ARCHITECTURE 324
Structures II

Recitation 07

Sections 04&05

Instructor
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GSI
Alireza Fazel

Feb 28, 2025

Office Hours

→ Office Hours

→ Day: Fridays, 12:00 PM - 1:00 PM

→ Location Options:

- In-person meetings: [2223B]
- Virtual meetings via Zoom

Please make sure to sign up at least 24 hours in advance to allow for proper scheduling via this link:

https://docs.google.com/forms/d/e/1FAIpQLSdOb4gAc6SoCdsMAZP4zKrn3ecPyGt6dwVahVcOD3EqXGG-oA/viewform?usp=dialog

If the slots are fully booked or if you have a time conflict, please email me directly to find an alternative time (arfazel@umich.edu)

Contents

Structural Continuity

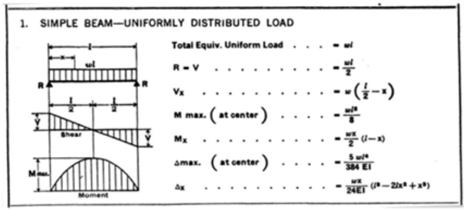
Continuous Beams

- Continuous over one or more supports
  - Most common in monolithic concrete
  - Steel: continuous or with moment connections
  - Wood: as continuous beams, affected long Glulam spans
- Statically indeterminate
  - Cannot be solved by the three equations of statics alone
  - Internal forces (shear & moment) as well as reactions are affected by movement or settlement of the supports

two spans - simply supported

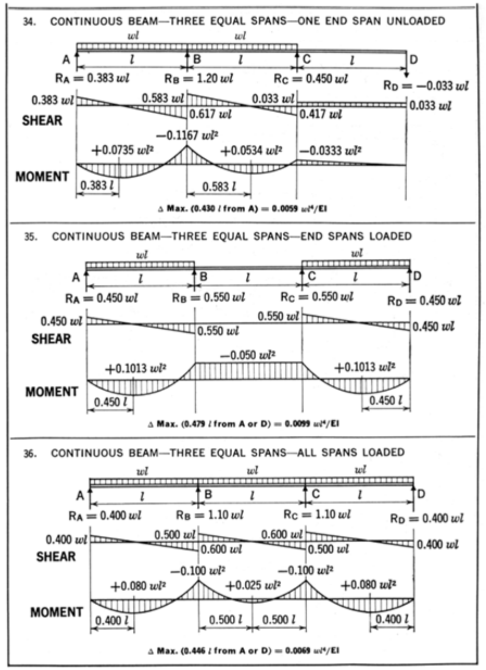
two spans - continuous

Simple vs. Continuous Beams



- Simple Beam
  - End moments = 0
  - Mmax at C.L = $wL^2/8 = 0.125wL^2$
- Continuous Beam
  - Exterior end moments = 0
  - Interior support moments are usually negative
  - Mid-span moments are usually positive

Note: moments shown reversed



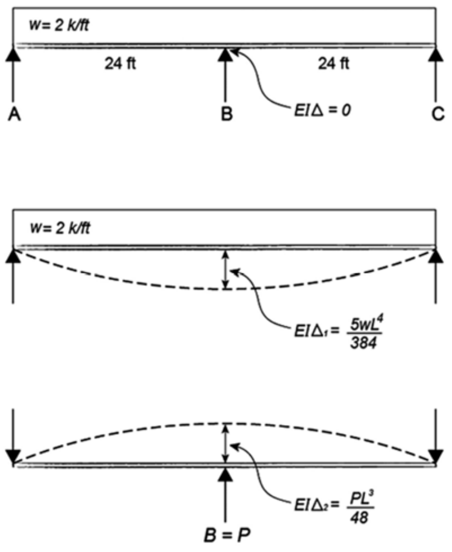
Structural Continuity

Deflection Method

- Two continuous, symmetric spans
- Symmetric Load

Procedure:

1. Remove the center support and calculate the center deflection for each load case as a simple span.
2. Remove the applied loads and replace the center support. Set the deflection equation for this case (center point load) equal to the deflection from step 1.
3. Solve the resulting equation for the center reaction force. (upward point load)
4. Calculate the remaining two end reactions.
5. Draw shear and moment diagrams as usual.

