ARCHITECTURE 324
Structures II

Recitation 04

Sections 04&05

Instructor
Peter von Buelow

GSI
Alireza Fazel
Feb 06, 2025

Office Hours

→ Office Hours

→ Day: Fridays, 12:00 PM - 1:00 PM

→ Location Options:

- In-person meetings: [2223B]
- Virtual meetings via Zoom

Please make sure to sign up at least 24 hours in advance to allow for proper scheduling via this link:

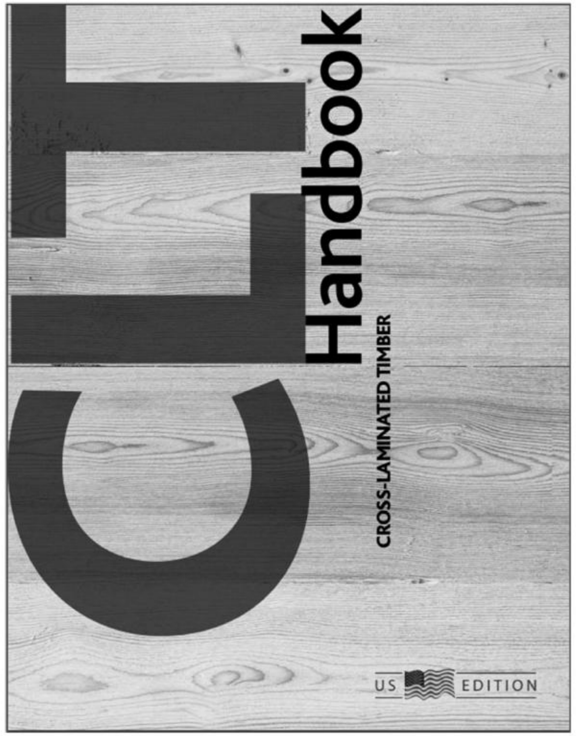
https://docs.google.com/forms/d/e/1FAIpQLSdOb4gAc6SoCdsMAZP4zKrn3ecPyGt6dwVahVcOD3EqXGG-oA/viewform?usp=dialog

If the slots are fully booked or if you have a time conflict, please email me directly to find an alternative time (arfazel@umich.edu)

Contents

CLT

Cross-Laminated Timber



Cross-Laminated Timber - Flexure Example

To find S = **I**/c = 2 **I**/h, first find **I** (or E **I** eff)

Calculation of section stiffness – E **I** eff

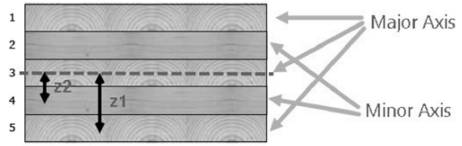


Figure 6
Cross-section of a 5-layer CLT panel

For a 5-layer, E1 panel:

h_i = Thickness of an individual layer = 1 3/8 in.
b = Design width = 12 in.

values for all layers

Major strength axis (parallel to grain)
$F_{b,0}$ = Bending strength = 1950 psi
E_0 = Modulus of elasticity = 1.7×10^6 psi

values from Table 1 PRG-320

Minor strength axis (perpendicular to grain)
$F_{b,90}$ = Bending strength = 500 psi
E_0 = Modulus of elasticity = 1.2×10^6 psi

$$EI_{eff} = \sum_{i=1}^{n} E_i \cdot b_i \cdot \frac{h_i^3}{12} + \sum_{i=1}^{n} E_i \cdot A_i \cdot z_i^2 \qquad [24]$$

Table 3
Parallel axis theorem calculations for EI_{eff}

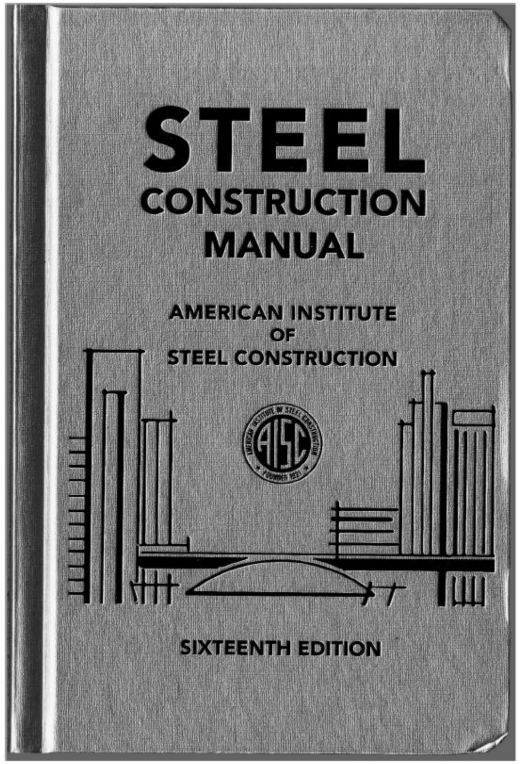
Layer	E (x 10⁶ psi)	z (in.)	Ebh³/12 (lb.-in.²)	EAz² (lb.-in.²)	Sum of Layer
1	1.7	2.75	4.4	212.1	216.5
2	1.2/30=0.04	1.375	0.1	1.2	1.4
3	1.7	0.0	4.4	0.0	4.4
4	0.04	1.375	0.1	1.2	1.4
5	1.7	2.75	4.4	212.1	216.5
				Total	**440**

EI_{eff}

Steel manual

Current AISC Manual

Specification and Manual for both ASD and LRFD



Nomenclature of steel shapes

Standard section shapes:

W – wide flange

S – American standard beam

C – American standard channel

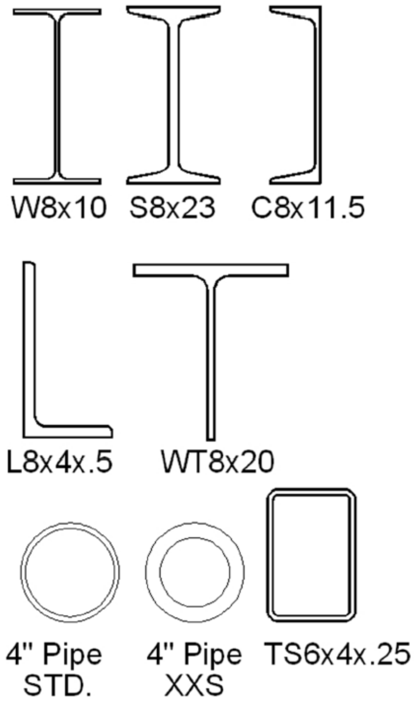
L – angle

WT or **ST** – structural T

STD, **XS** or **XXS** – Pipe

HSS – Hollow Structural Sections
Rectangular, Square, Round

LLBB , SLBB - Double Angles

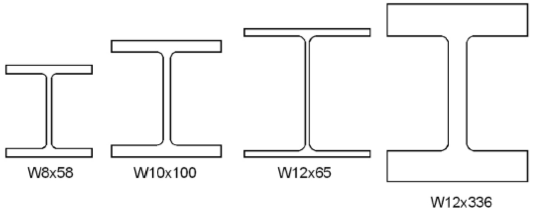


Steel manual

Steel W-sections for beams and columns

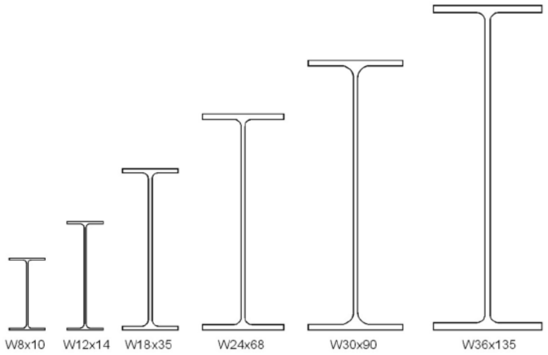
Columns:

Closer to square
Thicker web & flange



Beams:

Deeper sections
Flange thicker than web



LRFD Analysis

Load & Resistance Factored Design (LRFD)

- Use loads with safety factor γ
- Use forces with strength factor ϕ

$$P_{load} = \gamma \cdot P_{applied} \qquad P_{load} \le P_{resisting} \qquad P_{resisting} = \phi \cdot P_{material}$$

