ARCHITECTURE 324
Structures II

Recitation 09

Sections 04&05

Instructor
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GSI
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March 20, 2025

Office Hours

→ Office Hours

→ Day: Fridays, 12:00 PM - 1:00 PM

→ Location Options:

- In-person meetings: [2223B]
- Virtual meetings via Zoom

Please make sure to sign up at least 24 hours in advance to allow for proper scheduling via this link:

https://docs.google.com/forms/d/e/1FAIpQLSdOb4gAc6SoCdsMAZP4zKrn3ecPyGt6dwVahVcOD3EqXGG-oA/viewform?usp=dialog

If the slots are fully booked or if you have a time conflict, please email me directly to find an alternative time (arfazel@umich.edu)

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→ Summary

  → Intro to Concrete

  → Concrete Beams pt1

→ Problem Set

  → Problem set 08 (Concrete Beam Analysis)

→ Lab

  → Flexural Strain

Constituents of Concrete

Constituents of Concrete

- **Aggregate**
- Cement
- Water

Fine aggregate
(Sand)
≤ 1/4”

coarse aggregate
~ 3/8” (small)

coarse aggregate
~ 1/2” to 1” (medium)

coarse aggregate
~ 1.5” (large)

Photos by Emadrazo

Constituents of Concrete

- Aggregate
- **Cement**
- Water

Ingredients:

- Limestone
- Cement rock
- Clay
- Iron ore
- + (after firing and grinding)
- gypsum

Cement Types

- **Type 1**
  normal portland cement. Type 1 is a **general use** cement.
- **Type 2**
  is used for structures in water or soil containing **moderate amounts of sulfate**, or when heat build-up is a concern.
- **Type 3**
  high early strength. Used when high strength are desired at very early periods.
- **Type 4**
  low heat portland cement. Used where the amount and rate of heat generation must be kept to a minimum.
- **Type 5**
  Sulfate resistant portland cement. Used where water or soil is high in alkali.
- Types IA, IIA and IIIA are cements used to make **air-entrained** concrete.

Workability

Workability

Measured in inches of "slump" of a molded cone of fresh mix.

- range 1" to 4" with vibration
- 2" to 6" without vibration

Water/Cement Ratio

- range 0.4 to 0.7
- for strength: higher is weaker
- for workability: higher is more workable

Cement Content

- LBS per cubic yard
- range 400-800 lbs/yd^3
- dependent on aggregate
- increases cost

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Constituents of Concrete

W/C ratios - Strength

Concrete Strength and w/c Ratio:

Water-Cement Ratio (w/c)	Approximate Compressive Strength (psi)	Notes
0.8	2000	Fairly weak concrete, more water
0.50	—	Maximum for normal strength concrete
0.45	—	Maximum for high strength concrete
0.30 - 0.35	—	Minimum w/c ratio
0.3	Too stiff to handle	Requires superplasticizers

Reinforcing & Curing

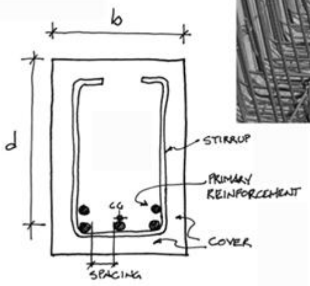
Reinforcing

Grade = Yield strength
- gr. 40 is 40 ksi
- gr. 60 is 60 ksi
- gr. 75 is 75 ksi

Size in 1/8 inch increments
- #4 is ½ inch dia.
- #6 is ¾ inch dia.

Deformation Patterns
- add to bond with concrete

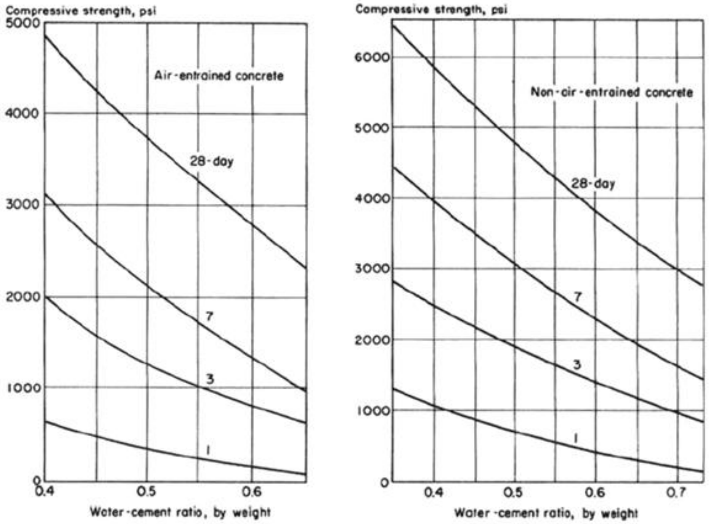
Minimum Spacing
- between bars (horizontal) the greatest of the 3 is the minimum
  - Bar diameter
  - 1”
  - 5/4 x max aggregate size
- between layers (vertical) 1”
- cover
  - 3” against soil
  - 1.5”-2” exterior
  - 3/4” interior



Reinforcement of Weidatalbrücke photo by Störfix

Curing

Strength increases with age. The “design” strength is 28 days.