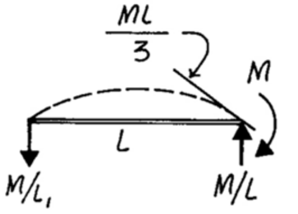


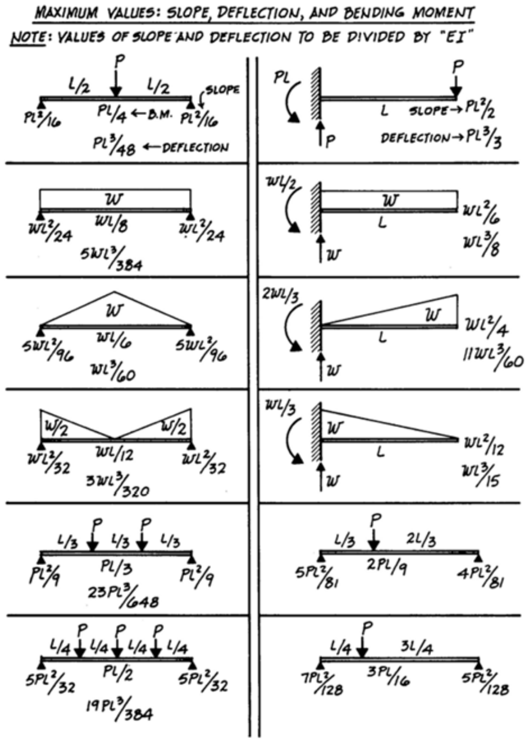
$$EI\Delta_1 + EI\Delta_2 = 0$$

Slope Method

Slope equations:

$$M = \frac{3}{L_1 + L_2}\left[EI\Theta_1 + EI\Theta_2\right]$$

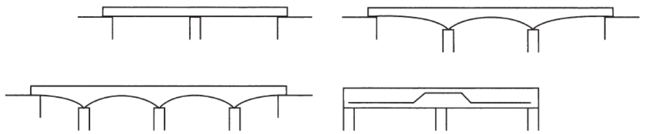




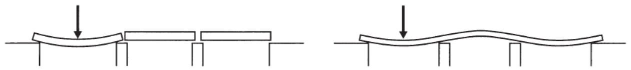
Structural Continuity

300 CHAPTER EIGHT

FIGURE 8.1 Continuous versus simple beams and typical types of continuous beams.

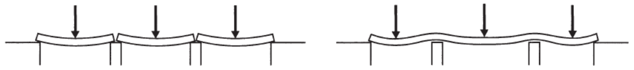


(a) Typical continuous beam structures



Simple beams. A load on any one span causes curvatures and bending moments to develop in that span alone. No other span is affected. The only part of the total assembly playing a role in carrying the load is the member directly beneath the load.

Continuous beam. A load on any one span causes curvatures and bending moments to develop in all spans. Consequently, all spans share in carrying the load on any one span. Member sizes can be smaller than those in a simply supported system because comparable design moments are smaller and rigidity greater.



Simple beams. Each beam deflects independently.

Continuous beams. Reverse curvatures are common. The system is typically stiffer (deflections are less) than in a simply supported system.

(b) General behavior of continuous beam structures

loads are often smaller than in comparable determinate structures. Thus, member sizes can often be reduced. Smaller amounts of material also are more efficiently used. Disadvantages in structures of this type include their sensitivity to support settlements and thermal effects. Support settlements, for example, can cause undesirable bending moments to develop in continuous beams over several supports, while not necessarily affecting a comparable series of simply supported beams.

Continuous Structures: Beams 317

FIGURE 8.14 Use of construction joints in continuous members. Construction joints often facilitate construction. Creating a condition of zero moment by design at points of inflection models the behavior of a continuous member by a series of statically determinate members.

(a) Loading

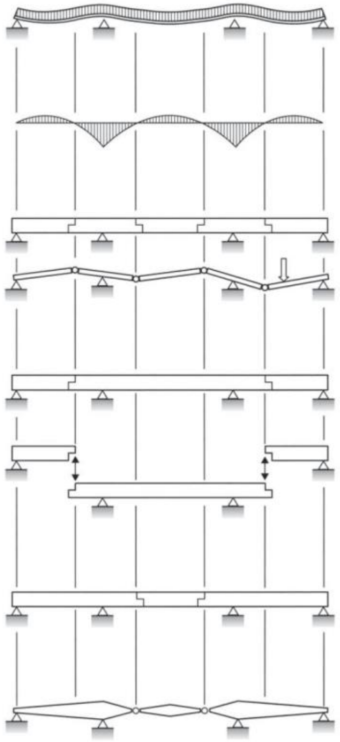
(b) Moment diagram

(c) Use of construction joints (typically pinned connections) at each point of inflection. The resultant structure is unstable when partially loaded. Rigid construction joints capable of carrying some moment could be used to achieve stability. Otherwise, one of the options shown must be adopted.