Design Strength $\quad P_u \le \phi\, P_n \quad$ Required (Nominal) Strength

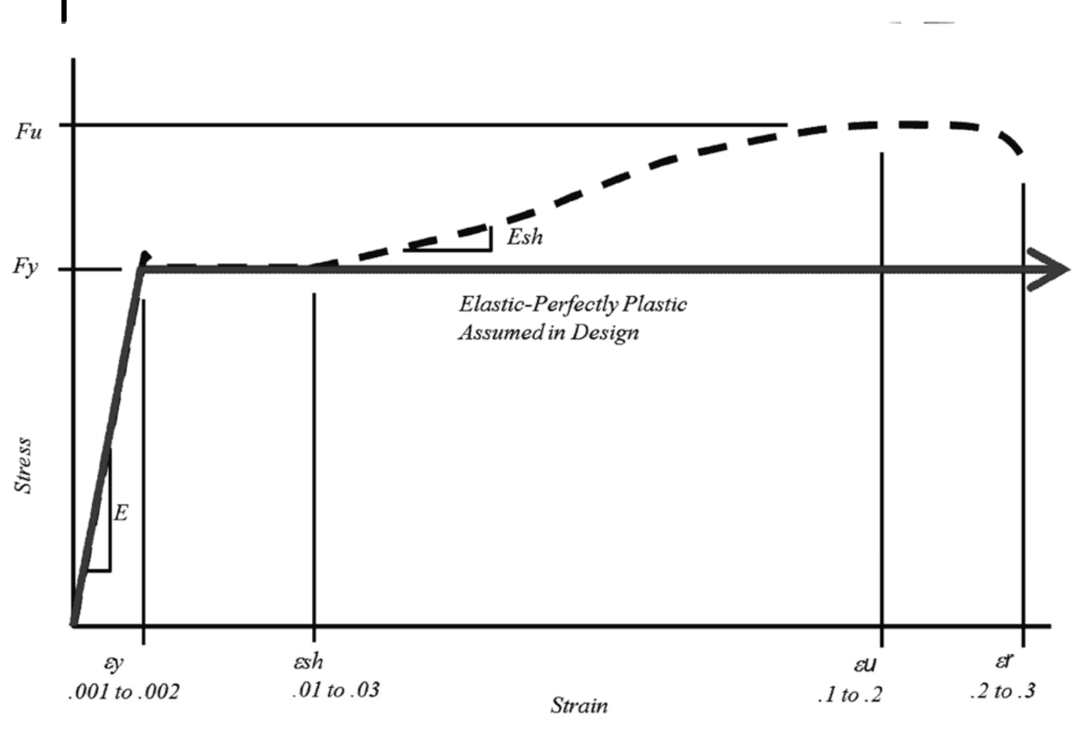
2.3 LOAD COMBINATIONS FOR STRENGTH DESIGN

1a. $1.4D$
2a. $1.2D + 1.6L + (0.5L_r \text{ or } 0.3S \text{ or } 0.5R)$
3a. $1.2D + (1.6L_r \text{ or } 1.0S \text{ or } 1.6R) + (L \text{ or } 0.5W)$
4a. $1.2D + 1.0(W \text{ or } W_T) + L + (0.5L_r \text{ or } 0.3S \text{ or } 0.5R)$
5a. $0.9D + 1.0(W \text{ or } W_T)$

ASCE 7 - 2022

Wood Column Design

Stress vs. Strain – AISC design curve



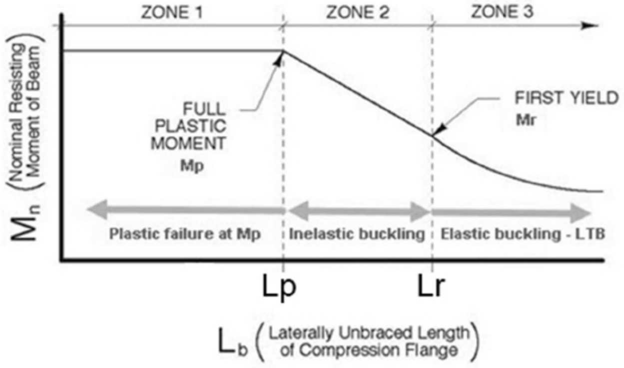
Steel Beams by LRFD

Yield Stress Values

- A36 Carbon Steel $F_y = 36$ ksi
- A992 High Strength $F_y = 50$ ksi

Elastic Analysis for Bending

- **Plastic Behavior (zone 1)**
  $M_n = M_p = F_y Z < 1.5 M_y$
  - Braced against LTB ($L_b < L_p$)
- **Inelastic Buckling "Decreased" (zone 2)**
  $M_n = C_b (M_p-(M_p-M_r) [(L_b-L_p)/(L_r-L_p)] < M_p$
  - $L_p < L_b < L_r$
- **Elastic Buckling "Decreased Further" (zone 3)**
  $M_{cr} = C_b * \pi/L_b \sqrt{(E*I_y*G*J + (\pi*E/L_b)^2 * I_y C_w)}$
  - $L_b > L_r$

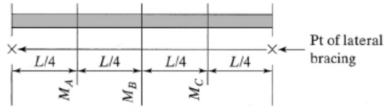


$L_p = 1.76\, r_y \sqrt{E/Fy}$

$M_p = F_y Z_x$

$M_r = 0.7 F_y S_x$

C_b is LTB modification factor



$$C_b = \frac{12.5\ \text{Mmax}}{2.5\ \text{Mmax} + 3\ \text{MA} + 4\text{MB} + 3\text{MC}}$$

Problem Set

No problem set for today :)

Lab

→ No Lab for today! :)

Tower project

Architecture 324
Structures II

Prof. Peter von Buelow
Winter 2025

Tower Project

Description

This project gives students the chance to apply concepts learned in column analysis to the design of a structural system that carries primarily a compression load – a tower. Work is to be done in groups of up to four people. The project is divided into 3 parts: 1) initial conceptual design, 2) design development and testing, 3) final analysis and documentation.

Goals

- to explore design parameters of geometry and material under compression.
- to develop a design of a compression member to meet the criteria below.
- to make some rough hand calculation to estimate the expected performance.
- to test the compression member and record the results.
- to document the results in a well organized and clear report format.

Criteria

- The tower is to be made of wood. Either linear wood (sticks) or wood panels (sheets) can be used. Glue can be used to connect the elements. Gusset plates at the joints are allowed and can also be glued. But **no steel pins** or fasteners may be used.
- Wood: **any species**. **maximum cross-sectional dimension = 1/4".**
- **NO** paper, mylar or plastic or string or dental floss.
- If a member is made by laminating multiple pieces together, the maximum cross-sectional dimension or thickness still cannot exceed 1/4".
- The **height of the tower = 48"**.
- The tower **must hold at least 50 lbs**.
- The entire tower **can weigh no more than 4 oz**.
- The top of the tower must be loadable. The weights will be stacked on top of the tower, but you may optionally use a loose piece of MDF or plywood as a tray under the weights. (It will not be counted in either weight or load)
- Towers will be graded on their low weight, high load-carrying capacity, and the load/weight ratio. The evaluation formula is:

  (4/weight in OZ) + (load in LBS/50) + (load LBS/weight OZ)x1.5
- The score will be normalized to a range of 50 to 100. It is used together with report scores to assess your project (a detailed evaluation form is given separately).

Procedure

1. Develop a structural concept for a tower meeting the above criteria.
2. Analyze the design concept with **either** hand calculations or a computer program (e.g. Dr. Frame)
3. Determine the capacity of the major members and of the overall tower (total capacity in LBS)
4. Estimate your expected score using the formula above.
5. Write the preliminary report.
6. Construct the structural model.
7. Test the model. 5-pound steel bars will be placed on top of the model, until the model fails. (bar size: 1 ½" x 2" x 5 13/16").
8. Produce final report documenting requirements and process. See also score sheet.

Due Dates
See Course Schedule

Scoring

Preliminary Report	40 pts
Testing	60 pts
Final Report	150 pts

Architecture 324
Winter 2025

ARCHITECTURAL STRUCTURES II (3)
Lecture and Assignment Schedule

DATE	TOPIC	Text Reading	PROBLEMS (due dates online)
JAN 8	Course Intro	Onouye, Schodek	
JAN 13	1 - Wood Properties	NDS	
JAN 15	2 - Wood Beam Analysis	Schodek 6.4.2	
JAN 17	Recitation **[1-Wood Beams]**		
		Topic Quiz 1	1. Wood Beam Analysis
JAN 20	**Martin Luther King Day **** No Class **** Martin Luther King Day **** No Class**		
JAN 22	3 - Wood Beam Design	Onouye 8	
JAN 24	Recitation		
		Topic Quiz 2	
JAN 27	4 - Wood Column Analysis	Onouye 9.1-9.2 & 9.4, Schodek 7.4.3	
JAN 29	5 - Wood Column Design	NDS	Tower Intro
JAN 31	Recitation **[2-Wood Columns]**		
		Topic Quiz 3	2. Wood Column Analysis
FEB 3	6 - Cross Laminated Timbers	CLT Handbook	
FEB 5	7 - Steel Properties	AISC, Onouye 8.7	
FEB 7	Recitation – Tower Project		
		Topic Quiz 4	
FEB 10	8 - Steel Beam Analysis	Schodek 6.4.3	
FEB 12	9 - Steel Beam Analysis	Schodek 6.4.3	
FEB 14	Recitation **[3-Steel Beams]**		**Prelim. Tower Report Due**
		Topic Quiz 5	3 Steel Beam Analysis
FEB 17	10 - Steel Beam Design	Schodek 6.4.3	
FEB 19	11 - Steel Column Analysis	Onouye 9.3, Schodek 7.4.4	
FEB 21	Recitation **[4-Steel Columns]**		
		Topic Quiz 6	4. Steel Beam Design
FEB 24	12 - Steel Column Design	Onouye 9.3, Schodek 7.4.4	
FEB 26	"Skyscrapers" David Macaulay video		
FEB 28	Recitation		
			5. Steel Column Analysis
MAR 3	**WINTER RECESS **** NO CLASS **** WINTER RECESS **** NO CLASS ******		
MAR 5	**WINTER RECESS **** NO CLASS **** WINTER RECESS **** NO CLASS ******		
MAR 7	**WINTER RECESS **** NO CLASS **** WINTER RECESS **** NO CLASS ******		
MAR 10	13 - Continuous Beams	I. Engel Ch. 17, Schodek 8	
MAR 12	14 - Gerber Beams	Schodek 8.4.4	
MAR 14	Recitation **[5-Continuous Beams]**		
		Topic Quiz 7	6. Three Moment Theorem
MAR 17	15 - Intro to Concrete – PCA video.		
MAR 19	16 - Concrete Beams	Schodek 6.4.4 – 6.4.6	
MAR 21	Recitation **[6-Stress vs Strain]**		
		Topic Quiz 8	
MAR 24	**Tower Testing **** Tower Testing **** Tower Testing **** Tower Testing ******		
MAR 26	17 - Concrete Beams	I. Engel Ch.15	
MAR 28	Recitation		
		Topic Quiz 9	7. Concrete Beam Analysis
MAR 31	18 - Concrete Beam Design		
APR 2	19 - Concrete Columns	Schodek 7.4.5	
APR 4	Recitation **[7-Concrete Reinforcing]**		
		Topic Quiz 10	8. Concrete Beam Design
APR 7	20 - Composite Sections		
APR 9	21 - Composite Sections		
APR 11	Recitation **[8-Composite Sections]**		
		Topic Quiz 11	9. Composite Sections
APR 14	22 - Masonry Intro.	TMS 402	
APR 16	23 - Masonry Walls	TMS 402	
APR 18	Recitation **[9-Lateral Stability]**		**Final Tower Report Due**
		Topic Quiz 12	10. Masonry Walls
APR 21	24 - Masonry Walls	TMS 402	

