ARCHITECTURE 324
Structures II

Recitation 01

Sections 04&05

Instructor
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GSI
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Office Hours

→ Office Hours

→ Day: Fridays, 12:00 PM - 1:00 PM

→ Location Options:

- In-person meetings: [2223B]
- Virtual meetings via Zoom

Please make sure to sign up at least 24 hours in advance to allow for proper scheduling via this link:

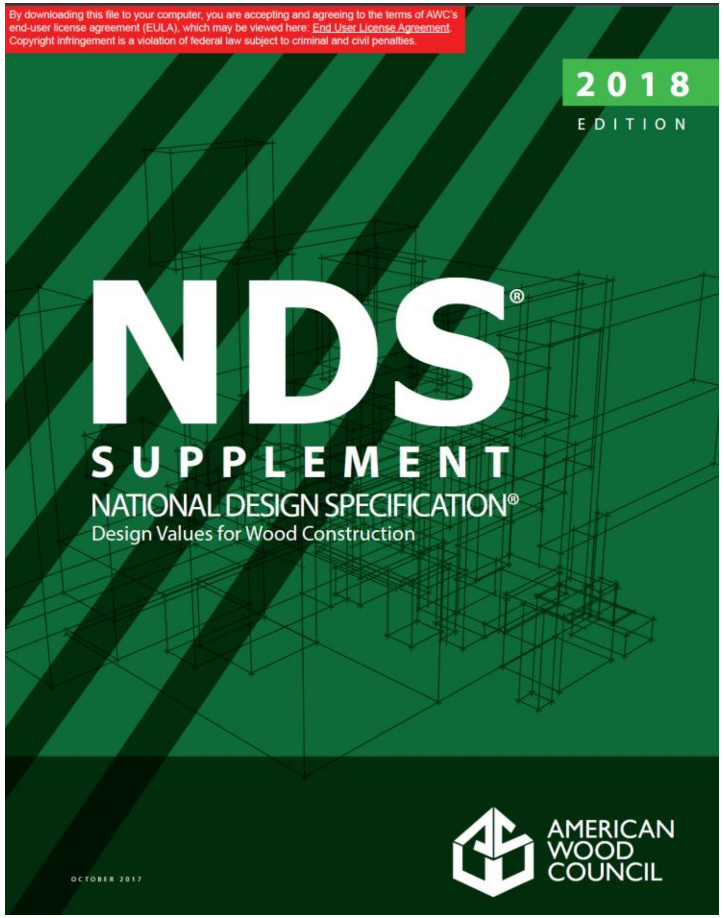
https://docs.google.com/forms/d/e/1FAIpQLSdOb4gAc6SoCdsMAZP4zKrn3ecPyGt6dwVahVcOD3EqXGG-oA/viewform?usp=dialog

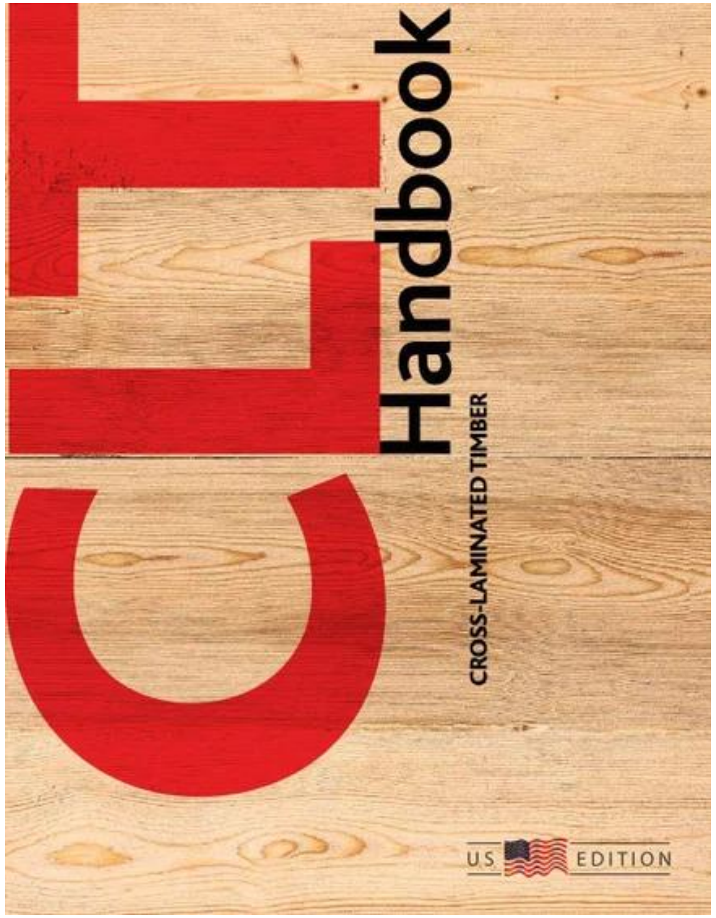
If the slots are fully booked or if you have a time conflict, please email me directly to find an alternative time (arfazel@umich.edu)

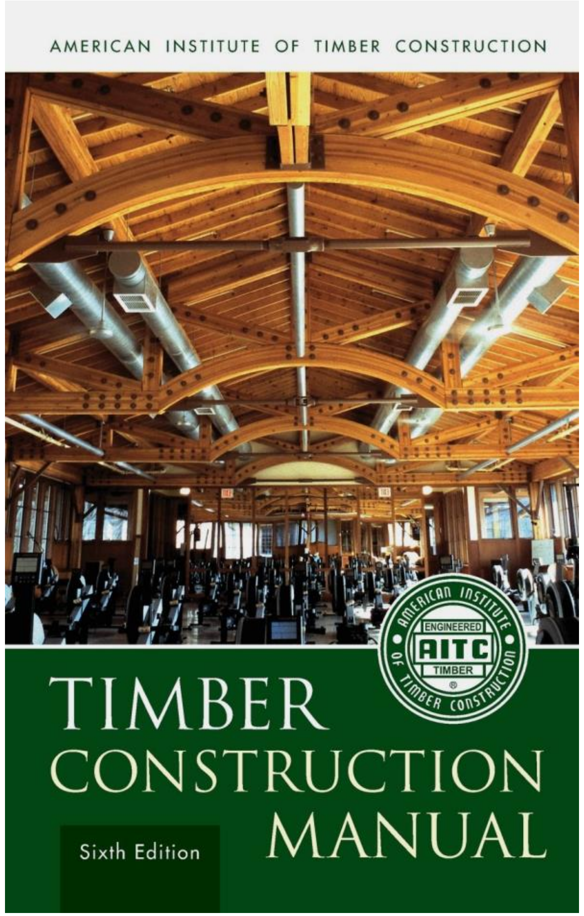
Contents

Wood types

Standards & Manuals







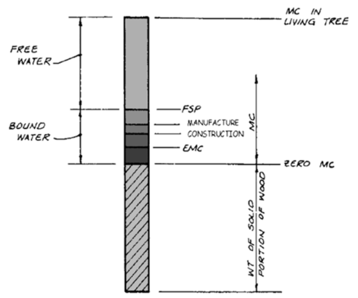
Wood Properties

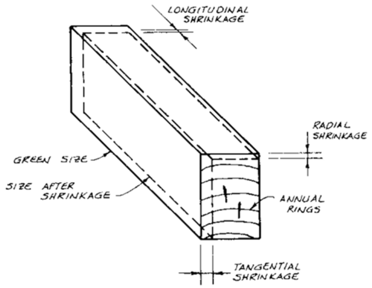
Moisture Content

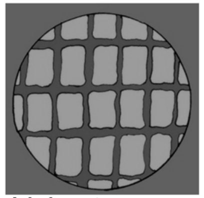
- MC = %water to oven dry wood
- In a living tree, MC can be 200%
- “free water” is contained in cell cavity
- “bound water” is within the cell wall
- Fiber Saturation Point (FSP) is the MC at 0% free and 100% bound water FSP is about 30%
- Equilibrium Moisture Content (EMC) is reached in service

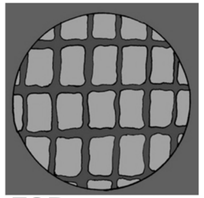
Shrinkage

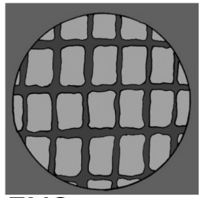
- Shrinkage begins once MC<FSP
- Shrinkage is not the same in each direction
- Uncontrolled shrinkage results in splits