(d) Use of construction joints in end span only. The resultant configuration is stable under any loading. The joints facilitate construction. The center span would be put in place followed by the end spans.

(e) Use of construction in middle span. Again, the resultant structure is stable under any loading. The end spans would be put in place first followed by the middle span.

(f) Use of construction joints in middle span is coupled with shaping of structure to reflect moments present.



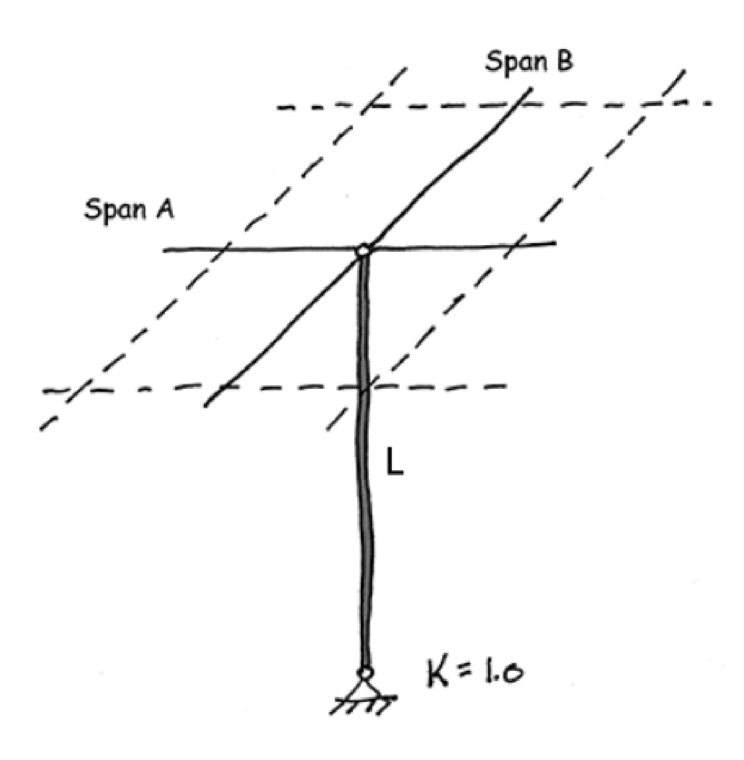
Problem Set 06

6. Steel Column Analysis

For the given axially loaded steel W-section, determine the maximum floor live load capacity, P LL. Assume the column is pinned top and bottom: K = 1.0, and there is no intermediate bracing. Use AISC-LRFD steel equations to determine phi Pn and the load. E = 29000 ksi.

DATASET: 1 -2- -3-

W-section	W8X31
Fy	36 KSI
Span A	32 FT
Span B	30 FT
Height L	17 FT
Floor Dead Load	39 PSF



Problem Set 06

#Q1: Total unfactored floor dead load on the column

6. Steel Column Analysis

For the given axially loaded steel W-section, determine the maximum floor live load capacity, P LL. Assume the column is pinned top and bottom: K = 1.0, and there is no intermediate bracing. Use AISC-LRFD steel equations to determine phi Pn and the load. E = 29000 ksi.