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Architecture 324
Structures II

Prof. Peter von Buelow
Winter 2025

Tower Project – Preliminary Report Requirements

Explanation – describe how the design was developed, the basis of the structural concept, and how the principles of column behavior influenced the design decisions.

Illustration – include diagrams/drawings that describe the structure in its entirety. **At least a horizontal cross-section and an elevation of the tower are required.** Dimensions are to be included and the member sizes labeled.

Analysis – the report should include the following:

- **Choose wood type and stress properties**. Either use values below for typical model grade Basswood or use values in the NDS or find test values online. Indicate in the report which values you choose.
- **Determine the cross-sectional area of each member**. Find the axial force P and the allowable stress F'c. The force P can be determined either by a hand calculated truss analysis or as a second order analysis in Dr. Frame or STAAD.Pro. The stress F'c should be found using the NDS equations for C_P and F'c. Other NDS stress adjustment factors (C_D, C_M, C_t, C_F and C_i) can be taken equal to 1.0. Size members based on the predicted load, P and the allowable stress F'c. Target (or predict) some total capacity load for the tower. A minimum of 50 LBS is required. Then size the members based on the force in each member.
- **Predict the total weight of the tower**. Provide a table with each member type showing, length, section and weight for each. Make an estimate of the weight added by glue joints and/or gusset plates. The total weight should be under 4 OZ.
- **Predict Capacity**. Predict the ultimate capacity in pounds that the entire tower can carry based on the actual cross-sections chosen. Produce a utilization table to show for each member type (e.g. main vertical, horizontal tie, diagonal brace) the utilization ratio fc/F'c based on the predicted total capacity load. This ratio should be below 1.0 for all members.
- **Calculate the buckling capacity of the tower as a whole**. This is done by treating the tower as one column loaded at the top, made up in cross section of multiple columns. Show the moment of inertia of the tower cross-section, and use it to calculate the critical buckling load using the Euler equation. An example of this calculation is given in the slides from the class lecture. The ultimate capacity is the lower of the two capacities (critical member or tower as a whole).

Note: If an excel spreadsheet is used to make calculations, show the equations being used for each cell or column in the table. If STAAD.Pro or Dr. Frame is used to do any of the above, include print-outs showing the applied loads and resulting member forces.

Format - Reports should be formatted for **8½ X 11** paper. 11X17 format reports will not be accepted. Once returned to you graded, **save the original copy of the preliminary report** for submission together with the Final Report.

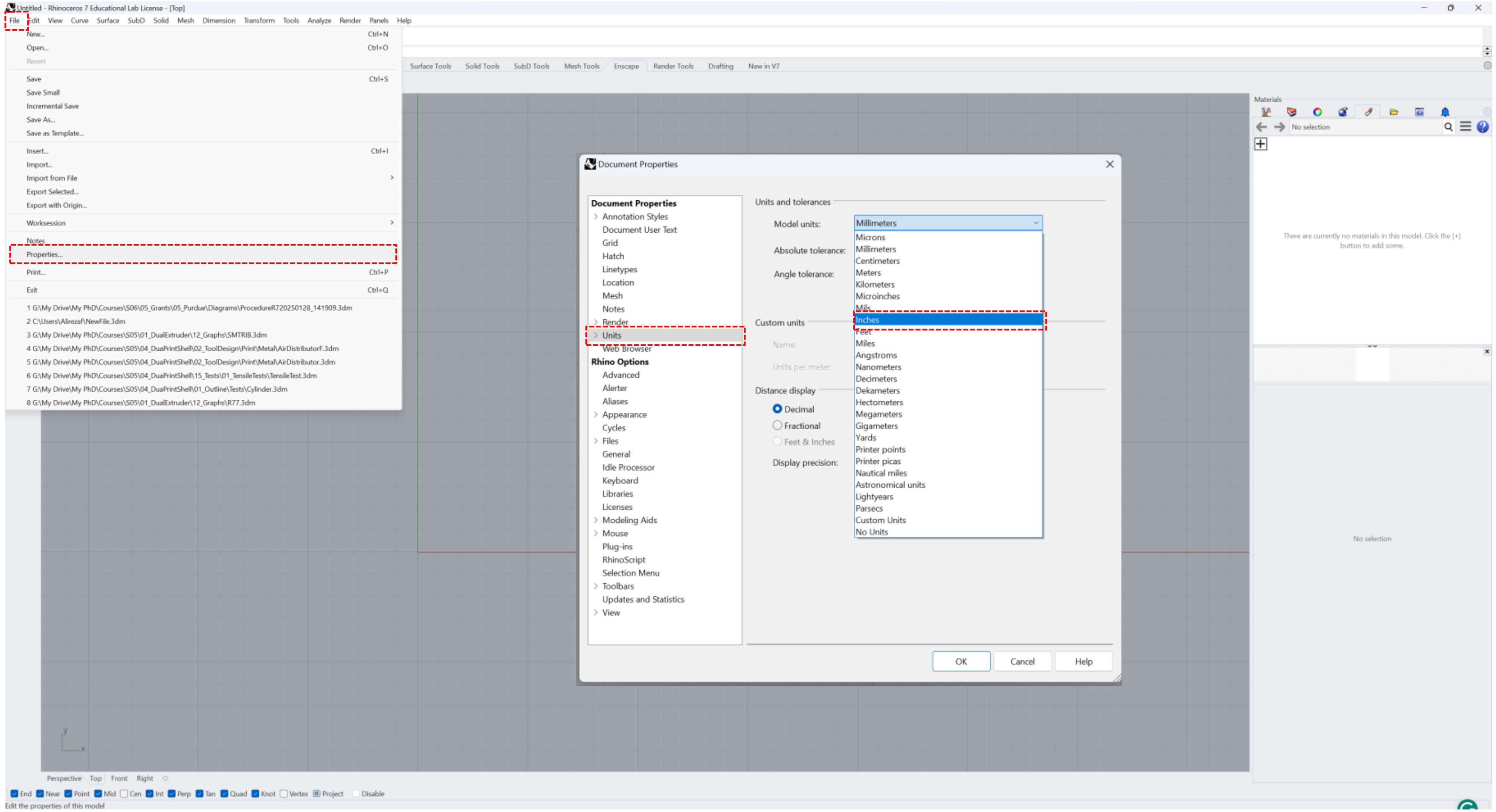
The report is a professional document. Text should be clear, grammatically correct, and language should be appropriate and professional. All calculations should be legible and clearly described – not just numbers or results, but with a clear description of what is being calculated included.

Properties of Basswood: (like in the Media Center)