Living tree


FSP

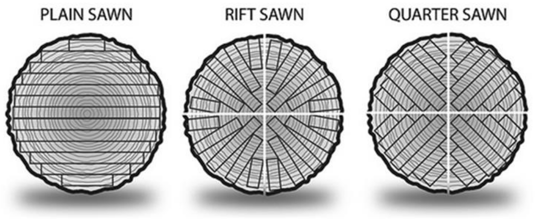

EMC

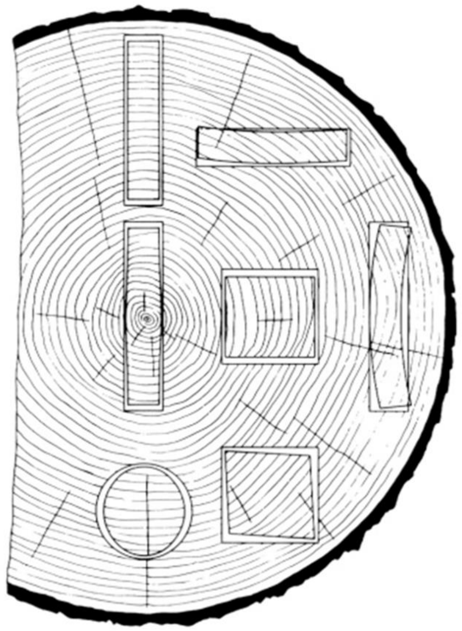
Shrinkage

- Is different in different directions
- Longitudinal is the least
- Across the grain is more
- Circumferential is greatest

Cut

- Plain Sawn – most economical and common
- Quarter Sawn – less warping
- Rift Sawn – least warping but more waste





Wood Properties

Yard Dry

- Initial free water is removed
- stacked separated by 1” stickers
- Air dried outdoors or under cover
- Dry rate depends on humidity and circulation
- Coating ends reduces splitting
- Takes ~ weeks to months

Kiln Dry - KD

- Enclosed in humidity controlled chamber
- Introduction of controlled heat
- Air circulation
- Dried to < %18

Heat Treated - HT

- temperature raised to 53° C (127° F) for 30 min.
- kills organisms
- requirement for imports

GRADING

Visual Grading

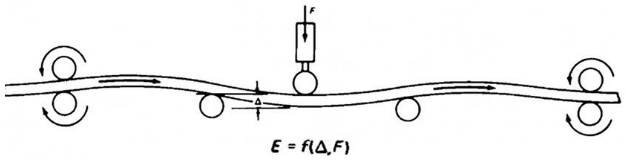
Each member is assessed for visual defects. (splits, knots, density, etc.)

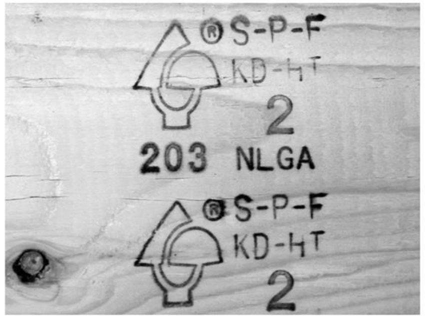
Machine Evaluated Lumber (MEL)

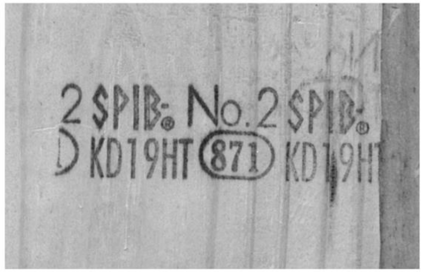
Each member is assessed for density using x-ray technology.

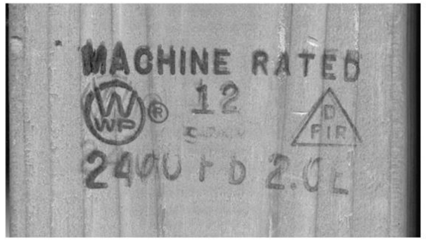
Machine Stress Rated (MSR)

Each member is stressed by running it through rollers which measure the deflection and stiffness. The E modulus in bending can be calculated from the deflection.









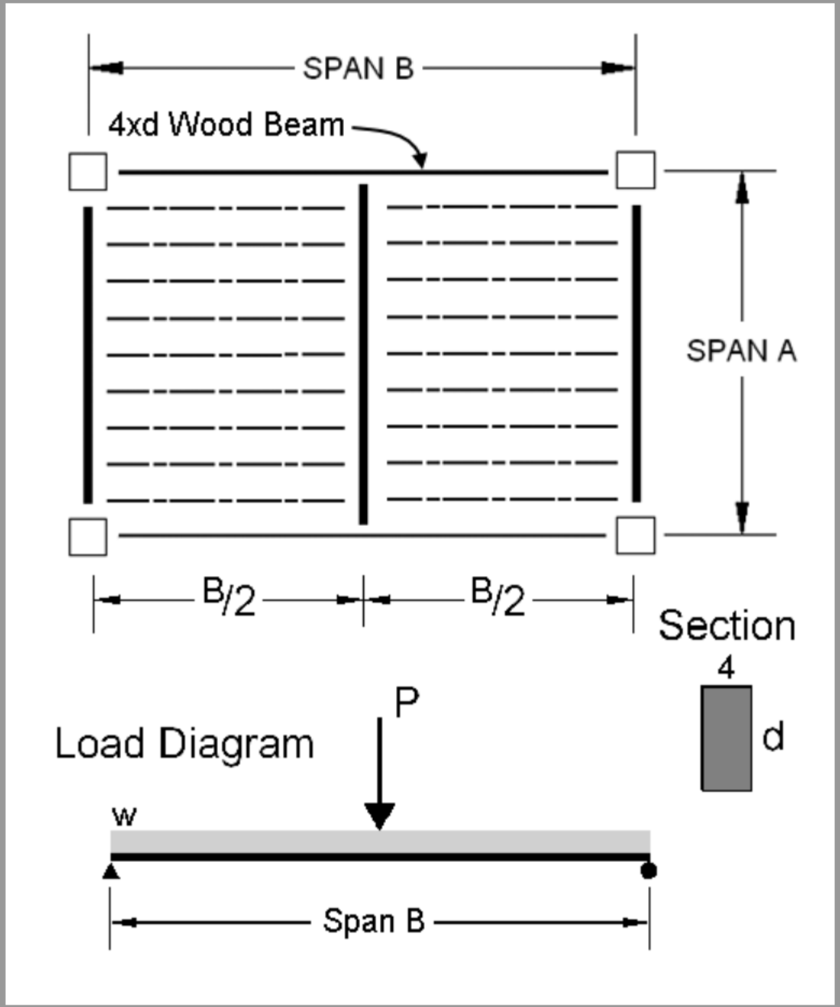
Problem Set 01

1. Wood Beam Analysis

Analyze the given 4x dimensioned lumber beam to determine if it passes or fails the NDS code criteria. The beam carries both dead and live floor load plus its own selfweight. Check the actual shear and bending stresses against the factored allowable stresses including all applicable factors from the NDS. Load duration is based on the live load (CD = 1.0). Assume normal temperature, and no incising (Ct = Ci = 1.0). Find the beam selfweight including the given moisture content. The beam is braced at the ends and the C.L. (meets criteria in 4.4.1) so CL = 1.0.