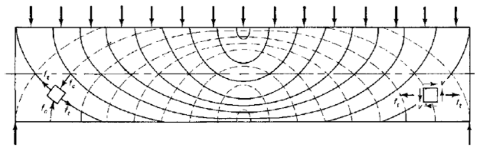


Portland Cement Association

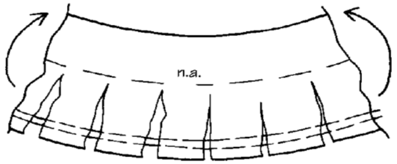
Flexure

Flexure

The stress trajectories in this simple beam, show principal tension as solid lines.

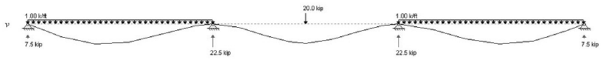


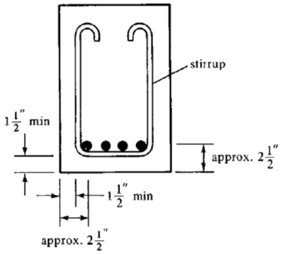
Reinforcement must be placed to resist these tensile forces

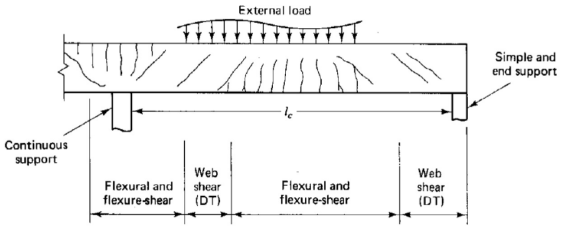


In beams continuous over supports, the stress reverses (negative moment).
In such areas, tensile steel is on top.

Shear reinforcement is provided by vertical or sloping stirrups.







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Ultimate Strength – (ACI 318-14)

Reduced Nominal Strength ≥ Factored Load Effects

ΦSn ≥ U

γ Factored Loads (see ACSE 7)

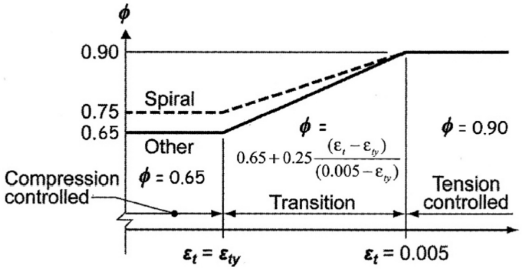
1) 1.4D
2) 1.2D + 1.6L + 0.5(Lr or S or R)
3) 1.2D + 1.6(Lr or S or R) + (1.0L or 0.5W)
4) 1.2D + 1.0W + 1.0L + 0.5(Lr or S or R)
5) 1.2D + 1.0E + 1.0L + 0.2S
6) 0.9D + 1.0W
7) 0.9D + 1.0E

D = service dead loads
L = service live load
Lr = service roof live load
S = snow loads
W = wind loads
R = rainwater loads
E = earthquake loads

Strength Reduction Factors, Φ

Mn	Flexural (ε > 0.005)	0.90
Vn	Shear	0.75
Pn	Compression (spiral)	0.75
Pn	Compression (other)	0.65
Bn	Bearing	0.65
Tn	Torsion	0.75
Nn	Tension	0.90
Combined stress		0.65 to 0.90

ACI 318 21.2.2



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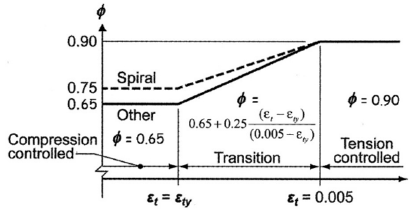
Failure modes