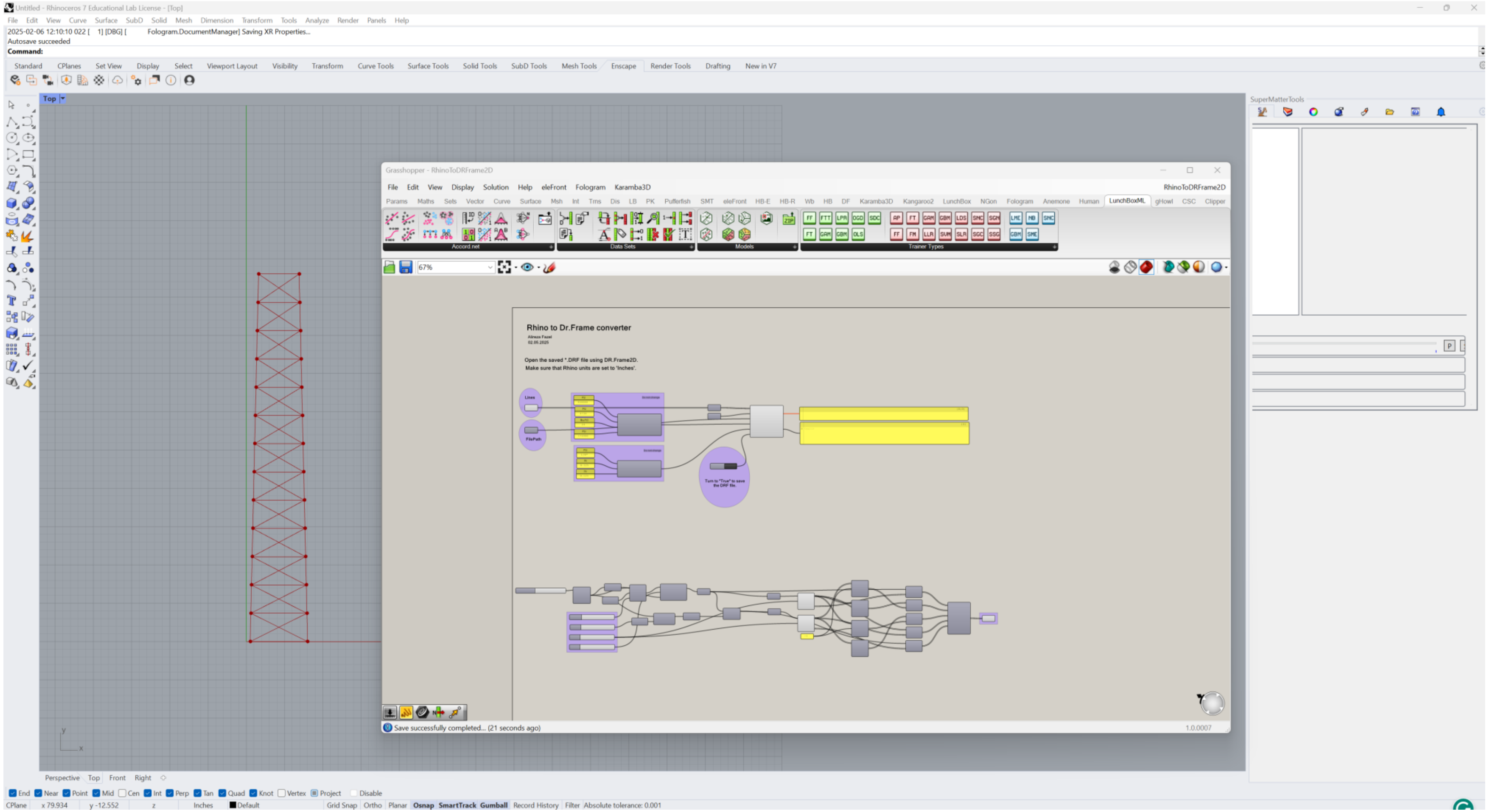
Density (oven dry)	29 pcf **
E (buckling)	1,650,000 psi **
F (Compression ‖ to grain)	4745 psi *
F (Compression ⊥ to grain)	377 psi *
F (Tension ‖ to grain)	4500 psi (estimate)
F (Tension ⊥ to grain)	348 psi *
F (Shear ‖ to grain)	986 psi *
F (Flexure)	5900 psi *

* from http://www.matweb.com/ ** tested by PvB (small pieces in compression)

Tower project

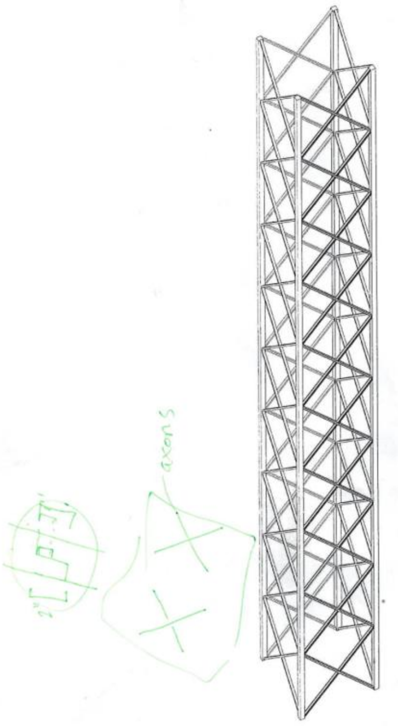


Tower project



Tower project

Tower Project: Preliminary Report



Paul Ligeti & Yinying Chen
"Tower Group"
Structures II
01/31/2016

Explanation:

We started by looking at precedents - both experimental and real-life implementations. Previous winning groups - Beam Me Up Scotty, Tower 2015, Take a Pisa My Heart - all seemed to use the same method: four long members supported by diagonal bracing and (except for Beam Me Up Scotty) horizontal bracing members. These members serve an important purpose: they shorten the effective buckling length of the four vertical members.

Radio towers use a similar method: vertical supporting masts, supported by diagonal bracing (and guy wires). While these do not experience compression outside of weight, they do experience a lot of wind - lateral forces - that can lead to buckling if not designed correctly.

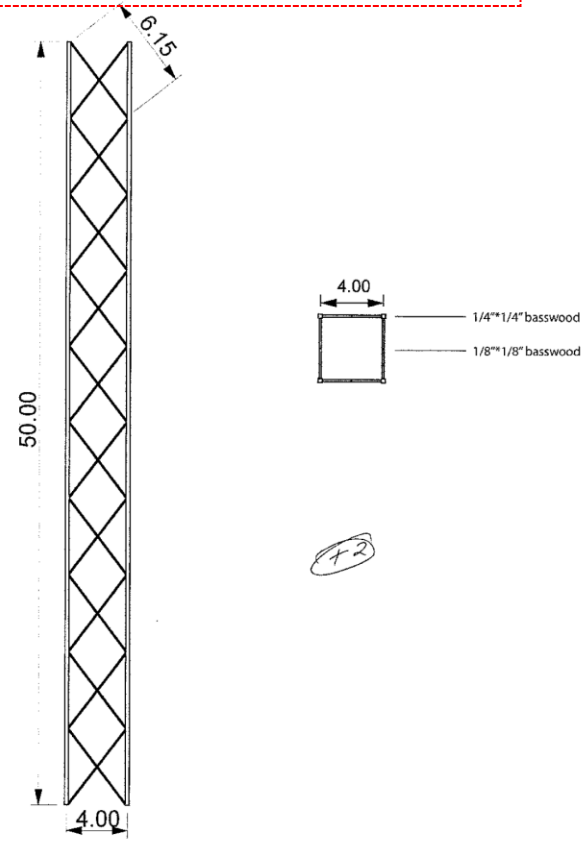
Finally, the optimization sheet given to us also shows similar structures with diagonal bracing. So it was clear that this was the way to go.

We chose to divide the tower into 10 "levels," one for every 5 inches. We believe (based on the equations) that this should effectively combat buckling, and in an ideal situation with perfect craft, the tower may be able to support up to **296** pounds!

+2

overall clarity +1

Illustration (Elevation/Cross-Section):



Tower project

Analysis:

Derivation of cross-sectional areas of each member:

We began our design by aiming at the capacity of 200 lb, *not* 50 lb. Using the Euler buckling equation, we can solve for a required slenderness ratio:

$KL/r=\sqrt{\pi^2 EA/P}=\sqrt{\pi^2 \cdot 1{,}650{,}000 \cdot 0.625/50}=142.67$

This is a very high slenderness ratio, and it is very attainable. Assuming K=1.0, we can get the radius of gyration, r:

$r=KL/142.67=1 \cdot 5/142.67=0.035$

Checking whether the member we are using is appropriate:

$r=h/\sqrt{12}$ $h=0.035 \cdot \sqrt{12}=0.12$ in.

.12 in is smaller than .71268 in (the r for a 1/4"*1/4" basswood column), which confirms the viability of our choice of the 1/4"*1/4" basswood column.

Comparing crushing:

$P=F \cdot A=4745 \cdot 0.0625=296.5625$ lbs

Compared to our load of 50lbs for each vertical member, this crushing capacity is larger. Thus, our colums should be able to hold up.

+2

Predicted weight estimate of entire tower:

Basswood: 20 lb/ft^3, divided by (12^3) = .0116 lb/in^3, multipled by 16 =

.185 oz/in^3

Total weight: [.185*({4 vertical members * 50in * .25in * .25 in}
+ {80 diagonal members * 6.4in * .125in * .125in})] + [.25 oz of glue)] =

4.04 oz. (can be adjusted) +1

Just keep an eye on it and you should be ok.

Predicted Capacity...

...Of vertical members:

Length = 50"/10 spaces = 5"; Area = .0625"

$r = \sqrt{I/A}$ = ... width/$\sqrt{12}$ <== Based off of JY recitation notes on wood columns
= .25/$\sqrt{12}$
= .07217

Vertical crushing: $P = F_c A$ = (4745 *psi*)(.0625in^2) = **296.56 lb**

Vertical buckling: P = $(\pi^2)AE/(KL/r)^2$
= $1017802.95/((1 \cdot 5)/.07217)^2$
= $1017802.95/(65.252)^2$
= **212.05 lb**

*This is spread out over 4 members, making the buckling capacity a whopping **848** lbs (of course, assuming perfect craft and materials, and no other factors). Thus, crushing will probably happen first.

...Of the tower as whole:

Moment of interia:

Using the subtractive method (subtracting void of "column" from 4" * 4" occupied area of "column":

$I = \Sigma I_{total} - I_{void}$

I = [((4in)(4in)^3)/12] - [2*((.25in)(3.5in)^3)/12] - [((3.5in)(4in)^3)/12]

I = .88 in^4

+2

Critical buckling load:

Assume K = 1

r = $\sqrt{(.88in^4/(4 \cdot .0625in^2))}$ = 1.876 in

KL/R = 1(50in)/1.876 = 26.6524

$F_{cr} = (\pi^2)(1{,}650{,}000psi)/(26.6542)^2$ = 22,921.95 psi

$P_{cr} = F_{cr}A$ = (22,921.95 psi)(4*.0625in^2) = **5,730.4875 lb**

Of course, this is assuming that the tower is one large column with perfect craft. One can only dream...