DATASET: 2 -1- -3-

Wood Species	NORTHERN WHITE CEDAR
Wood Grade	Select Structural
Span A	14 FT
Span B	10 FT
Nominal Depth of Beam, d	8 IN
Moisture Content, m.c.	15 %
Floor DL	9 PSF
Floor LL	35 PSF



Problem Set 01

#Q1: Tabulated Allow. Bending Stress, Fb

#Q2: Tabulated Allow. Shear Stress, Fv

#Q3: Tabulated Wood Dry Density

DATASET: 2 -1- -3-

Wood Species	NORTHERN WHITE CEDAR
Wood Grade	Select Structural
Span A	14 FT
Span B	10 FT
Nominal Depth of Beam, d	8 IN
Moisture Content, m.c.	15 %
Floor DL	9 PSF
Floor LL	35 PSF

According to the table, for “Northern White Cedar grade, select structural, we have the following:

$$F_b = 775\ PSI$$

$$F_V = 120\ PSI$$

$$G = 0.31$$

Table 4A (Cont.) **Reference Design Values for Visually Graded Dimension Lumber (2" - 4" thick)[1,2,3]**

(All species except Southern Pine — see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

USE WITH TABLE 4A ADJUSTMENT FACTORS

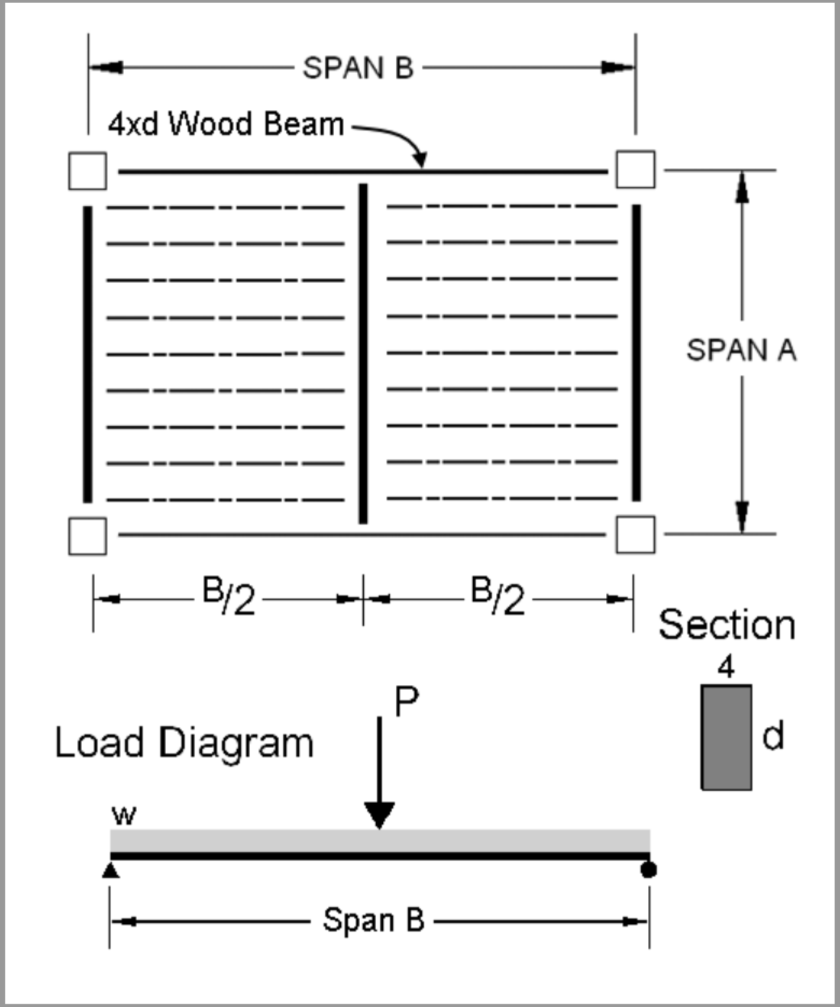
Species and commercial grade	Size classification	Design values in pounds per square inch (psi): Bending F_b	Tension parallel to grain F_t	Shear parallel to grain F_v	Compression perpendicular to grain $F_{c\perp}$	Compression parallel to grain F_c	Modulus of Elasticity E	Modulus of Elasticity E_{min}	Specific Gravity[4] G	Grading Rules Agency
MIXED MAPLE										
Select Structural	2" & wider	1,000	600	195	620	875	1,300,000	470,000	0.55	NELMA
No. 1	2" & wider	725	425	195	620	700	1,200,000	440,000		
No. 2	2" & wider	700	425	195	620	550	1,100,000	400,000		
No. 3	2" & wider	400	250	195	620	325	1,000,000	370,000		
Stud	2" & wider	550	325	195	620	350	1,000,000	370,000		
Construction	2" - 4" wide	800	475	195	620	725	1,100,000	400,000		
Standard	2" - 4" wide	450	275	195	620	575	1,000,000	370,000		
Utility	2" - 4" wide	225	125	195	620	375	900,000	330,000		
NORTHERN WHITE CEDAR										
Select Structural	2" & wider	775	450	120	370	750	800,000	290,000	0.31	NELMA
No. 1	2" & wider	575	325	120	370	600	700,000	260,000		
No. 2	2" & wider	550	325	120	370	475	700,000	260,000		
No. 3	2" & wider	325	175	120	370	275	600,000	220,000		
Stud	2" & wider	425	250	120	370	300	600,000	220,000		
Construction	2" - 4" wide	625	375	120	370	625	700,000	260,000		
Standard	2" - 4" wide	350	200	120	370	475	600,000	220,000		
Utility	2" - 4" wide	175	100	120	370	325	600,000	220,000		
RED MAPLE										
Select Structural	2" & wider	1,300	750	210	615	1,100	1,700,000	620,000	0.58	NELMA
No. 1	2" & wider	925	550	210	615	900	1,600,000	580,000		
No. 2	2" & wider	900	525	210	615	700	1,500,000	550,000		
No. 3	2" & wider	525	300	210	615	400	1,300,000	470,000		
Stud	2" & wider	700	425	210	615	450	1,300,000	470,000		
Construction	2" - 4" wide	1,050	600	210	615	925	1,400,000	510,000		
Standard	2" - 4" wide	575	325	210	615	725	1,300,000	470,000		
Utility	2" - 4" wide	275	150	210	615	475	1,200,000	440,000		

Problem Set 01

#Q4: Total Actual Applied Point Load, P

Steps:

1- Find the load path
2- Find the tributary area
3- Calculate the live load and dead load

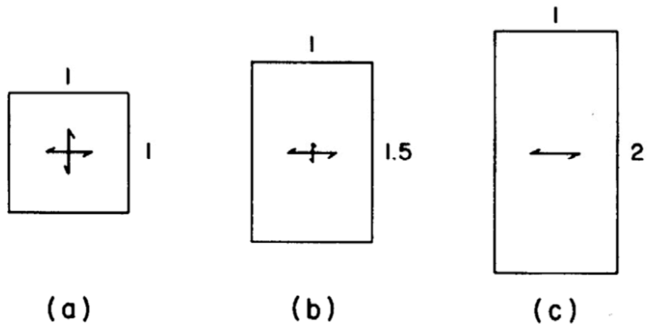


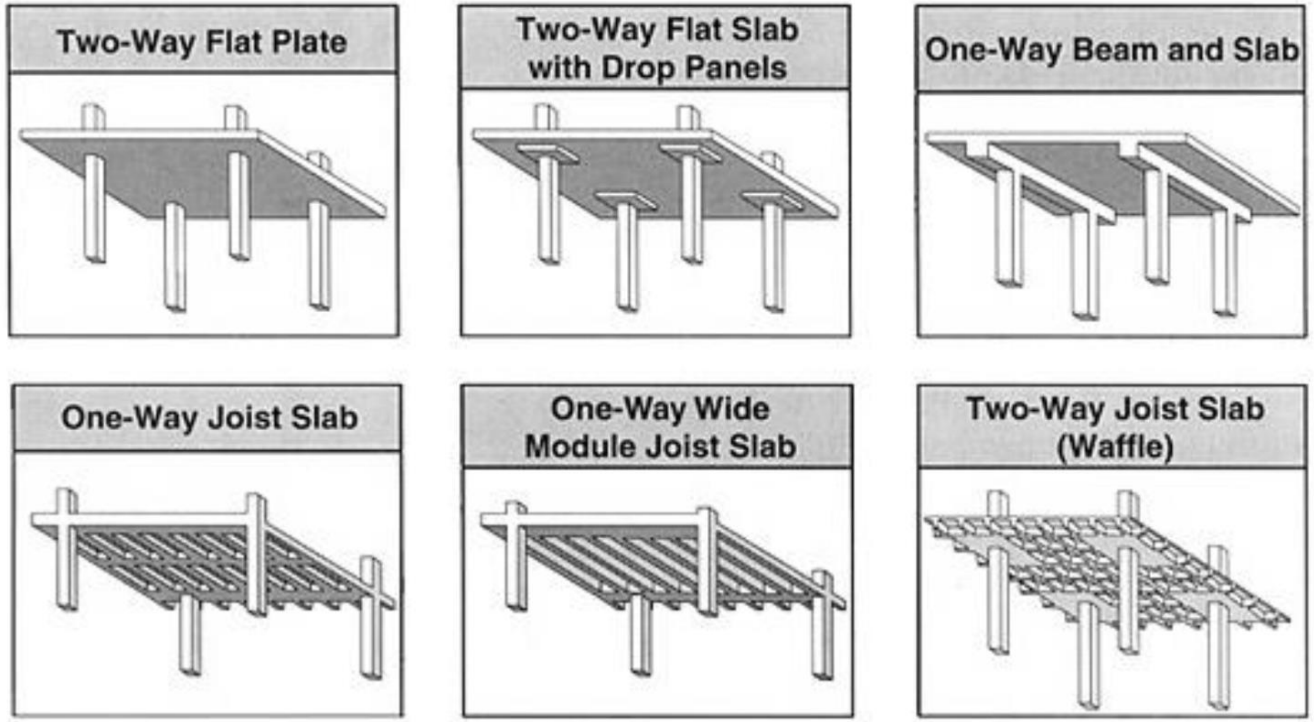
Quick refresher

Load Paths (slabs)

One-way slabs should span the shortest direction.

