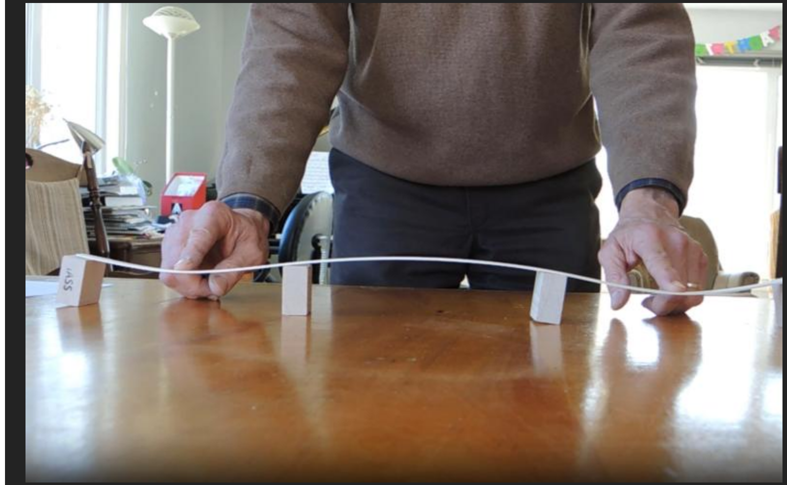
Procedure

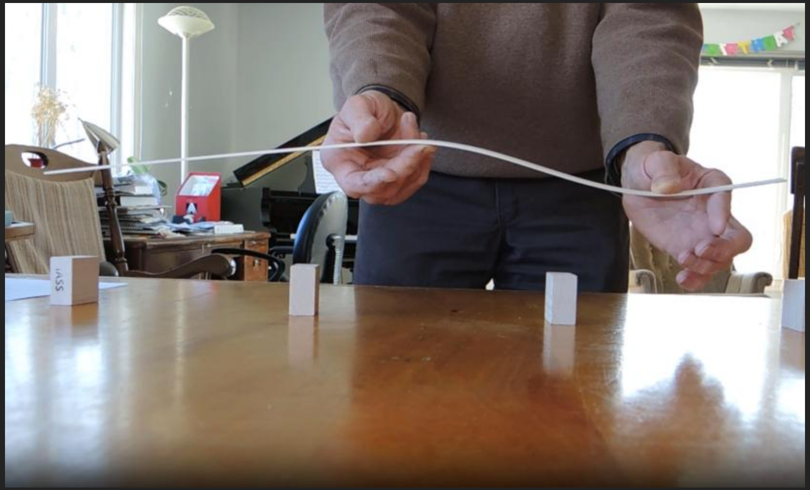
1. Using the 24 inch stick, position the supports and loads (with your finger) as shown in the diagrams below. Hold the beam down on the reactions if it lifts up.
2. For each case observe and draw the elastic curve.
3. Label + and – curvature (moment) and points of contraflexure.
4. Estimate the effective lengths, ℓ_e, across the beam. (between points of M=0)











Lab05

→ Group work instructions

Please form groups of 2 to 4 students.

Please do not forget to write all group members' names on both sheets.

Return the completed sheets to me at the end of the session.

Please ensure that you attend the recitation sessions.

If you are unable to attend a session, send me an email so that we can discuss how to proceed. *Email: arfazel@umich.edu*