Failure Modes Based on As

- **No Reinforcing**
  - Less than As_{min}
  - Brittle failure
- **Reinforcing < balance (use this)**
  - Steel yields before concrete fails
  - Ductile failure
  - $\varepsilon_t \geq$ 0.005 for tension controlled
- **Reinforcing = balance**
  - Concrete fails just as steel yields
  - r_{bal}
- **Reinforcing > balance**
  - Concrete fails before steel yields
  - Low ductility
  - Sudden failure



$$\rho = \frac{A_s}{bd}$$

As,min:
greater of a and b

(a) $\frac{3\sqrt{f'_c}}{f_y} b_w d$

(b) $\frac{200}{f_y} b_w d$

$$\rho_{max} = 0.75\,\rho_b$$

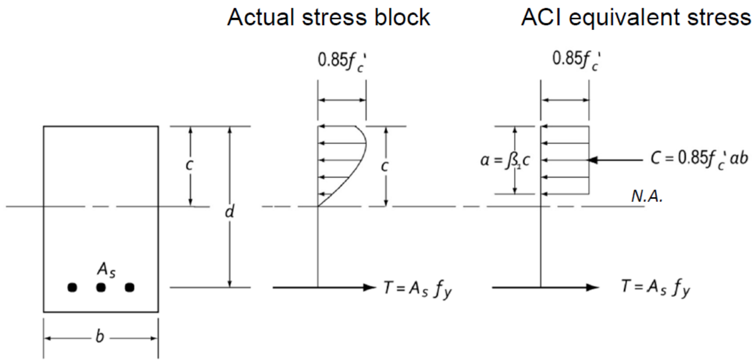
$$\rho_{bal} = \left(\frac{0.85\beta_1 f'_c}{f_y}\right)\left(\frac{87000}{87000 + f_y}\right)$$

$As > As_{max}$ SuddenDeath!!

ACI Stress Block

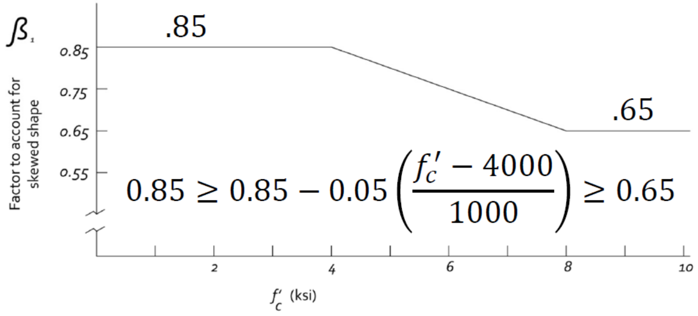
β_1 is a factor to account for the non-linear shape of the compression stress block.

$$a = \beta_1 c$$



psi

f'c	β1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65



$$0.85 \geq 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \geq 0.65$$

Problem Set 08

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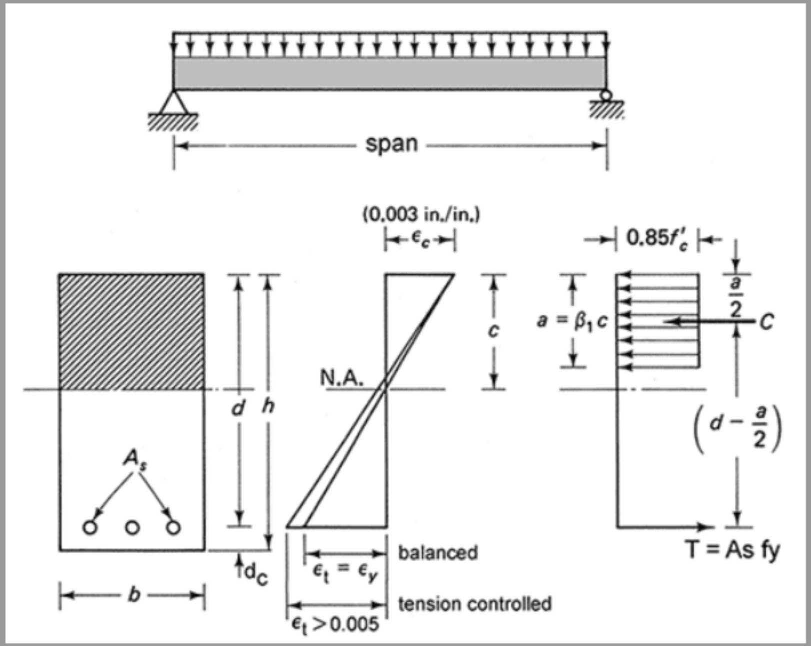
8. Concrete Beam Analysis

Analyze the given composite floor system. Using a transformed section, determine peak stress values in both concrete and steel.