Two-way slabs span in both directions. Aspect ratios should be between 1:1 and 2:1. The load path divides at 45° from corner.



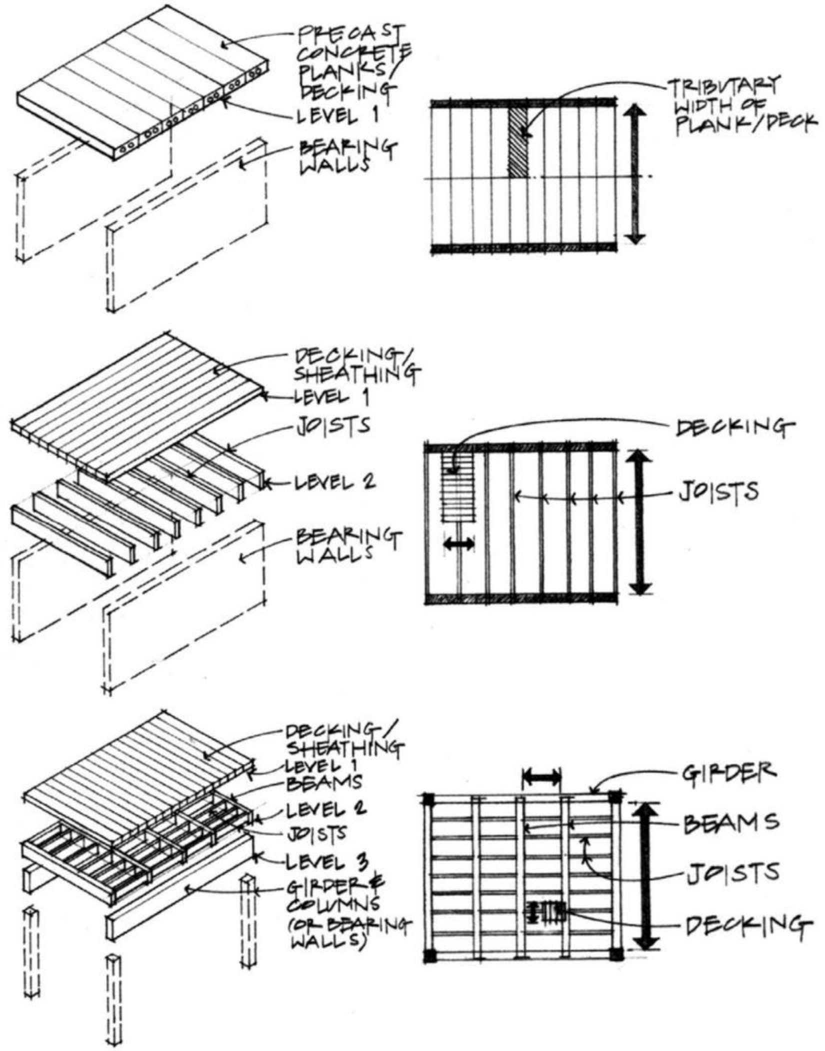


Quick refresher

Tributary Area

The tributary area is an **area that corresponds to the load on a member**.

If geometry and loading is symmetric, then load paths and reactions are also symmetric.

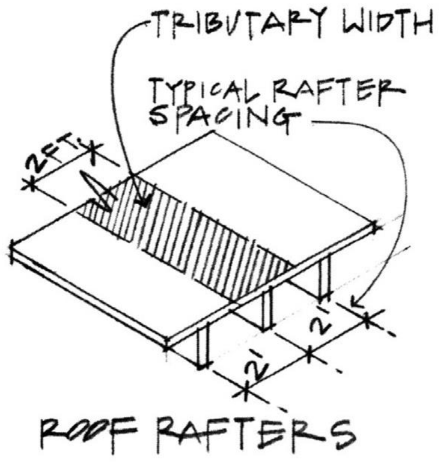


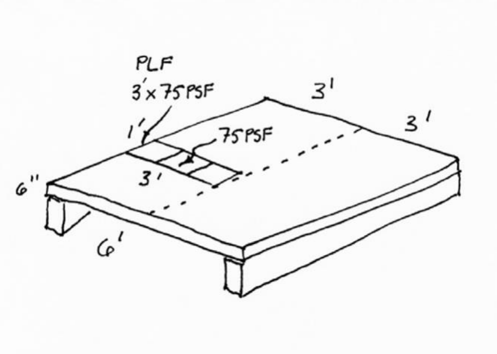
Quick refresher

Tributary Area

The tributary area is an **area that corresponds to the load on a member**.

Each member has a tributary area that can be used to find the total load on that member.





Problem Set 01

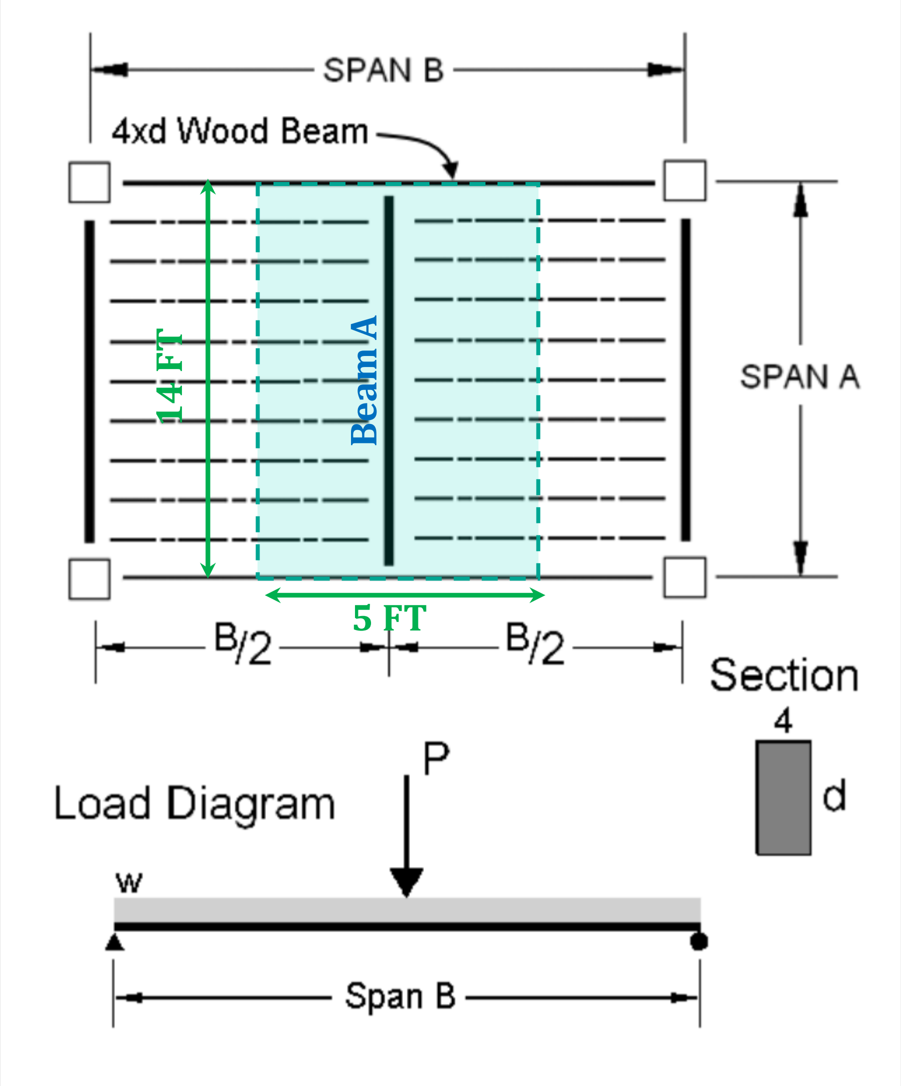
#Q4: Total Actual Applied Point Load, P

Span A	14 FT
Span B	10 FT
Nominal Depth of Beam, d	8 IN
Moisture Content, m.c.	15 %
Floor DL	9 PSF
Floor LL	35 PSF

$$\boldsymbol{Tributary\ Area} = SpanA \times SpanB/2 = 14 \times \frac{10}{2} = 70\ FT^2$$

$$\boldsymbol{Total\ Load\ \_beam\ A} = 70 \times (DL + LL) = 70(9 + 35) = 3080\ LBS$$

$$\boldsymbol{Point\ load\ P} = Load\ on\ beam\frac{A}{2} = \frac{3080}{2} = \mathbf{1540\ LBS}$$



Problem Set 01

#Q5: Wood Density (Including M.C.)