Tower project

Analysis (Determine the cross-sectional area of each member)

$Step\ 1: finding\ l_e$

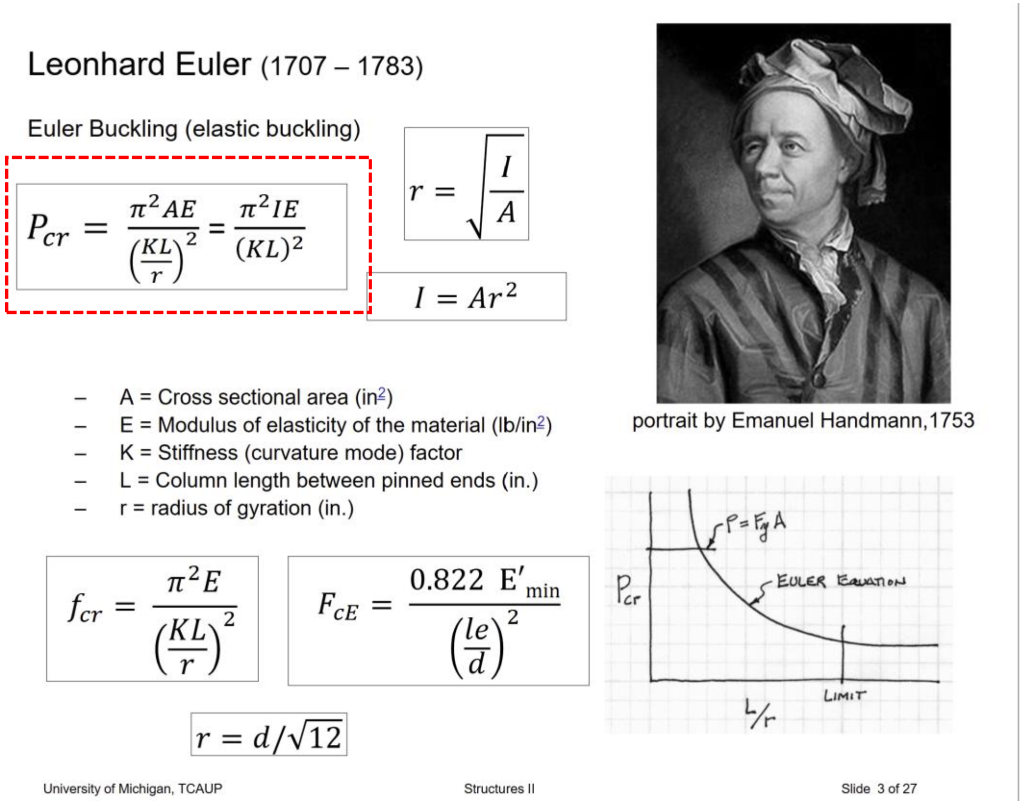
$This\ parameter\ depends\ on\ your\ design, if\ we\ devide\ the\ tower\ height\ into\ 10\ equal\ elements,\ we\ have:$

$$l_e = \frac{48}{10} = 4.8\ IN$$

$Step\ 2: assuming\ maximum\ weight$

$The\ minimum\ weight\ is\ considered\ 50\ LBS. So, you\ can\ choose\ any\ number\ higher\ than\ this, for\ now\ we\ assume\ 100LBS, and\ if\ we\ have\ a\ rectangular\ tower, each\ column\ should\ carry \frac{1}{4} of\ the\ load.$

$$P = \frac{100}{4} = 25\ LBS$$



Tower project

Analysis (Determine the cross-sectional area of each member)

Step 3: finding the radious of gyration

now, using Euler equation, we can calculate the designed radius of gyration, in this example, we assume we are using 1/4" basswood for the columns.

$E_{min} = 1{,}650{,}000\ psi$

$A = (0.25)^2 = 0.0625\ IN^2$

$K = 1$

$$\boldsymbol{P_{cr} = \frac{\pi^2 AE}{\left(\frac{KL}{r}\right)^2} \quad \Rightarrow \quad \frac{KL}{r} = \sqrt{\frac{\pi^2 AE}{P_{cr}}}}$$

$$\Rightarrow \frac{\boldsymbol{L}}{\boldsymbol{r}} = \sqrt{\frac{\pi^2(0.0625)1650000}{25}} = \mathbf{201.77}$$

$$\Rightarrow \boldsymbol{r} = \frac{L}{201.77} = \frac{4.8}{201.77} = \mathbf{0.02378}$$

$$\boldsymbol{r} = \frac{\boldsymbol{d}}{\sqrt{\mathbf{12}}} \Rightarrow d = r\sqrt{12} \quad \rightarrow \boldsymbol{d} = 0.02378\sqrt{12} = \mathbf{0.0824\ IN}$$

Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

$$P_{cr} = \frac{\pi^2 AE}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 IE}{(KL)^2}$$

$$r = \sqrt{\frac{I}{A}}$$

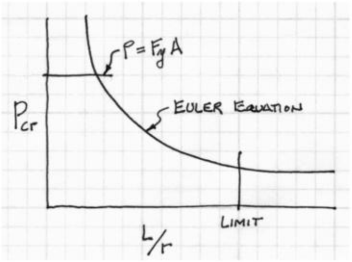
$$I = Ar^2$$

portrait by Emanuel Handmann,1753

- A = Cross sectional area (in2)
- E = Modulus of elasticity of the material (lb/in2)
- K = Stiffness (curvature mode) factor
- L = Column length between pinned ends (in.)
- r = radius of gyration (in.)

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

$$F_{cE} = \frac{0.822\ E'_{min}}{\left(\frac{le}{d}\right)^2}$$



$$r = d/\sqrt{12}$$

University of Michigan, TCAUP Structures II Slide 3 of 27

Tower project

Analysis (Determine the cross-sectional area of each member)

$\boldsymbol{Step\ 4: comparing\ radius\ of\ gyration\ r, (or\ d)}$

$now, whith\ the\ assumed\ loading\ condition, we\ found$
$r\ and\ , so\ we\ can\ compare\ them\ with\ the\ actual\ numbers$

$d_c = 0.0824\ IN : what\ we\ found\ under\ 25\ LBS\ load$

$d_d = 0.25\ IN : what\ we\ selected$

$\boldsymbol{d_d > d_c \quad \rightarrow \quad Pass}$

Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

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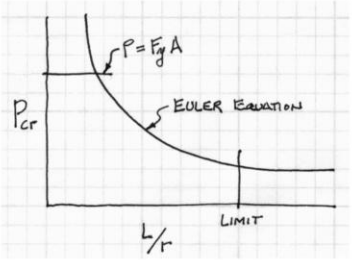


$$r = d/\sqrt{12}$$

University of Michigan, TCAUP

Structures II

Slide 3 of 27

Analysis (Determine the cross-sectional area of each member)

Step 5: comparing crushing

now, whith the assumed loading condition, we found r and , so we can compare them with the actual numbers

$f_c = 4745\ psi$

$$\boldsymbol{f_c = \frac{P}{A}} \rightarrow \boldsymbol{P = f_c A} = 4745\ (0.0625) = 296.5625\ LBS$$

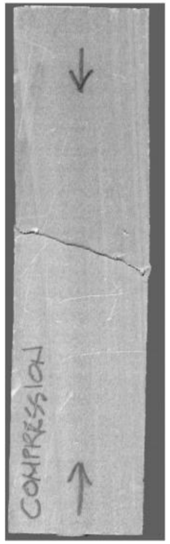
The maximum crushing load is 296.56 LBS, and we are applying 25 LBS, so → ***pass***

Failure Mode - Strength

Short Columns – fail by crushing

Analysis	Design
$f_c = \frac{P}{A} \leq F'_c$	$A = \frac{P}{F'_c}$

- f_c = Actual compressive stress
- A = Cross-sectional area of column (in[2])
- P = Load on the column
- F'_c = Allowable compressive stress per codes



Properties of Basswood: (like in the Media Center)

Density (oven dry)	29 pcf **
E (buckling)	1,650,000 psi **
F (Compression ∥ to grain)	4745 psi *
F (Compression ⊥ to grain)	377 psi *
F (Tension ∥ to grain)	4500 psi (estimate)
F (Tension ⊥ to grain)	348 psi *
F (Shear ∥ to grain)	986 psi *
F (Flexure)	5900 psi *

* from http://www.matweb.com/ ** tested by PvB (small pieces in compression)