DATASET: 1 -2- -3-

simple span	26 FT
section width, b	16 IN
section height, h	23 IN
max. aggrigate size	0.75 IN
bar size number	8
the number of bars	6
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	6500 PSI
steel yield strength, fy	60000 PSI



Problem Set 08

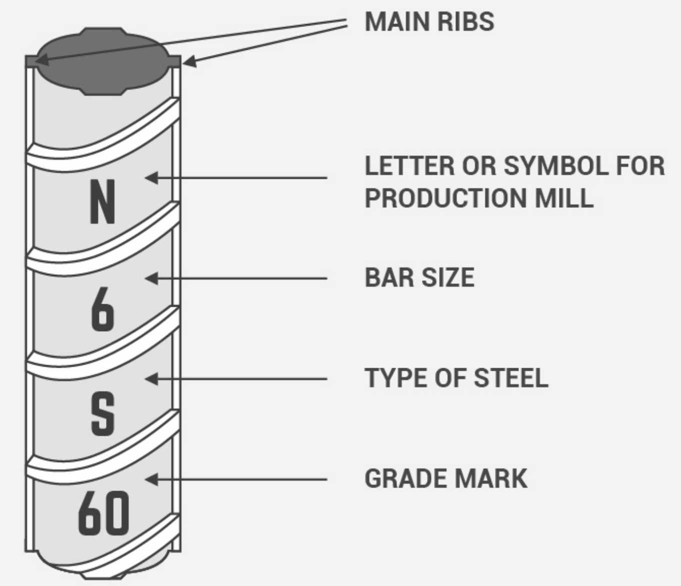
#Q1: flexural steel bar diameter, db
#Q2: stirrup bar diameter

DATASET: 1 -2- -3-	
simple span	26 FT
section width, b	16 IN
section height, h	23 IN
max. aggrigate size	0.75 IN
bar size number	8
the number of bars	6
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	6500 PSI
steel yield strength, fy	60000 PSI

$$D_b = Bar\ size\ \times 1/8$$
$$= 8 \times \frac{1}{8} = \mathbf{1}\ IN$$

$$D_{stirrup} = 4 \times \frac{1}{8} = \mathbf{0.5}\ IN$$

HOW TO READ REBAR MARKINGS



ASTM STANDARD INCH-POUND REINFORCING BARS

BAR SIZE DESIGNATION	NOMINAL DIMENSIONS AREA (in²)	WEIGHT (lb/ft)	DIAMETER (in.)
#3	0.11	0.376	0.375
#4	0.20	0.668	0.500
#5	0.31	1.043	0.625
#6	0.44	1.502	0.750
#7	0.60	2.044	0.875
#8	0.79	2.670	1.000
#9	1.00	3.400	1.128
#10	1.27	4.303	1.270
#11	1.56	5.313	1.410
#14	2.25	7.65	1.693
#18	4.00	13.60	2.257

The current A615 specification covers bar sizes #14 and #18 in Grade 60, and bar sizes #11, #14 and #18 in Grade 75. The current A706 specification also covers bar sizes #14 and #18. Bar sizes #9 through #18 are not included in the A996 specification.

Problem Set 08

#Q3: distance from lower beam edge to center of flexural steel, dc
#Q4: distance from top beam edge to center of flexural steel, d