Span A	14 FT
Span B	10 FT
Nominal Depth of Beam, d	8 IN
Moisture Content, m.c.	15 %
Floor DL	9 PSF
Floor LL	35 PSF

$$\boldsymbol{Density} = 62.4\left[\frac{G}{1+G(0.009)m.c.}\right]\left[1+\frac{m.c.}{100}\right]$$

$$G = 0.31\,, m.c = 15\% \rightarrow$$

$$\boldsymbol{Density} = 62.4\left[\frac{0.31}{1+0.31(0.009)\,15}\right]\left[1+\frac{15}{100}\right]$$

$$= \boldsymbol{21.352\ PCF}$$

3.1 Section Properties of Sawn Lumber and Structural Glued Laminated Timber

3.1.1 Standard Sizes of Sawn Lumber

Details regarding the dressed sizes of various species of lumber in the grading rules of the agencies which formulate and maintain such rules. The dressed sizes in Table 1A conform to the sizes set forth in U.S. Department of Commerce Voluntary Product Standard PS 20-10 (American Softwood Lumber Standard). While these sizes are generally available on a commercial basis, it is good practice to consult the local lumber dealer to determine what sizes are on hand or can be readily secured.

Dry lumber is defined as lumber which has been seasoned to a moisture content of 19% or less. Green lumber is defined as lumber having a moisture content in excess of 19%.

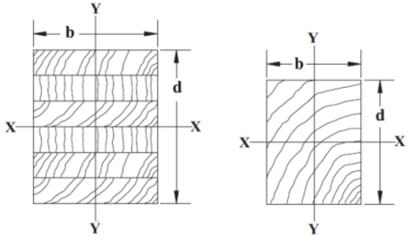
3.1.2 Properties of Standard Dressed Sizes

Certain mathematical expressions of the properties or elements of sections are used in design calculations for various member shapes and loading conditions. The section properties for selected standard sizes of boards, dimension lumber, and timbers are given in Table 1B. Section properties for selected standard sizes of structural glued laminated timber are given in Tables 1C and 1D.

3.1.3 Definitions

NEUTRAL AXIS, in the cross section of a beam, is the line on which there is neither tension nor compression stress.

Figure 1A Dimensions for Rectangular Cross Section



Structural Glued Laminated Timber   Sawn Lumber

MOMENT OF INERTIA, I, of the cross section of a beam is the sum of the products of each of its elementary areas multiplied by the square of their distance from the neutral axis of the section.

SECTION MODULUS, S, is the moment of inertia divided by the distance from the neutral axis to the extreme fiber of the section.

CROSS SECTION is a section taken through the member perpendicular to its longitudinal axis.

The following symbols and formulas apply to rectangular beam cross sections:

X-X = neutral axis for edgewise bending (load applied to narrow face)

Y-Y = neutral axis for flatwise bending (load applied to wide face)

b = breadth (thickness) of rectangular bending member, in.

d = depth (width) of rectangular bending member, in.

A = bd = area of cross section, in.^{2}

c = distance from neutral axis to extreme fiber of cross section, in.

$I_x = bd^3/12$ = moment of inertia about the X-X axis, in.^{4}

$I_y = db^3/12$ = moment of inertia about the Y-Y axis, in.^{4}

$r_x = \sqrt{I_x/A} = d/\sqrt{12}$ = radius of gyration about the X-X axis, in.

$r_y = \sqrt{I_y/A} = b/\sqrt{12}$ = radius of gyration about the Y-Y axis, in.

$S_x = I_x/c = bd^2/6$ = section modulus about the X-X axis, in.^{3}

$S_y = I_y/c = db^2/6$ = section modulus about the Y-Y axis, in.^{3}

The following formula shall be used to determine the density in lbs/ft^3 of wood:

$$\text{density} = 62.4\left[\frac{G}{1+G(0.009)(m.c.)}\right]\left[1+\frac{m.c.}{100}\right]$$

where:

G = specific gravity of wood

m.c. = moisture content of wood, %

Problem Set 01

#Q6: Beam Selfweight (Including M.C.), w

Span A	14 FT
Span B	10 FT
Nominal Depth of Beam, d	8 IN
Moisture Content, m.c.	15 %
Floor DL	9 PSF
Floor LL	35 PSF

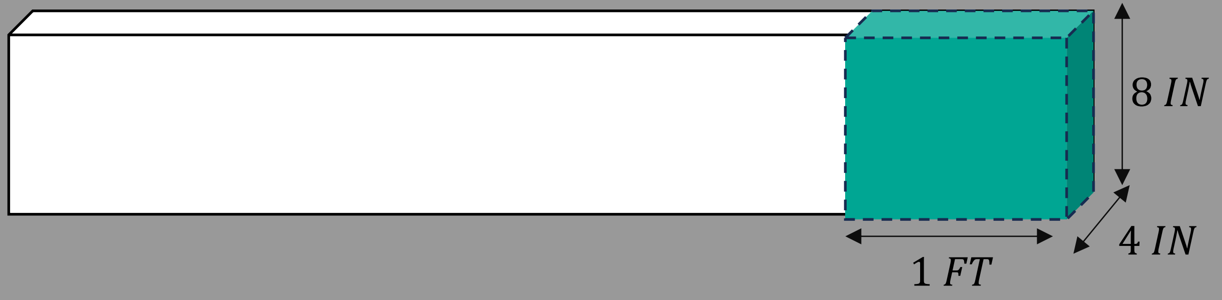
$$PLF = Volume \times Density \times 1\ Ft$$

$$PLF = \left(25.38\ IN^2 \times \frac{1\ FT}{144\ IN^2} \times 21.352\ \times 1\ FT\right)$$