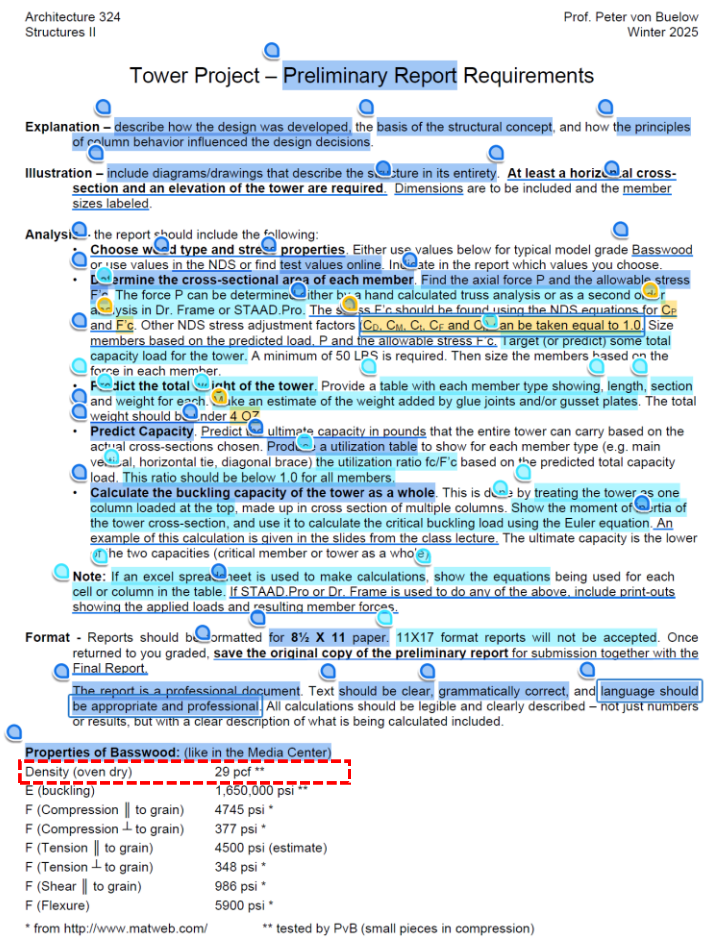
Tower project

Analysis (Predict the total weight of the tower)

$$\boldsymbol{Total\ weight} = \sum V_i \times Density + W_{Glue}$$

$\boldsymbol{Note}: density\ is\ geven\ in\ PCF\ and\ you\ need\ to\ convert\ it\ to\ oz./IN^3$

Density (oz/in^3)=Density (pcf)×0.009259 oz/in^3



Architecture 324
Structures II

Prof. Peter von Buelow
Winter 2025

Tower Project – Preliminary Report Requirements

Explanation – describe how the design was developed, the basis of the structural concept, and how the principles of column behavior influenced the design decisions.

Illustration – include diagrams/drawings that describe the structure in its entirety. **At least a horizontal cross-section and an elevation of the tower are required.** Dimensions are to be included and the member sizes labeled.

Analysis - the report should include the following:

- **Choose wood type and stress properties**. Either use values below for typical model grade Basswood or use values in the NDS or find test values online. Indicate in the report which values you choose.
- **Determine the cross-sectional area of each member**. Find the axial force P and the allowable stress F'c. The force P can be determined either by a hand calculated truss analysis or as a second order analysis in Dr. Frame or STAAD.Pro. The stress F'c should be found using the NDS equations for C_P and F'c. Other NDS stress adjustment factors (C_D, C_M, C_t, C_F and C_i) can be taken equal to 1.0. Size members based on the predicted load, P and the allowable stress F'c. Target (or predict) some total capacity load for the tower. A minimum of 50 LBS is required. Then size the members based on the force in each member.
- **Predict the total weight of the tower**. Provide a table with each member type showing, length, section and weight for each. Make an estimate of the weight added by glue joints and/or gusset plates. The total weight should be under 4 OZ.
- **Predict Capacity**. Predict the ultimate capacity in pounds that the entire tower can carry based on the actual cross-sections chosen. Produce a utilization table to show for each member type (e.g. main vertical, horizontal tie, diagonal brace) the utilization ratio fc/F'c based on the predicted total capacity load. This ratio should be below 1.0 for all members.
- **Calculate the buckling capacity of the tower as a whole**. This is done by treating the tower as one column loaded at the top, made up in cross section of multiple columns. Show the moment of inertia of the tower cross-section, and use it to calculate the critical buckling load using the Euler equation. An example of this calculation is given in the slides from the class lecture. The ultimate capacity is the lower of the two capacities (critical member or tower as a whole).

Note: If an excel spreadsheet is used to make calculations, show the equations being used for each cell or column in the table. If STAAD.Pro or Dr. Frame is used to do any of the above, include print-outs showing the applied loads and resulting member forces.

Format - Reports should be formatted for **8½ X 11** paper. 11X17 format reports will not be accepted. Once returned to you graded, **save the original copy of the preliminary report** for submission together with the Final Report.

The report is a professional document. Text should be clear, grammatically correct, and language should be appropriate and professional. All calculations should be legible and clearly described – not just numbers or results, but with a clear description of what is being calculated included.

Properties of Basswood: (like in the Media Center)

Density (oven dry)	29 pcf **
E (buckling)	1,650,000 psi **
F (Compression ‖ to grain)	4745 psi *
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F (Flexure)	5900 psi *

* from http://www.matweb.com/ ** tested by PvB (small pieces in compression)

Analysis (Predict Capacity.)

this is similar to what we did in determining the cross-sectional area of each member, you should create a table in excel with appropriate labeling format and predict the maximumcapacity

Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

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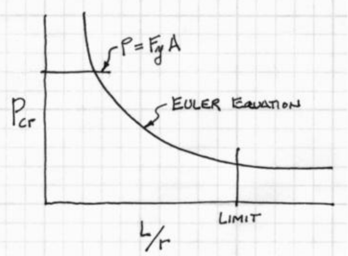

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Analysis (Calculate the buckling capacity of the tower as a whole.)

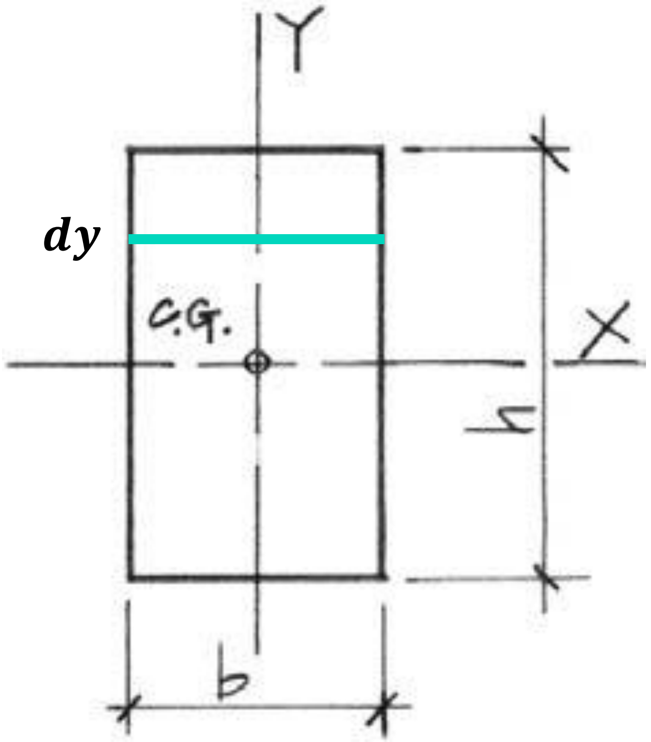
Find Moment of Inertia

$$I_x = \int y^2 dA = \int_{-\frac{h}{2}}^{h/2} y^2 dA = \int_{-\frac{h}{2}}^{h/2} by^2 dy$$

$$= b\left[\frac{y^3}{3}\right]_{-h/2}^{h/2} = \frac{b}{3}\left[\frac{h^3}{8} - (-\frac{h^3}{8})\right] = \frac{b}{3}(\frac{2h^3}{8})$$

$$I_x = \frac{bh^3}{12}$$

$$I_y = \frac{hb^3}{12}$$



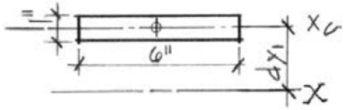
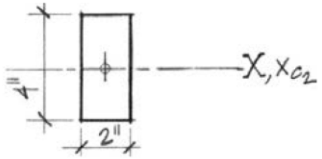
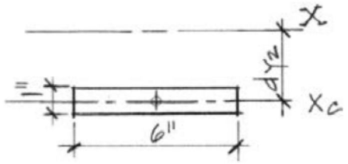
Tower project

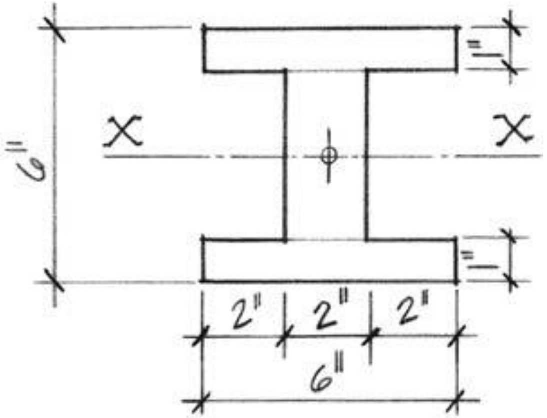
Analysis (Calculate the buckling capacity of the tower as a whole.)