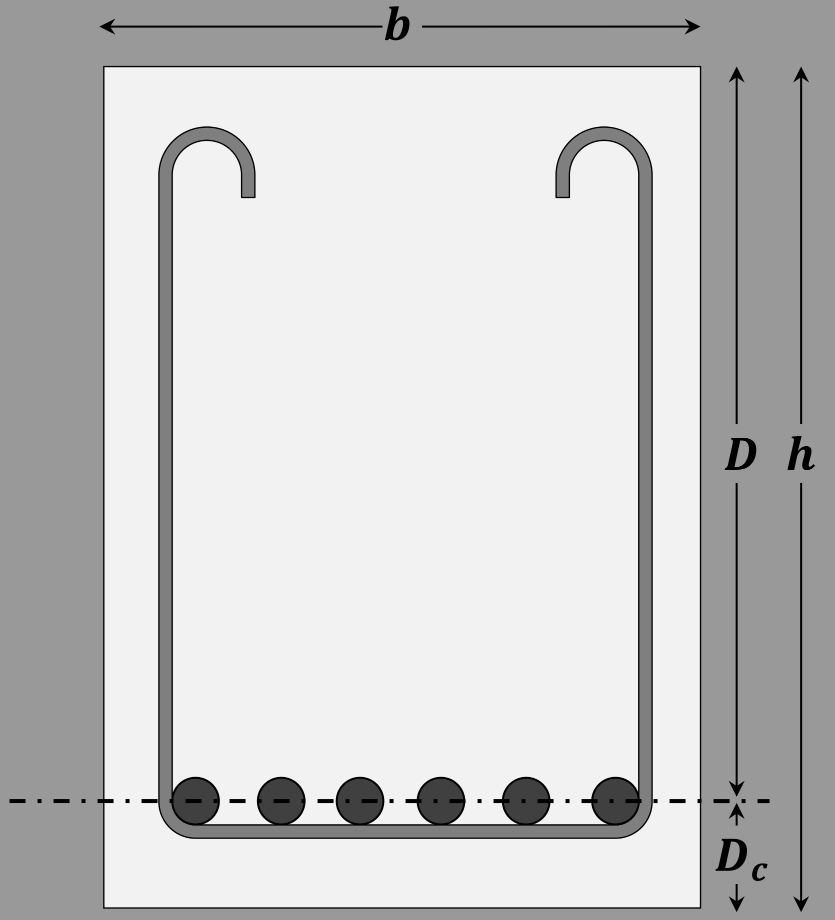
DATASET: 1 -2- -3-

simple span	26 FT
section width, b	16 IN
section height, h	23 IN
max. aggrigate size	0.75 IN
bar size number	8
the number of bars	6
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	6500 PSI
steel yield strength, fy	60000 PSI

$$\boldsymbol{D_c = concrete\ cover + D_{Stirrup} + \frac{D_b}{2}}$$

$$= 1.5 + 0.5 + \frac{1}{2} = \mathbf{2.5\ IN}$$

$$\boldsymbol{D = h - D_c} = 23 - 2.5 = \mathbf{20.5\ IN}$$

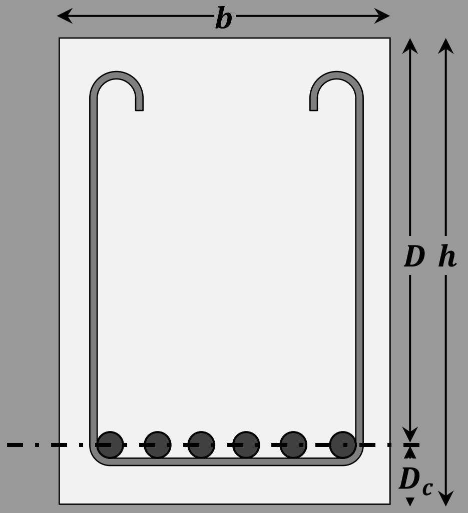


Problem Set 08

#Q5: minimum required area of steel, As,min (GREATER of the 2 criteria)
#Q6: actual area of flexural steel, As

DATASET: 1 -2- -3-

simple span	26 FT
section width, b	16 IN
section height, h	23 IN
max. aggrigate size	0.75 IN
bar size number	8
the number of bars	6
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	6500 PSI
steel yield strength, fy	60000 PSI

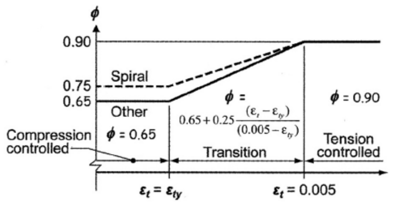


$$As_{min} = max \begin{cases} \dfrac{3\sqrt{f'_c}}{f_y}\, b.d = \dfrac{3\sqrt{6{,}500}}{60{,}000} \times 16 \times 20.5 = \mathbf{1.32}\ IN^2 \\ \dfrac{200}{F_y}\, b.d = \dfrac{200}{60{,}000} \times 16 \times 20.5 = 1.09\ IN^2 \end{cases}$$

$$As = 6\left(\frac{\pi D_b^2}{4}\right) = 6\,\frac{3.14\,(1^2)}{4} = \mathbf{4.74}\ IN^2$$

Failure Modes Based on As

- **No Reinforcing**
  - Less than As_{min}
  - Brittle failure
- **Reinforcing < balance (use this)**
  - Steel yields before concrete fails
  - Ductile failure
  - $\varepsilon_t \geq$ 0.005 for tension controlled
- **Reinforcing = balance**
  - Concrete fails just as steel yields
  - r_{bal}
- **Reinforcing > balance**
  - Concrete fails before steel yields
  - Low ductility
  - Sudden failure



$$\rho = \frac{A_s}{bd}$$

As,min: greater of a and b

(a) $\frac{3\sqrt{f'_c}}{f_y} b_w d$

(b) $\frac{200}{f_y} b_w d$

$$\rho_{max} = 0.75\,\rho_b$$

$$\rho_{bal} = \left(\frac{0.85\beta_1 f'_c}{f_y}\right)\left(\frac{87000}{87000 + f_y}\right)$$

As > As_{max} SuddenDeath!!