$$= \mathbf{3.763\ \ PLF}$$

Table 1B Section Properties of Standard Dressed (S4S) Sawn Lumber

Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A in.^{2}	X-X AXIS Section Modulus S_{xx} in.^{3}	X-X AXIS Moment of Inertia I_{xx} in.^{4}	Y-Y AXIS Section Modulus S_{yy} in.^{3}	Y-Y AXIS Moment of Inertia I_{yy} in.^{4}	Approximate weight in pounds per linear foot (lbs/ft) of piece when density of wood equals: 25 lbs/ft^3	30 lbs/ft^3	35 lbs/ft^3	40 lbs/ft^3	45 lbs/ft^3	50 lbs/ft^3
Dimension Lumber (see NDS 4.1.3.2) and Decking (see NDS 4.1.3.5)												
2 x 3	1-1/2 x 2-1/2	3.750	1.56	1.953	0.938	0.703	0.651	0.781	0.911	1.042	1.172	1.302
2 x 4	1-1/2 x 3-1/2	5.250	3.06	5.359	1.313	0.984	0.911	1.094	1.276	1.458	1.641	1.823
2 x 5	1-1/2 x 4-1/2	6.750	5.06	11.39	1.688	1.266	1.172	1.406	1.641	1.875	2.109	2.344
2 x 6	1-1/2 x 5-1/2	8.250	7.56	20.80	2.063	1.547	1.432	1.719	2.005	2.292	2.578	2.865
2 x 8	1-1/2 x 7-1/4	10.88	13.14	47.63	2.719	2.039	1.888	2.266	2.643	3.021	3.398	3.776
2 x 10	1-1/2 x 9-1/4	13.88	21.39	98.93	3.469	2.602	2.409	2.891	3.372	3.854	4.336	4.818
2 x 12	1-1/2 x 11-1/4	16.88	31.64	178.0	4.219	3.164	2.930	3.516	4.102	4.688	5.273	5.859
2 x 14	1-1/2 x 13-1/4	19.88	43.89	290.8	4.969	3.727	3.451	4.141	4.831	5.521	6.211	6.901
3 x 4	2-1/2 x 3-1/2	8.75	5.10	8.932	3.646	4.557	1.519	1.823	2.127	2.431	2.734	3.038
3 x 5	2-1/2 x 4-1/2	11.25	8.44	18.98	4.688	5.859	1.953	2.344	2.734	3.125	3.516	3.906
3 x 6	2-1/2 x 5-1/2	13.75	12.60	34.66	5.729	7.161	2.387	2.865	3.342	3.819	4.297	4.774
3 x 8	2-1/2 x 7-1/4	18.13	21.90	79.39	7.552	9.440	3.147	3.776	4.405	5.035	5.664	6.293
3 x 10	2-1/2 x 9-1/4	23.13	35.65	164.9	9.635	12.04	4.015	4.818	5.621	6.424	7.227	8.030
3 x 12	2-1/2 x 11-1/4	28.13	52.73	296.6	11.72	14.65	4.883	5.859	6.836	7.813	8.789	9.766
3 x 14	2-1/2 x 13-1/4	33.13	73.15	484.6	13.80	17.25	5.751	6.901	8.051	9.201	10.35	11.50
3 x 16	2-1/2 x 15-1/4	38.13	96.90	738.9	15.89	19.86	6.619	7.943	9.266	10.59	11.91	13.24
4 x 4	3-1/2 x 3-1/2	12.25	7.15	12.51	7.146	12.51	2.127	2.552	2.977	3.403	3.828	4.253
4 x 5	3-1/2 x 4-1/2	15.75	11.81	26.58	9.188	16.08	2.734	3.281	3.828	4.375	4.922	5.469
4 x 6	3-1/2 x 5-1/2	19.25	17.65	48.53	11.23	19.65	3.342	4.010	4.679	5.347	6.016	6.684
4 x 8	3-1/2 x 7-1/4	25.38	30.66	111.1	14.80	25.90	4.405	5.286	6.168	7.049	7.930	8.811
4 x 10	3-1/2 x 9-1/4	32.38	49.91	230.8	18.89	33.05	5.621	6.745	7.869	8.993	10.12	11.24
4 x 12	3-1/2 x 11-1/4	39.38	73.83	415.3	22.97	40.20	6.836	8.203	9.570	10.94	12.30	13.67
4 x 14	3-1/2 x 13-1/4	46.38	102.41	678.5	27.05	47.34	8.051	9.661	11.27	12.88	14.49	16.10
4 x 16	3-1/2 x 15-1/4	53.38	135.66	1034	31.14	54.49	9.266	11.12	12.97	14.83	16.68	18.53



Problem Set 01

#Q7: Actual Beam Bending Moment, M

#Q8: Actual Maximum Shear Force (at reaction) , V

Steps:

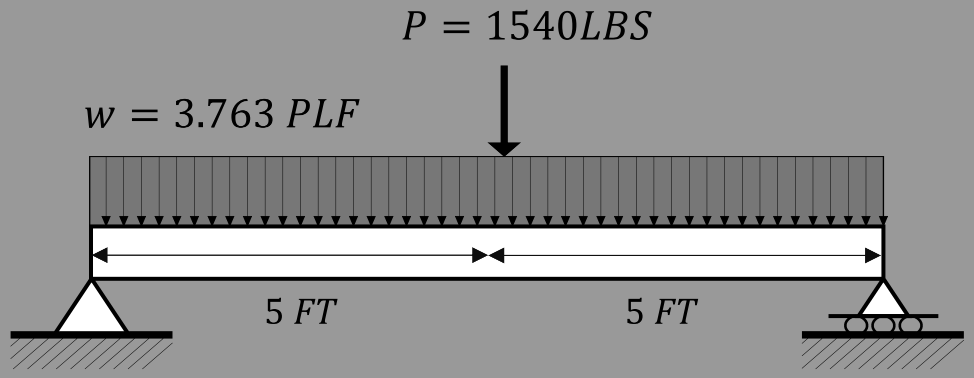
1- Draw free body diagram
2- Find maximum moments and shear
3- Draw moment and shear diagrams

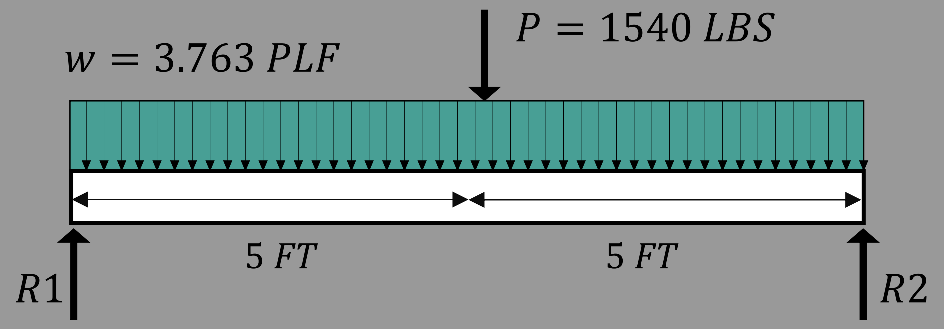
$$M_{max} = \frac{P.L}{4} + \frac{w.L^2}{8} = \frac{1540 \times 10}{4} + \frac{3.763 \times 10^2}{8}$$

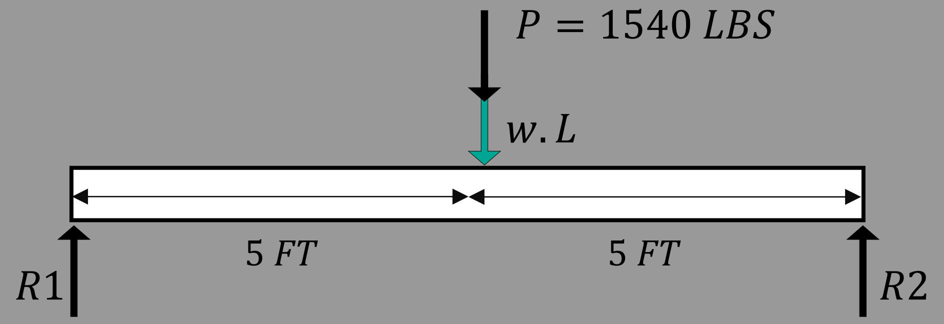
$$= 3850 + 47.03 = \mathbf{3897.03FT - LB}$$

$$V_{max} = \frac{P}{2} + \frac{w.L}{2} = \frac{1540}{2} + \frac{3.763 \times 10}{2}$$

$$= 3850 + 47.03 = \mathbf{788.81\ LBS}$$







Problem Set 01

#Q7: Actual Beam Bending Moment, M

#Q8: Actual Maximum Shear Force (at reaction) , V

Steps:

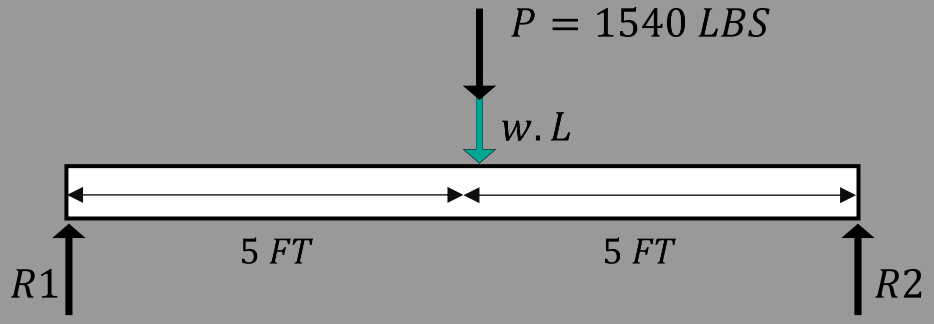
1- Draw free body diagram
2- Find maximum moments and shear
3- Draw moment and shear diagrams

$$\boldsymbol{M_{max}} = \frac{P.L}{4} + \frac{w.L^2}{8} = \frac{1540 \times 10}{4} + \frac{3.763 \times 10^2}{8}$$