Example

Determine the moment of inertia I_x about the centroidal x axis.

$$\boldsymbol{I_x = \sum I_{x_c} + \sum A.y^2} = 11.67 + 75 = \boldsymbol{86.67\ IN^4}$$

Component	I_{x_c} (in.^{4})	A (in.^{2})	$y\ (in)$	$Ay^2\ (in^4)$
	$\frac{bh^3}{12} = \frac{6''(1'')^3}{12} = 0.5$ in.^{4}	6 in.^{2}	2.5″	37.5 in.^{4}
	$\frac{2''(4'')^3}{12} = 10.67$ in.^{4}	8 in.^{2}	0″	0
	$= 0.5$ in.^{4}	6 in.^{2}	–2.5″	37.5 in.^{4}
	$\Sigma I_{x_c} = 11.67$ in.^{4}			$\Sigma A d_y^2 = 75$ in.^{4}

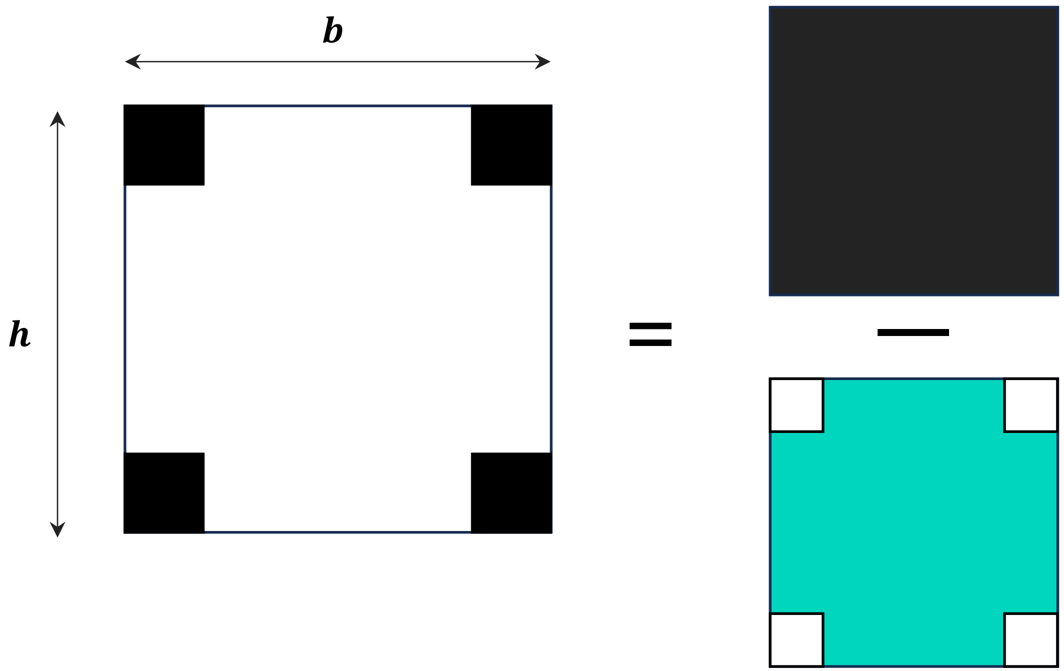


Tower project

Analysis (Calculate the buckling capacity of the tower as a whole.)

$\boldsymbol{Step\ 1: finding\ I\ for\ the\ whole\ column}$

$$\boldsymbol{Total\ I} = \sum I_{total} - I_{Void}$$



Tower project

Analysis (Calculate the buckling capacity of the tower as a whole.)

$\boldsymbol{Step\ 2: using\ the\ Euler\ equation, you\ can\ calculate\ the\ buckling\ load\ and\ compare\ to\ what\ you\ designed}$

$\boldsymbol{If}$:

$\boldsymbol{P_{cr}\ (whole\ column) > P_{cr}(design)\ \rightarrow\ \ Pass}$

$\boldsymbol{otherwise}$:

$\rightarrow\ \ \boldsymbol{Fail}$

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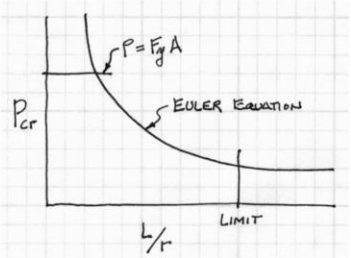

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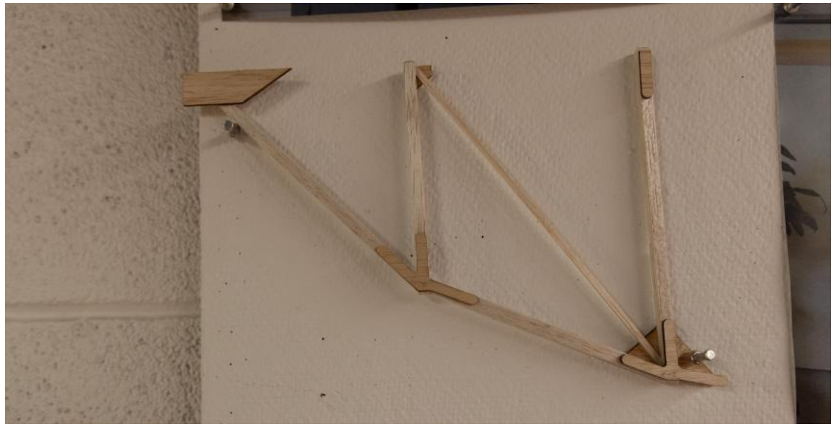
Tower project



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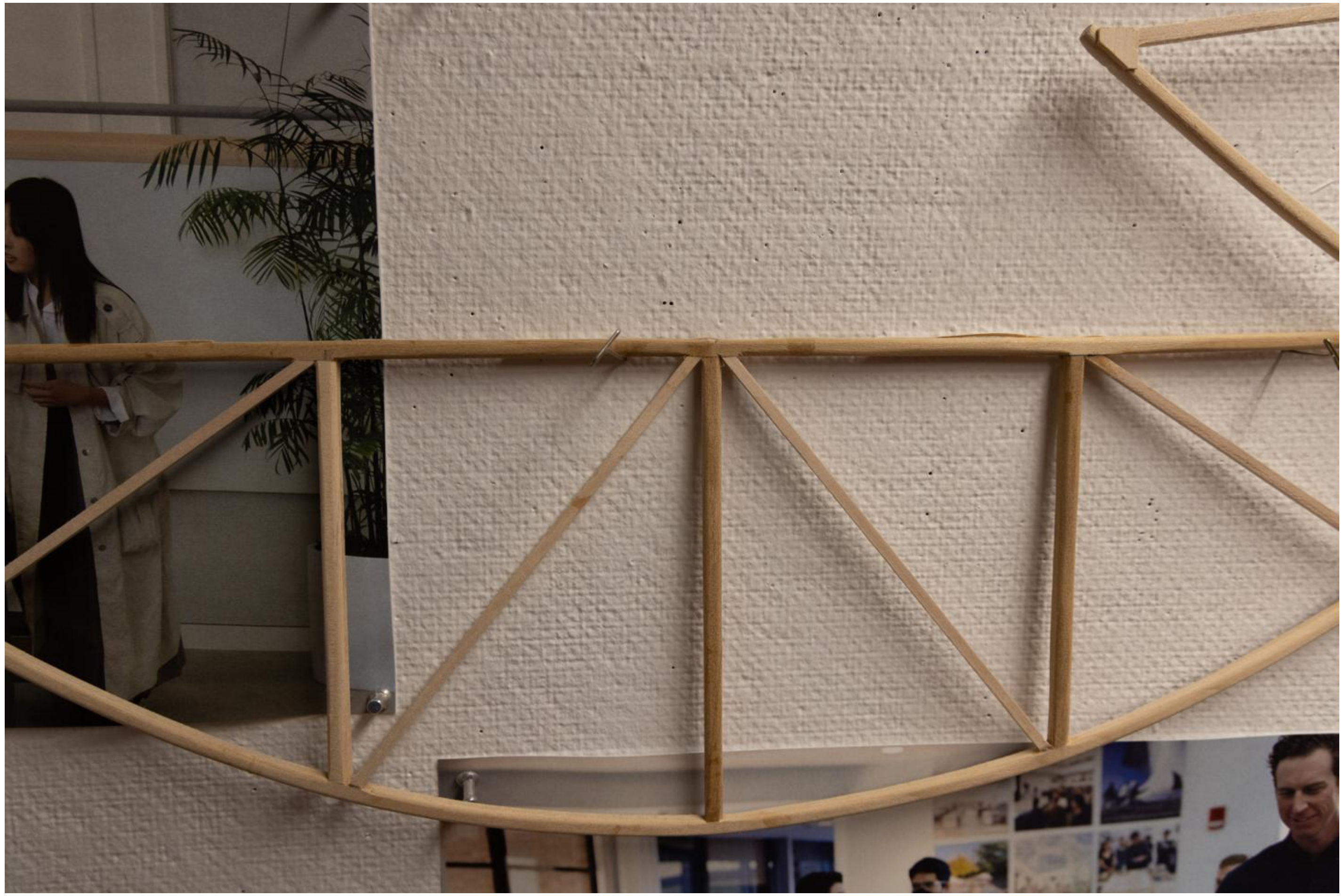