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Problem Set 08

#Q7: depth of concrete stress block, a
#Q8: factor beta_1
#Q9: distance to Neutral Axis from top of beam, c

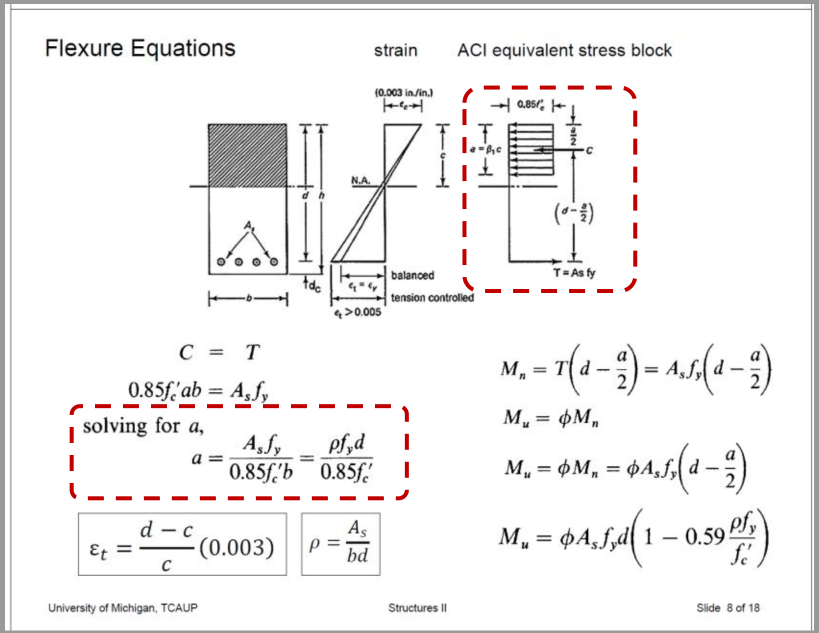
DATASET: 1 -2- -3-

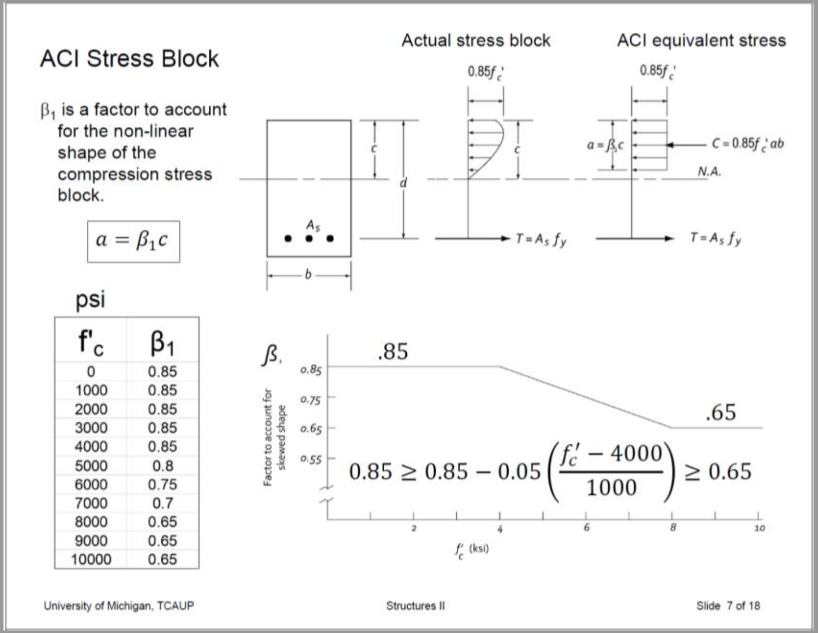
simple span	26 FT
section width, b	16 IN
section height, h	23 IN
max. aggrigate size	0.75 IN
bar size number	8
the number of bars	6
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	6500 PSI
steel yield strength, fy	60000 PSI

$$a = \frac{A_s f_y}{0.85\, f'_c\, b} = \frac{4.74\ (60{,}000)}{0.85\ (6500)\ (16)} = 3.21\ IN$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) = 0.725$$

$$c = \frac{a}{\beta_1} = \frac{3.21}{0.725} = 4.43\ IN$$





Problem Set 08

#Q10: strain in flexural steel, epsilon_t
#Q11: strength reduction factor, phi
#Q12: tensile force in the flexural steel, T