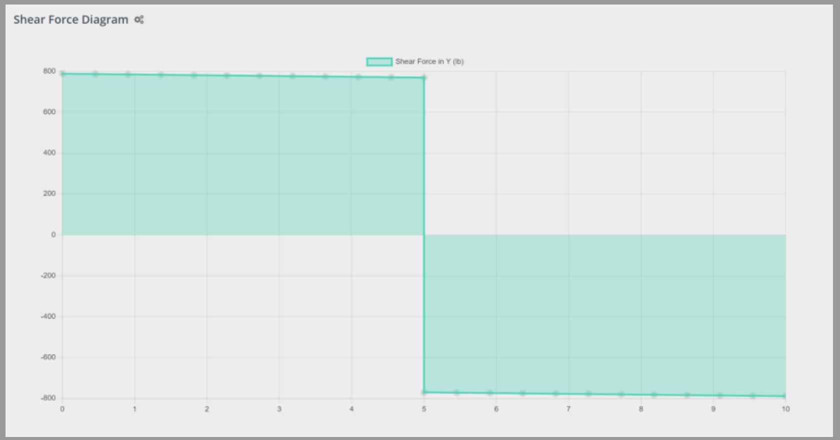
$$= 3850 + 47.03 = \mathbf{3897.03 FT - LB}$$

$$\boldsymbol{V_{max}} = \frac{P}{2} + \frac{w.L}{2} = \frac{1540}{2} + \frac{3.763 \times 10}{2}$$

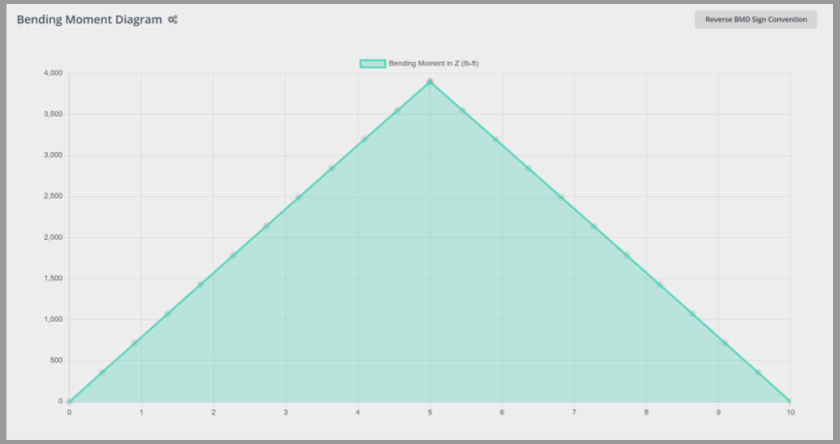
$$= 3850 + 47.03 = \mathbf{788.81\ LBS}$$



Shear diagram



Moment diagram



Problem Set 01

#Q9: Size Factor, CF

#Q10: Wet Service Factor for Fb, CM_b

#Q11: Wet Service Factor for Fv, CM_v

According to the table, for 4×8 in lumber with 15% moisture content:

$$C_F = 1.3$$

$$C_{Mb} = 1$$

$$C_{Mv} = 1$$

Table 4A Adjustment Factors

Repetitive Member Factor, C_r

Bending design values, F_b, for dimension lumber 2" to 4" thick shall be multiplied by the repetitive member factor, $C_r = 1.15$, when such members are used as joists, truss chords, rafters, studs, planks, decking, or similar members which are in contact or spaced not more than 24" on center, are not less than 3 in number and are joined by floor, roof, or other load distributing elements adequate to support the design load.

Wet Service Factor, C_M

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors, C_M

F_b	F_t	F_v	$F_{c\perp}$	F_c	E and E_{min}
0.85*	1.0	0.97	0.67	0.8**	0.9

* when $(F_b)(C_F) \le 1{,}150$ psi, $C_M = 1.0$

** when $(F_c)(C_F) \le 750$ psi, $C_M = 1.0$

Flat Use Factor, C_{fu}

Bending design values adjusted by size factors are based on edgewise use (load applied to narrow face). When dimension lumber is used flatwise (load applied to wide face), the bending design value, F_b, shall also be permitted to be multiplied by the following flat use factors:

Flat Use Factors, C_{fu}

Width (depth)	Thickness (breadth) 2" & 3"	4"
2" & 3"	1.0	—
4"	1.1	1.0
5"	1.1	1.05
6"	1.15	1.05
8"	1.15	1.05
10" & wider	1.2	1.1

NOTE

To facilitate the use of Table 4A, shading has been employed to distinguish design values based on a 4" nominal width (Construction, Standard, and Utility grades) or a 6" nominal width (Stud grade) from design values based on a 12" nominal width (Select Structural, No.1 & Btr, No.1, No.2, and No.3 grades).

Size Factor, C_F

Tabulated bending, tension, and compression parallel to grain design values for dimension lumber 2" to 4" thick shall be multiplied by the following size factors:

Size Factors, C_F

Grades	Width (depth)	F_b Thickness (breadth) 2" & 3"	F_b Thickness (breadth) 4"	F_t	F_c
Select Structural, No.1 & Btr, No.1, No.2, No.3	2", 3", & 4"	1.5	1.5	1.5	1.15
	5"	1.4	1.4	1.4	1.1
	6"	1.3	1.3	1.3	1.1
	8"	1.2	1.3	1.2	1.05
	10"	1.1	1.2	1.1	1.0
	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
Stud	2", 3", & 4"	1.1	1.1	1.1	1.05
	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3 Grade tabulated design values and size factors			
Construction, Standard	2", 3", & 4"	1.0	1.0	1.0	1.0
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	—	0.4	0.6

Problem Set 01

#Q12: Factored Allow. Bending Stress, F'b

#Q13: Factored Allow. Shear Stress, F'v

$$\boldsymbol{F'_b} = F_b . C_F . C_{Mb} = 775 \times 1.3 \times 1 = \mathbf{1,007.5}\ \boldsymbol{PSI}$$

$$\boldsymbol{F'_V} = F_V . C_{MV} = 120 \times 1 = \mathbf{120}\ \boldsymbol{PSI}$$

Allowable Flexure Stress F_v'

F_v from tables determined by species and grade

F_v' = F_v (usage factors)

Usage factors for shear:

- C_D Load Duration Factor
- C_M Moisture Factor
- C_t Temperature Factor
- C_i Incising Factor

Adjustment Factors

Allowable Flexure Stress F_b'

F_b from tables determined by species and grade

F_b' = F_b (C_D C_M C_t C_L C_F C_{fu} C_i C_r)

Usage factors for flexure:

- **C_D** Load Duration Factor

Problem Set 01

#Q14: Actual Bending Stress, fb_actual

#Q15: Actual Shear Stress, fv_actual

$$\boldsymbol{F_b} = \frac{M}{S} = \frac{3897.03\ FTLB\ \times 12}{30.66} = \boldsymbol{1,525.25\ PSI}$$

$$\boldsymbol{F_v} = 1.5\,\frac{V}{A} = 1.5\,\frac{788.81}{25.38} = \boldsymbol{46.62\ PSI}$$