DATASET: 1 -2- -3-

W-section	W8X31
Fy	36 KSI
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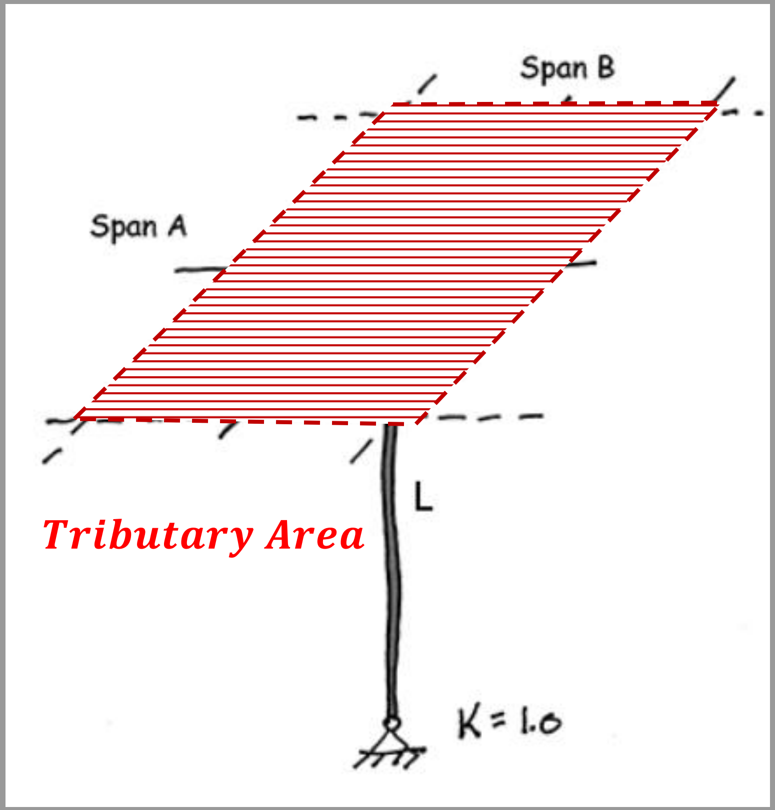


$$\boldsymbol{Tributary\ Area} = \textbf{spanA}\ (\boldsymbol{SpanB}) = 32(30) = \mathbf{960}\ \boldsymbol{FT^2}$$

$$\boldsymbol{W_{DL}} = \boldsymbol{Floor_{DL}}(\boldsymbol{Tributary\ Area}) = 39(960) = 37{,}440\ \text{lbs} = \mathbf{37.44\ KIPS}$$

Problem Set 06

#Q2: Controlling slenderness ratio

6. Steel Column Analysis

For the given axially loaded steel W-section, determine the maximum floor live load capacity, P LL. Assume the column is pinned top and bottom: K = 1.0, and there is no intermediate bracing. Use AISC-LRFD steel equations to determine phi Pn and the load. E = 29000 ksi.

DATASET: 1 -2- -3-

W-section	W8X31
Fy	36 KSI
Span A	32 FT
Span B	30 FT
Height L	17 FT
Floor Dead Load	39 PSF