DATASET: 1 -2- -3-

simple span	26 FT
section width, b	16 IN
section height, h	23 IN
max. aggrigate size	0.75 IN
bar size number	8
the number of bars	6
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	6500 PSI
steel yield strength, fy	60000 PSI

$$\varepsilon_t = \frac{\boldsymbol{d-c}}{\boldsymbol{c}}(\mathbf{0.003}) = \frac{20.5-4.43}{4.43}(0.003) = \mathbf{0.0108}$$

$$0.0108 > 0.004 \ \rightarrow ok$$

$$0.0108 > 0.005 \ \rightarrow tension\ controlled$$

$$\Rightarrow \ \emptyset = \mathbf{0.9}$$

$$\boldsymbol{T = A_s f_y} = 4.74\ (60{,}000) = 284{,}400/1000 \ = \mathbf{284.4\ K}$$

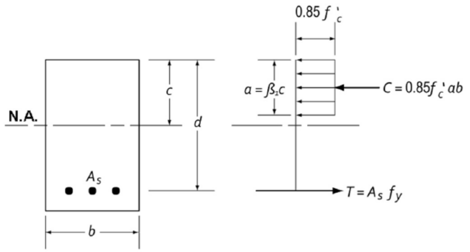
Rectangular Beam Analysis

Data:

- Section dimensions – b, h, (span)
- Steel area - As
- Material properties – f'c, fy

Required:

- Nominal Strength (of beam) Moment - Mn
- Required (by load) Design Moment – Mu
- Load capacity



As_{min}: greater of (a) and (b)

(a) $\frac{3\sqrt{f'_c}}{f_y} b_w d$

(b) $\frac{200}{f_y} b_w d$

$c = \frac{a}{\beta_1}$

$\varepsilon_t = \frac{d-c}{c} 0.003 \geq 0.005$

$a = \frac{A_s f_y}{0.85 f'_c b}$

$M_n = A_s f_y \left(d - \frac{a}{2}\right)$

$\varphi M_n \geq M_u$

$M_u = \frac{(1.2w_{DL} + 1.6w_{LL})l^2}{8}$

$1.6w_{LL} = \frac{M_u 8}{l^2} - 1.2w_{DL}$

1. Calculate d
2. Check As min
3. Calculate a
4. Determine c
5. Check that $\varepsilon_t \geq 0.005$ (tension controlled)
6. Find nominal moment, Mn
7. Calculate required moment, ϕ Mn ≥ Mu

   (if $\varepsilon_t \geq 0.005$ then $\phi = 0.9$)
8. Determine max. loading (or span)

Problem Set 08

#Q13: the nominal bending moment, Mn
#Q14: the factored bending resistance, phi Mn
#Q15: the factored design moment, Mu

DATASET: 1 -2- -3-	
simple span	26 FT
section width, b	16 IN
section height, h	23 IN
max. aggrigate size	0.75 IN
bar size number	8
the number of bars	6
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	6500 PSI
steel yield strength, fy	60000 PSI

$$\boldsymbol{M_n = A_s f_y\left(d - \frac{a}{2}\right)} = 4.74\ (60{,}000)\left(20.5 - \frac{3.21}{2}\right)$$

$$= 5{,}373{,}738\ /1000 = \boldsymbol{5373.738\ K - IN}$$

$$\emptyset\ \boldsymbol{M_n} = \boldsymbol{5373.738}\ (0.9) = 4836.36\ \text{K\_IN}$$

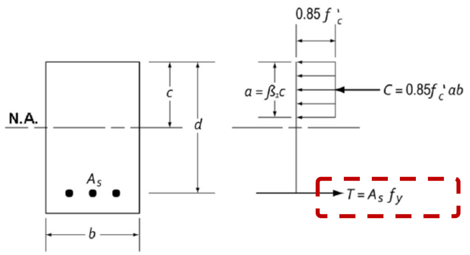
$$\emptyset\ \boldsymbol{M_n \geq M_u} \quad \rightarrow \boldsymbol{max:}\quad \emptyset\ \boldsymbol{M_n = M_u}$$

$$\boldsymbol{M_u} = 4836.36\ \text{K\_IN}\ \frac{1FT}{12\ IN} = \boldsymbol{403.03\ K\_FT}$$

Rectangular Beam Analysis

Data:
- Section dimensions – b, h, (span)
- Steel area - As
- Material properties – f'c, fy

Required:
- Nominal Strength (of beam) Moment - Mn
- Required (by load) Design Moment – Mu
- Load capacity



As_{min}: greater of (a) and (b)