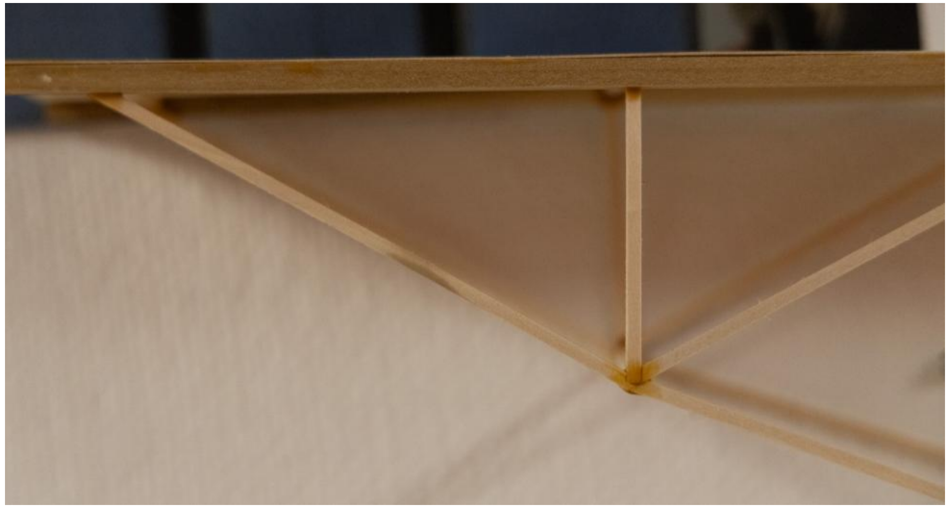

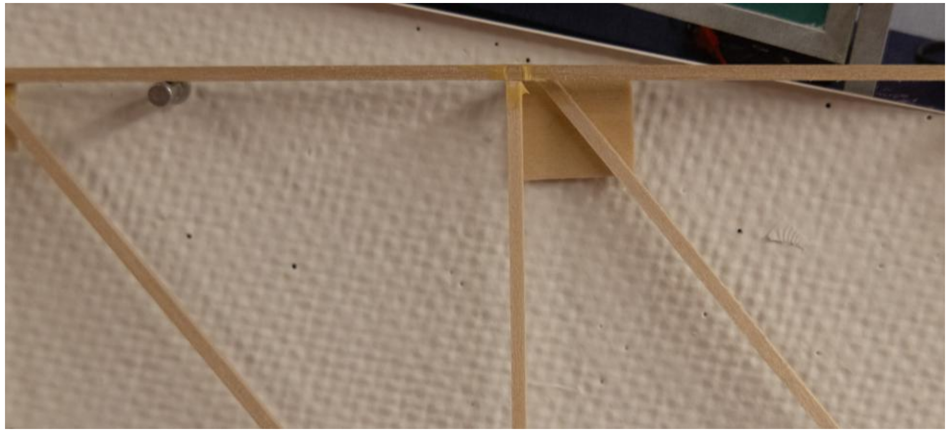

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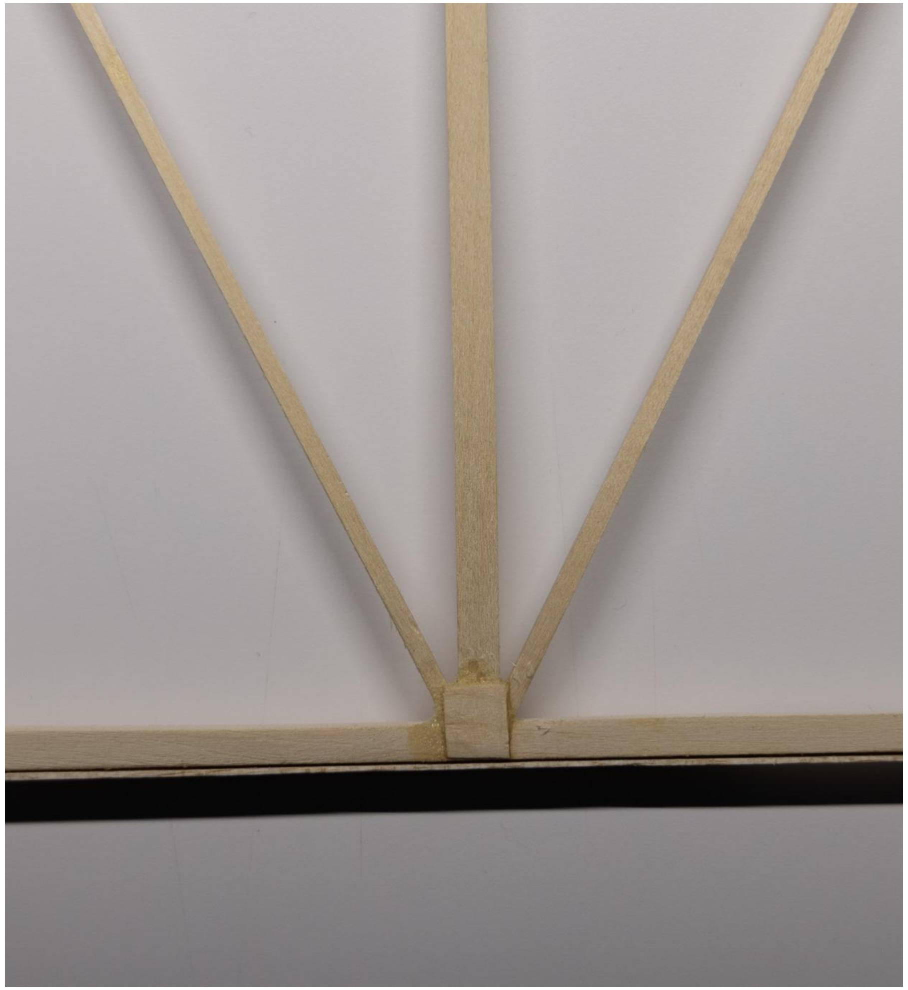

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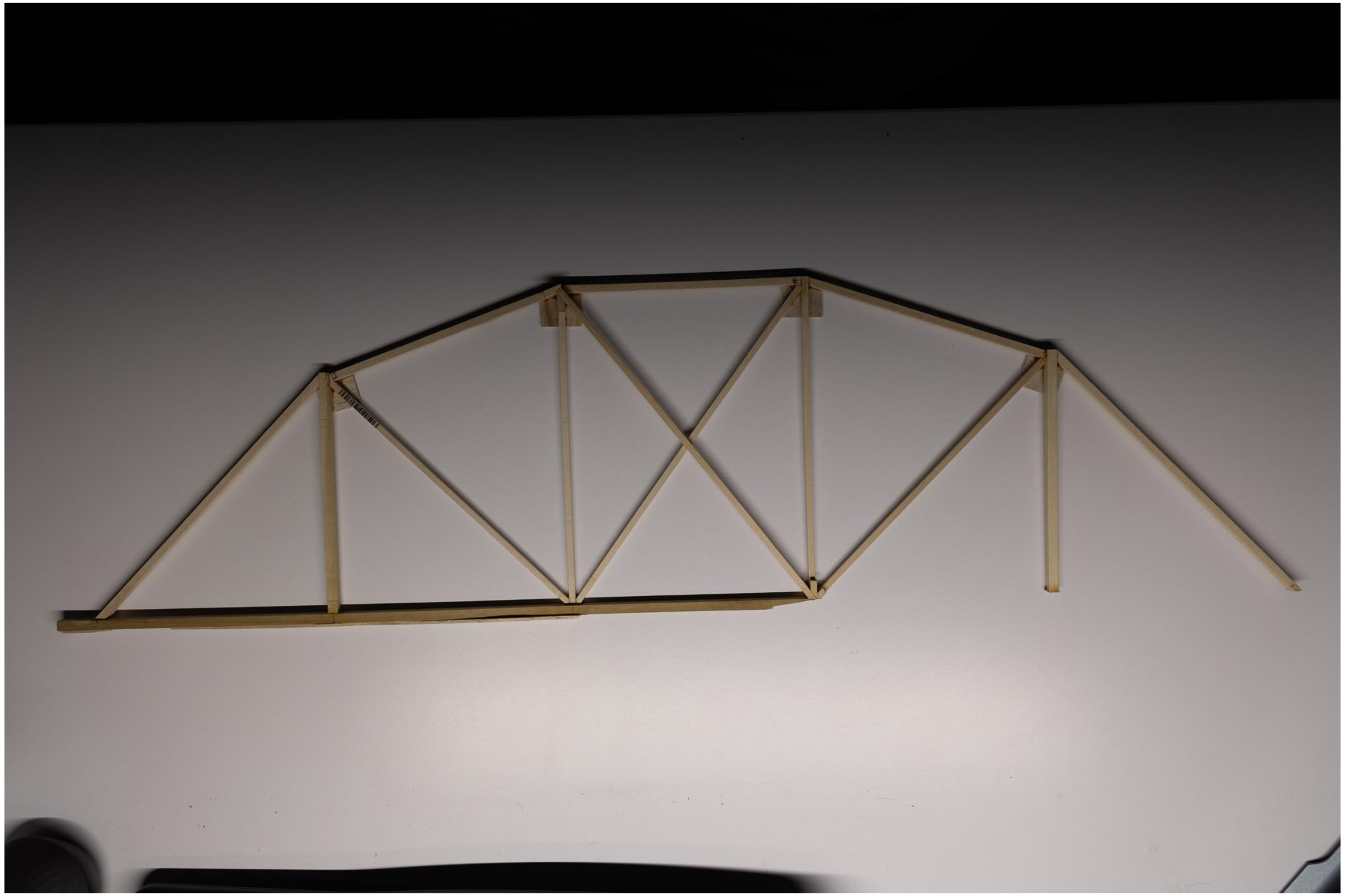

Tower project

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