1–28 DIMENSIONS AND PROPERTIES

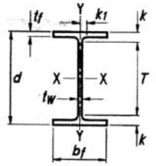


Table 1-1 (continued)
W-Shapes
Dimensions

Shape	Area, A	Depth, d		Web Thickness, t_w		Web $\frac{t_w}{2}$	Flange Width, b_f		Flange Thickness, t_f		Distance k k_{des}	Distance k k_{det}	k_1	T	Workable Gage
	in.^{2}	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W8×67	19.7	9.00	9	0.570	9/16	5/16	8.28	8 1/4	0.935	15/16	1.33	1 5/8	15/16	5 3/4	5 1/2
×58	17.1	8.75	8 3/4	0.510	1/2	1/4	8.22	8 1/4	0.810	13/16	1.20	1 1/2	7/8		
×48	14.1	8.50	8 1/2	0.400	3/8	3/16	8.11	8 1/8	0.685	11/16	1.08	1 3/8	13/16		
×40	11.7	8.25	8 1/4	0.360	3/8	3/16	8.07	8 1/8	0.560	9/16	0.954	1 1/4	13/16		
×35	10.3	8.12	8 1/8	0.310	5/16	3/16	8.02	8	0.495	1/2	0.889	1 3/16	13/16		
×31[f]	9.13	8.00	8	0.285	5/16	3/16	8.00	8	0.435	7/16	0.829	1 1/8	3/4	↓	↓
W8×28	8.25	8.06	8	0.285	5/16	3/16	6.54	6 1/2	0.465	7/16	0.859	15/16	5/8	6 1/8	4
×24	7.08	7.93	7 7/8	0.245	1/4	1/8	6.50	6 1/2	0.400	3/8	0.794	7/8	9/16	6 1/8	4
W8×21	6.16	8.28	8 1/4	0.250	1/4	1/8	5.27	5 1/4	0.400	3/8	0.700	7/8	9/16	6 1/2	2 3/4[g]
×18	5.26	8.14	8 1/8	0.230	1/4	1/8	5.25	5 1/4	0.330	5/16	0.630	13/16	9/16	6 1/2	2 3/4[g]
W8×15	4.44	8.11	8 1/8	0.245	1/4	1/8	4.02	4	0.315	5/16	0.615	13/16	9/16	6 1/2	2 1/4[g]
×13	3.84	7.99	8	0.230	1/4	1/8	4.00	4	0.255	1/4	0.555	3/4	9/16		
×10[c,f]	2.96	7.89	7 7/8	0.170	3/16	1/8	3.94	4	0.205	3/16	0.505	11/16	1/2	↓	↓
W6×25	7.34	6.38	6 3/8	0.320	5/16	3/16	6.08	6 1/8	0.455	7/16	0.705	15/16	9/16	4 1/2	3 1/2
×20	5.87	6.20	6 1/4	0.260	1/4	1/8	6.02	6	0.365	3/8	0.615	7/8	9/16		
×15[f]	4.43	5.99	6	0.230	1/4	1/8	5.99	6	0.260	1/4	0.510	3/4	9/16	↓	↓
W6×16	4.74	6.28	6 1/4	0.260	1/4	1/8	4.03	4	0.405	3/8	0.655	7/8	9/16	4 1/2	2 1/4[g]
×12	3.55	6.03	6	0.230	1/4	1/8	4.00	4	0.280	1/4	0.530	3/4	9/16		
×9[f]	2.68	5.90	5 7/8	0.170	3/16	1/8	3.94	4	0.215	3/16	0.465	11/16	1/2		
×8.5[f]	2.52	5.83	5 7/8	0.170	3/16	1/8	3.94	4	0.195	3/16	0.445	11/16	1/2	↓	↓
W5×19	5.56	5.15	5 1/8	0.270	1/4	1/8	5.03	5	0.430	7/16	0.730	13/16	7/16	3 1/2	2 3/4[g]
×16	4.71	5.01	5	0.240	1/4	1/8	5.00	5	0.360	3/8	0.660	3/4	7/16	3 1/2	2 3/4[g]
W4×13	3.83	4.16	4 1/8	0.280	1/4	1/8	4.06	4	0.345	3/8	0.595	3/4	1/2	2 5/8	2 1/4[g]