(a) $\frac{3\sqrt{f'_c}}{f_y} b_w d$

(b) $\frac{200}{f_y} b_w d$

$c = \frac{a}{\beta_1}$

$\varepsilon_t = \frac{d-c}{c} 0.003 \geq 0.005$

$a = \frac{A_s f_y}{0.85 f'_c b}$

$M_n = A_s f_y \left(d - \frac{a}{2}\right)$

$\varphi M_n \geq M_u$

$M_u = \frac{(1.2w_{DL} + 1.6w_{LL})l^2}{8}$

$1.6w_{LL} = \frac{M_u 8}{l^2} - 1.2w_{DL}$

1. Calculate d
2. Check As min
3. Calculate a
4. Determine c
5. Check that $\varepsilon_t \geq 0.005$ (tension controlled)
6. Find nominal moment, Mn
7. Calculate required moment, ϕ Mn ≥ Mu

   (if $\varepsilon_t \geq 0.005$ then $\phi = 0.9$)
8. Determine max. loading (or span)

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Lab06

Structures II   Name 1 ____________

Arch 324   Name 2 ____________

Name 3 ____________

Flexural Strain

Description

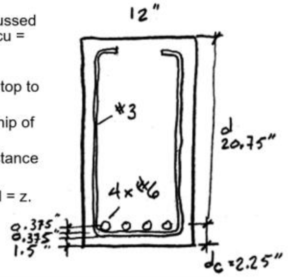
This project produces a graphic representation of the strain diagram for a tension controlled concrete beam.

Goals

To plot the compression and tension strain levels in a concrete beam
To graphically determine the neutral axis.
To draw the ACI "Whitney" stress block showing C and T forces.
To compare plotted and calculated results.

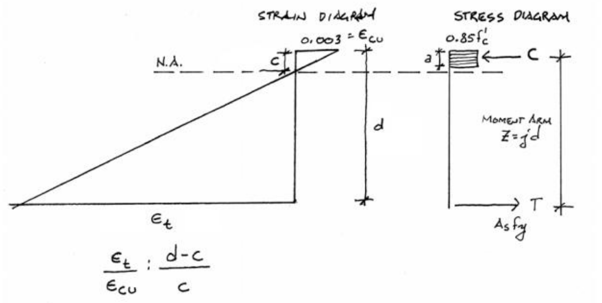
Procedure

1. For the tension controlled beam analysis discussed in lecture, construct the strain diagram with ϵ_{cu} = 0.003 and ϵ_t as calculated.
2. Use f'c = 6000 psi and fy = 60000 psi
3. Graphically determine the c distance from the top to the N.A on your diagram.
4. Make a second diagram to show the relationship of C & T forces to the strains.
5. Draw the ACI – Whitney stress block at "a" distance from the top.
6. Show the moment arm and calculate j using jd = z.



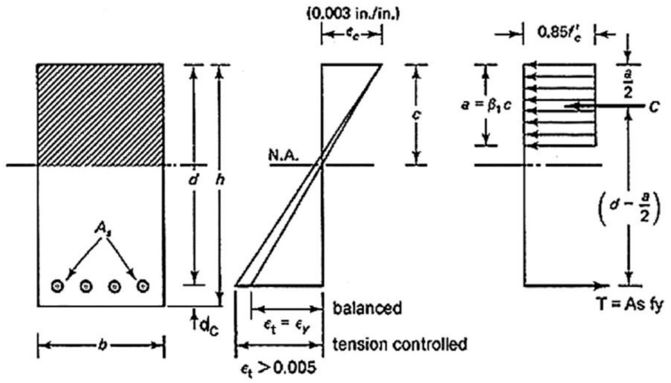
Due

Sunday, March 28



Flexure Equations

strain   ACI equivalent stress block



$$C = T$$

$$0.85f_c'ab = A_s f_y$$

solving for a,

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{\rho f_y d}{0.85 f_c'}$$

$$\varepsilon_t = \frac{d-c}{c}(0.003)$$

$$\rho = \frac{A_s}{bd}$$

$$M_n = T\left(d - \frac{a}{2}\right) = A_s f_y\left(d - \frac{a}{2}\right)$$

$$M_u = \phi M_n$$

$$M_u = \phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right)$$

$$M_u = \phi A_s f_y d\left(1 - 0.59\frac{\rho f_y}{f_c'}\right)$$

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Lab06

→ Group work instructions

Please form groups of 2 to 4 students.

Please do not forget to write all group members' names on both sheets.

Return the completed sheets to me at the end of the session.

Please ensure that you attend the recitation sessions.

If you are unable to attend a session, send me an email so that we can discuss how to proceed. *Email: arfazel@umich.edu*