DIMENSIONS AND PROPERTIES 1–29

Table 1-1 (continued)
W-Shapes
Properties

W8-W4

Nominal Wt.	Compact Section Criteria $\frac{b_f}{2t_f}$	Compact Section Criteria $\frac{h}{t_w}$	Axis X-X I	Axis X-X S	Axis X-X r	Axis X-X Z	Axis Y-Y I	Axis Y-Y S	Axis Y-Y r	Axis Y-Y Z	r_{ts}	h_o	Torsional Properties J	Torsional Properties C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
67	4.43	11.1	272	60.4	3.72	70.1	88.6	21.4	2.12	32.7	2.43	8.07	5.05	1440
58	5.07	12.4	228	52.0	3.65	59.8	75.1	18.3	2.10	27.9	2.39	7.94	3.33	1180
48	5.92	15.9	184	43.2	3.61	49.0	60.9	15.0	2.08	22.9	2.35	7.82	1.96	931
40	7.21	17.6	146	35.5	3.53	39.8	49.1	12.2	2.04	18.5	2.31	7.69	1.12	726
35	8.10	20.5	127	31.2	3.51	34.7	42.6	10.6	2.03	16.1	2.28	7.63	0.769	619
31	9.19	22.3	110	27.5	3.47	30.4	37.1	9.27	2.02	14.1	2.26	7.57	0.536	530
28	7.03	22.3	98.0	24.3	3.45	27.2	21.7	6.63	1.62	10.1	1.84	7.60	0.537	312
24	8.12	25.9	82.7	20.9	3.42	23.1	18.3	5.63	1.61	8.57	1.81	7.53	0.346	259
21	6.59	27.5	75.3	18.2	3.49	20.4	9.77	3.71	1.26	5.69	1.46	7.88	0.282	152
18	7.95	29.9	61.9	15.2	3.43	17.0	7.97	3.04	1.23	4.66	1.43	7.81	0.172	122
15	6.37	28.1	48.0	11.8	3.29	13.6	3.41	1.70	0.876	2.67	1.06	7.80	0.137	51.8
13	7.84	29.9	39.6	9.91	3.21	11.4	2.73	1.37	0.843	2.15	1.03	7.74	0.0871	40.8
10	9.61	40.5	30.8	7.81	3.22	8.87	2.09	1.06	0.841	1.66	1.01	7.69	0.0426	30.9
25	6.68	15.5	53.4	16.7	2.70	18.9	17.1	5.61	1.52	8.56	1.74	5.93	0.461	150
20	8.25	19.1	41.4	13.4	2.66	14.9	13.3	4.41	1.50	6.72	1.70	5.84	0.240	113
15	11.5	21.6	29.1	9.72	2.56	10.8	9.32	3.11	1.45	4.75	1.66	5.73	0.101	76.5
16	4.98	19.1	32.1	10.2	2.60	11.7	4.43	2.20	0.967	3.39	1.13	5.88	0.223	38.2
12	7.14	21.6	22.1	7.31	2.49	8.30	2.99	1.50	0.918	2.32	1.08	5.75	0.0903	24.7
9	9.16	29.2	16.4	5.56	2.47	6.23	2.20	1.11	0.905	1.72	1.06	5.69	0.0405	17.7
8.5	10.1	29.1	14.9	5.10	2.43	5.73	1.99	1.01	0.890	1.56	1.05	5.64	0.0333	15.8
19	5.85	13.7	26.3	10.2	2.17	11.6	9.13	3.63	1.28	5.53	1.45	4.72	0.316	50.9
16	6.94	15.4	21.4	8.55	2.13	9.63	7.51	3.00	1.26	4.58	1.43	4.65	0.192	40.6
13	5.88	10.6	11.3	5.46	1.72	6.28	3.86	1.90	1.00	2.92	1.16	3.82	0.151	14.0

According to the table, for W8 × 31, we have:

$$r_x = 3.47\ IN, \quad r_y = 2.02\ IN$$

$$L_C = KL \quad (K = 1) \rightarrow \quad L_C = L = 17\ FT$$

$$slenderness\ ratio_x = \frac{L_c}{r_x} = \frac{17\ FT \times \frac{12\ IN}{1FT}}{3.47} = 58.78$$

$$slenderness\ ratio_y = \frac{L_c}{r_y} = \frac{17 \times \frac{12\ IN}{1FT}}{2.02} = 100.99 > 58.78 \Rightarrow we\ pick\ the\ bigger\ one$$

Problem Set 06

#Q3: Transition slenderness value, 4.71(E/Fy)^.5

6. Steel Column Analysis

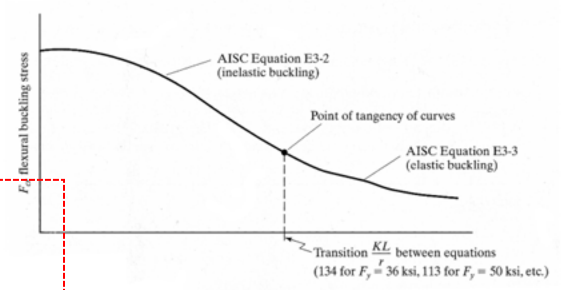
For the given axially loaded steel W-section, determine the maximum floor live load capacity, P LL. Assume the column is pinned top and bottom: K = 1.0, and there is no intermediate bracing. Use AISC-LRFD steel equations to determine phi Pn and the load. E = 29000 ksi.

DATASET: 1 -2- -3-

W-section	W8X31
Fy	36 KSI
Span A	32 FT
Span B	30 FT
Height L	17 FT
Floor Dead Load	39 PSF

Analysis of Steel Columns

Short columns
Fail by material crushing
Plastic behavior



Intermediate columns
Crush partially and then buckle
Inelastic behavior
Local buckling – flange or web
Flexural torsional buckling - twisting

$$slenderness = \frac{KL}{r}$$

short intermediate long

Long columns
Fail in Euler buckling
Elastic behavior

Transition Slenderness $4.71\sqrt{\frac{E}{F_y}}$

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$$\boldsymbol{Transition\ slenderness = 4.71\sqrt{\frac{E}{F_y}}} = 4.71\sqrt{\frac{29000}{36}} = \mathbf{134}$$

$$\mathbf{100.99 < 134 \Rightarrow}\ \boldsymbol{short}$$

Problem Set 06

#Q4: Euler stress, Fe

#Q5: Critical stress, Fcr

DATASET: 1 -2- -3-

W-section	W8X31
Fy	36 KSI
Span A	32 FT
Span B	30 FT
Height L	17 FT
Floor Dead Load	39 PSF

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{3.14^2(29000)}{(100.99)^2} = \mathbf{28.06\ KSI}$$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y = \left[0.658^{\frac{36}{28.06}}\right](36) = \mathbf{21.04\ KSI}$$

Analysis of Steel Columns - LRFD

Euler equation:

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

Short & Intermediate Columns:

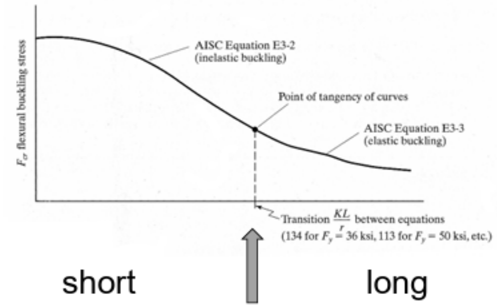
$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$$

Equation E3-2

Long Columns:

$$F_{cr} = 0.877 F_e$$

Equation E3-3



Transition Slenderness $4.71\sqrt{\frac{E}{F_y}}$

$$P_n = F_{cr} A_g$$

$$\phi_c P_n = \phi_c F_{cr} A_g$$

$$(\phi_c = 0.90)$$

University of Michigan, TCAUP Structures II Slide 10 of 20

Problem Set 06

#Q6: Nominal strength, Pn

#Q7: Factored nominal strength, phi Pn

DATASET: 1 | -2- | -3-

W-section	W8X31
Fy	36 KSI
Span A	32 FT
Span B	30 FT
Height L	17 FT
Floor Dead Load	39 PSF

$$\boldsymbol{A_g = 9.13\ IN^2}$$

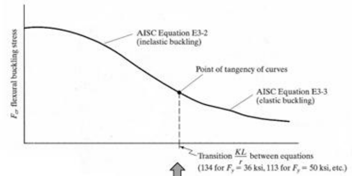
$$\boldsymbol{P_n = F_{cr}\ A_g} = 21.04\ (9.13) = \boldsymbol{192.09\ \text{KIPS}}$$

$$\boldsymbol{\varphi P_n = 192.09\ (0.9) = 172.88\ KIPS}$$

Analysis of Steel Columns - LRFD

Euler equation:

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$



Short & Intermediate Columns:

short | long

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$$

Equation E3-2

Transition Slenderness $4.71\sqrt{\frac{E}{F_y}}$

Long Columns:

$$F_{cr} = 0.877 F_e$$

Equation E3-3

$$P_n = F_{cr} A_g$$

$$\phi_c P_n = \phi_c F_{cr} A_g$$

$$(\phi_c = 0.90)$$

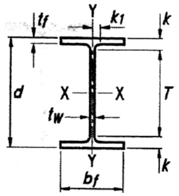


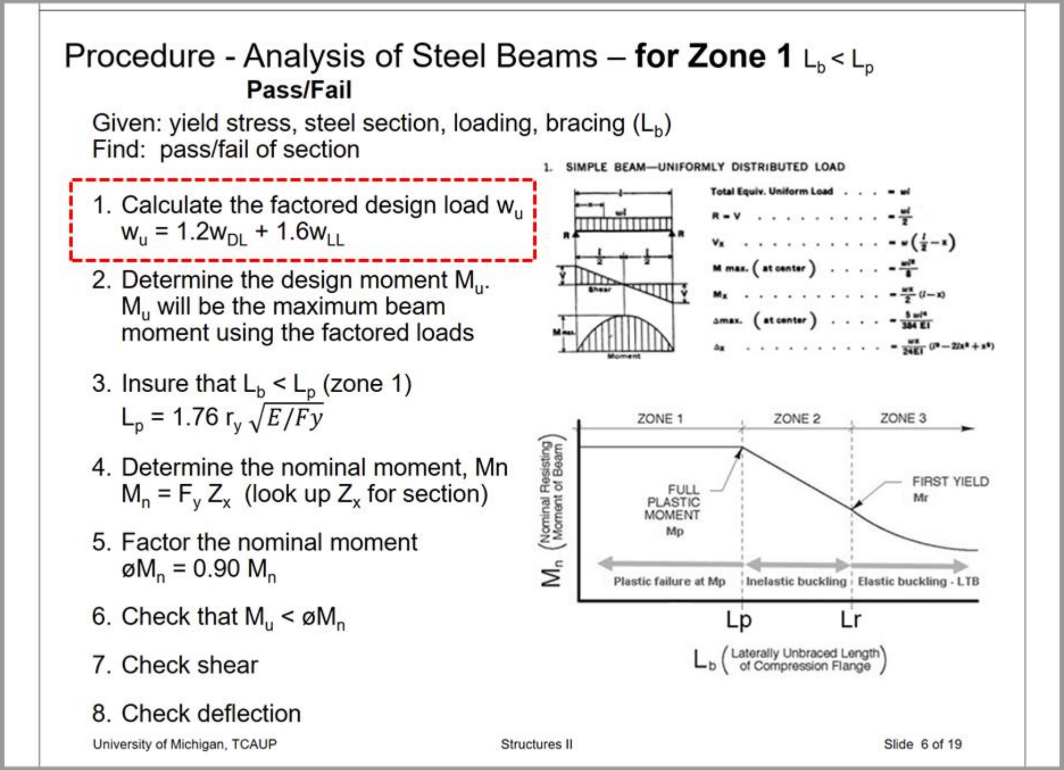
Table 1-1 (continued)
W-Shapes
Dimensions

Shape	Area, A	Depth, d		Web Thickness, t_w		Web $\frac{t_w}{2}$	Flange Width, b_f		Flange Thickness, t_f		Distance k k_{des}	Distance k k_{det}	k_1	T	Workable Gage
	in.^{2}	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W8×67	19.7	9.00	9	0.570	9/16	5/16	8.28	8 1/4	0.935	15/16	1.33	1 5/8	15/16	5 3/4	5 1/2
×58	17.1	8.75	8 3/4	0.510	1/2	1/4	8.22	8 1/4	0.810	13/16	1.20	1 1/2	7/8	↓	↓
×48	14.1	8.50	8 1/2	0.400	3/8	3/16	8.11	8 1/8	0.685	11/16	1.08	1 3/8	13/16	↓	↓
×40	11.7	8.25	8 1/4	0.360	3/8	3/16	8.07	8 1/8	0.560	9/16	0.954	1 1/4	13/16	↓	↓
×35	10.3	8.12	8 1/8	0.310	5/16	3/16	8.02	8	0.495	1/2	0.889	1 3/16	13/16	↓	↓
×31[f]	9.13	8.00	8	0.285	5/16	3/16	8.00	8	0.435	7/16	0.829	1 1/8	3/4	↓	↓
W8×28	8.25	8.06	8	0.285	5/16	3/16	6.54	6 1/2	0.465	7/16	0.859	15/16	5/8	6 1/8	4
×24	7.08	7.93	7 7/8	0.245	1/4	1/8	6.50	6 1/2	0.400	3/8	0.794	7/8	9/16	6 1/8	4
W8×21	6.16	8.28	8 1/4	0.250	1/4	1/8	5.27	5 1/4	0.400	3/8	0.700	7/8	9/16	6 1/2	2 3/4[g]

Problem Set 06

#Q8: UN-factored live load on column (actual total LL)
#Q9: Actual unfactored floor live load

DATASET: 1 -2- -3-	
W-section	W8X31
Fy	36 KSI
Span A	32 FT
Span B	30 FT
Height L	17 FT
Floor Dead Load	39 PSF



$$\boldsymbol{P_u = 1.2\,(DL) + 1.6\,(LL),\ \ (\text{max load})\ \ \rightarrow\ \ P_u = \varphi P_n}$$

$$172.88 = 1.2\,(37.44) + 1.6\,(LL)$$

$$\Rightarrow \boldsymbol{LL = 79.97\ KIPS}$$

$$\Rightarrow \boldsymbol{floor\ live\ load = \frac{LL}{Tributary\ Area}} = \frac{79.97\ * 1000}{960} = \boldsymbol{83.30\ PSF}$$

No lab for today ;)