Table 1B Section Properties of Standard Dressed (S4S) Sawn Lumber

Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A $in.^2$	X-X AXIS Section Modulus S_{xx} $in.^3$	X-X AXIS Moment of Inertia I_{xx} $in.^4$	Y-Y AXIS Section Modulus S_{yy} $in.^3$	Y-Y AXIS Moment of Inertia I_{yy} $in.^4$	Approximate weight in pounds per linear foot (lbs/ft) of piece when density of wood equals: 25 lbs/ft^3	30 lbs/ft^3	35 lbs/ft^3	40 lbs/ft^3	45 lbs/ft^3	50 lbs/ft^3
Boards[1]												
1 x 3	3/4 x 2-1/2	1.875	0.781	0.977	0.234	0.088	0.326	0.391	0.456	0.521	0.586	0.651
1 x 4	3/4 x 3-1/2	2.625	1.531	2.680	0.328	0.123	0.456	0.547	0.638	0.729	0.820	0.911
1 x 6	3/4 x 5-1/2	4.125	3.781	10.40	0.516	0.193	0.716	0.859	1.003	1.146	1.289	1.432
1 x 8	3/4 x 7-1/4	5.438	6.570	23.82	0.680	0.255	0.944	1.133	1.322	1.510	1.699	1.888
1 x 10	3/4 x 9-1/4	6.938	10.70	49.47	0.867	0.325	1.204	1.445	1.686	1.927	2.168	2.409
1 x 12	3/4 x 11-1/4	8.438	15.82	88.99	1.055	0.396	1.465	1.758	2.051	2.344	2.637	2.930
Dimension Lumber (see NDS 4.1.3.2) and Decking (see NDS 4.1.3.5)												
2 x 3	1-1/2 x 2-1/2	3.750	1.56	1.953	0.938	0.703	0.651	0.781	0.911	1.042	1.172	1.302
2 x 4	1-1/2 x 3-1/2	5.250	3.06	5.359	1.313	0.984	0.911	1.094	1.276	1.458	1.641	1.823
2 x 5	1-1/2 x 4-1/2	6.750	5.06	11.39	1.688	1.266	1.172	1.406	1.641	1.875	2.109	2.344
2 x 6	1-1/2 x 5-1/2	8.250	7.56	20.80	2.063	1.547	1.432	1.719	2.005	2.292	2.578	2.865
2 x 8	1-1/2 x 7-1/4	10.88	13.14	47.63	2.719	2.039	1.888	2.266	2.643	3.021	3.398	3.776
2 x 10	1-1/2 x 9-1/4	13.88	21.39	98.93	3.469	2.602	2.409	2.891	3.372	3.854	4.336	4.818
2 x 12	1-1/2 x 11-1/4	16.88	31.64	178.0	4.219	3.164	2.930	3.516	4.102	4.688	5.273	5.859
2 x 14	1-1/2 x 13-1/4	19.88	43.89	290.8	4.969	3.727	3.451	4.141	4.831	5.521	6.211	6.901
3 x 4	2-1/2 x 3-1/2	8.75	5.10	8.932	3.646	4.557	1.519	1.823	2.127	2.431	2.734	3.038
3 x 5	2-1/2 x 4-1/2	11.25	8.44	18.98	4.688	5.859	1.953	2.344	2.734	3.125	3.516	3.906
3 x 6	2-1/2 x 5-1/2	13.75	12.60	34.66	5.729	7.161	2.387	2.865	3.342	3.819	4.297	4.774
3 x 8	2-1/2 x 7-1/4	18.13	21.90	79.39	7.552	9.440	3.147	3.776	4.405	5.035	5.664	6.293
3 x 10	2-1/2 x 9-1/4	23.13	35.65	164.9	9.635	12.04	4.015	4.818	5.621	6.424	7.227	8.030
3 x 12	2-1/2 x 11-1/4	28.13	52.73	296.6	11.72	14.65	4.883	5.859	6.836	7.813	8.789	9.766
3 x 14	2-1/2 x 13-1/4	33.13	73.15	484.6	13.80	17.25	5.751	6.901	8.051	9.201	10.35	11.50
3 x 16	2-1/2 x 15-1/4	38.13	96.90	738.9	15.89	19.86	6.619	7.943	9.266	10.59	11.91	13.24
4 x 4	3-1/2 x 3-1/2	12.25	7.15	12.51	7.146	12.51	2.127	2.552	2.977	3.403	3.828	4.253
4 x 5	3-1/2 x 4-1/2	15.75	11.81	26.58	9.188	16.08	2.734	3.281	3.828	4.375	4.922	5.469
4 x 6	3-1/2 x 5-1/2	19.25	17.65	48.53	11.23	19.65	3.342	4.010	4.679	5.347	6.016	6.684
4 x 8	3-1/2 x 7-1/4	25.38	$\boxed{30.66}$	111.1	14.80	25.90	4.405	5.286	6.168	7.049	7.930	8.811
4 x 10	3-1/2 x 9-1/4	32.38	49.91	230.8	18.89	33.05	5.621	6.745	7.869	8.993	10.12	11.24
4 x 12	3-1/2 x 11-1/4	39.38	73.83	415.3	22.97	40.20	6.836	8.203	9.570	10.94	12.30	13.67
4 x 14	3-1/2 x 13-1/4	46.38	102.41	678.5	27.05	47.34	8.051	9.661	11.27	12.88	14.49	16.10
4 x 16	3-1/2 x 15-1/4	53.38	135.66	1034	31.14	54.49	9.266	11.12	12.97	14.83	16.68	18.53
Timbers (5" x 5" and larger)[2]												
Post and Timber (see NDS 4.1.3.4 and NDS 4.1.5.3)												
5 x 5	4-1/2 x 4-1/2	20.25	15.19	34.17	15.19	34.17	3.516	4.219	4.922	5.625	6.328	7.031
6 x 6	5-1/2 x 5-1/2	30.25	27.73	76.26	27.73	76.26	5.252	6.302	7.352	8.403	9.453	10.50
6 x 8	5-1/2 x 7-1/2	41.25	51.56	193.4	37.81	104.0	7.161	8.594	10.03	11.46	12.89	14.32
8 x 8	7-1/2 x 7-1/2	56.25	70.31	263.7	70.31	263.7	9.766	11.72	13.67	15.63	17.58	19.53
8 x 10	7-1/2 x 9-1/2	71.25	112.8	535.9	89.06	334.0	12.37	14.84	17.32	19.79	22.27	24.74
10 x 10	9-1/2 x 9-1/2	90.25	142.9	678.8	142.9	678.8	15.67	18.80	21.94	25.07	28.20	31.34
10 x 12	9-1/2 x 11-1/2	109.3	209.4	1204	173.0	821.7	18.97	22.76	26.55	30.35	34.14	37.93
12 x 12	11-1/2 x 11-1/2	132.3	253.5	1458	253.5	1458	22.96	27.55	32.14	36.74	41.33	45.92
12 x 14	11-1/2 x 13-1/2	155.3	349.3	2358	297.6	1711	26.95	32.34	37.73	43.13	48.52	53.91
14 x 14	13-1/2 x 13-1/2	182.3	410.1	2768	410.1	2768	31.64	37.97	44.30	50.63	56.95	63.28
14 x 16	13-1/2 x 15-1/2	209.3	540.6	4189	470.8	3178	36.33	43.59	50.86	58.13	65.39	72.66
16 x 16	15-1/2 x 15-1/2	240.3	620.6	4810	620.6	4810	41.71	50.05	58.39	66.74	75.08	83.42
16 x 18	15-1/2 x 17-1/2	271.3	791.1	6923	700.7	5431	47.09	56.51	65.93	75.35	84.77	94.18
18 x 18	17-1/2 x 17-1/2	306.3	893.2	7816	893.2	7816	53.17	63.80	74.44	85.07	95.70	106.3
18 x 20	17-1/2 x 19-1/2	341.3	1109	10813	995.3	8709	59.24	71.09	82.94	94.79	106.6	118.5
20 x 20	19-1/2 x 19-1/2	380.3	1236	12049	1236	12049	66.02	79.22	92.4	105.6	118.8	132.0
20 x 22	19-1/2 x 21-1/2	419.3	1502	16150	1363	13285	72.79	87.34	101.9	116.5	131.0	145.6
22 x 22	21-1/2 x 21-1/2	462.3	1656	17806	1656	17806	80.25	96.30	112.4	128.4	144.5	160.5
22 x 24	21-1/2 x 23-1/2	505.3	1979	23252	1810	19463	87.72	105.3	122.8	140.3	157.9	175.4
24 x 24	23-1/2 x 23-1/2	552.3	2163	25415	2163	25415	95.88	115.1	134.2	153.4	172.6	191.8

Problem Set 01

#Q16: Bending Stress Passing

#Q17: Shear Stress Passing

$$F'_b = 1{,}007.5\ PSI$$

$$F'_V = 120\ PSI$$

$$F_b = 1{,}525.25\ PSI$$

$$F_v = 46.62\ PSI$$

$$\boldsymbol{F'_b < F_b \Rightarrow Fail}$$

$$\boldsymbol{F'_V > F_V \Rightarrow Pass}$$

$F_b' \geq f_b$

Allowable Flexure Stress F_b'

F_b from NDS Supplement tables determined by species and grade

F_b' = F_b (usage factors)

usage factors for flexure:

- C_D Load Duration Factor
- C_M Moisture Factor
- C_t Temperature Factor
- C_L Beam Stability Factor
- C_F Size Factor
- C_{fu} Flat Use
- C_i Incising Factor
- C_r Repetitive Member Factor

$\geq$

Actual Flexure Stress f_b

$f_b = Mc/I = M/S$

$S = I/c = bd^2/6$

$F_v' \geq f_v$

Allowable Shear Stress Fv'

F_v from tables determined by species and grade

F_v' = F_v (usage factors)

usage factors for shear:

- C_D Load Duration Factor
- C_M Moisture Factor
- C_t Temperature Factor
- C_i Incising Factor

NDS Table 4.3.1

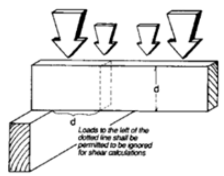
$\geq$

Actual Shear Stress fv

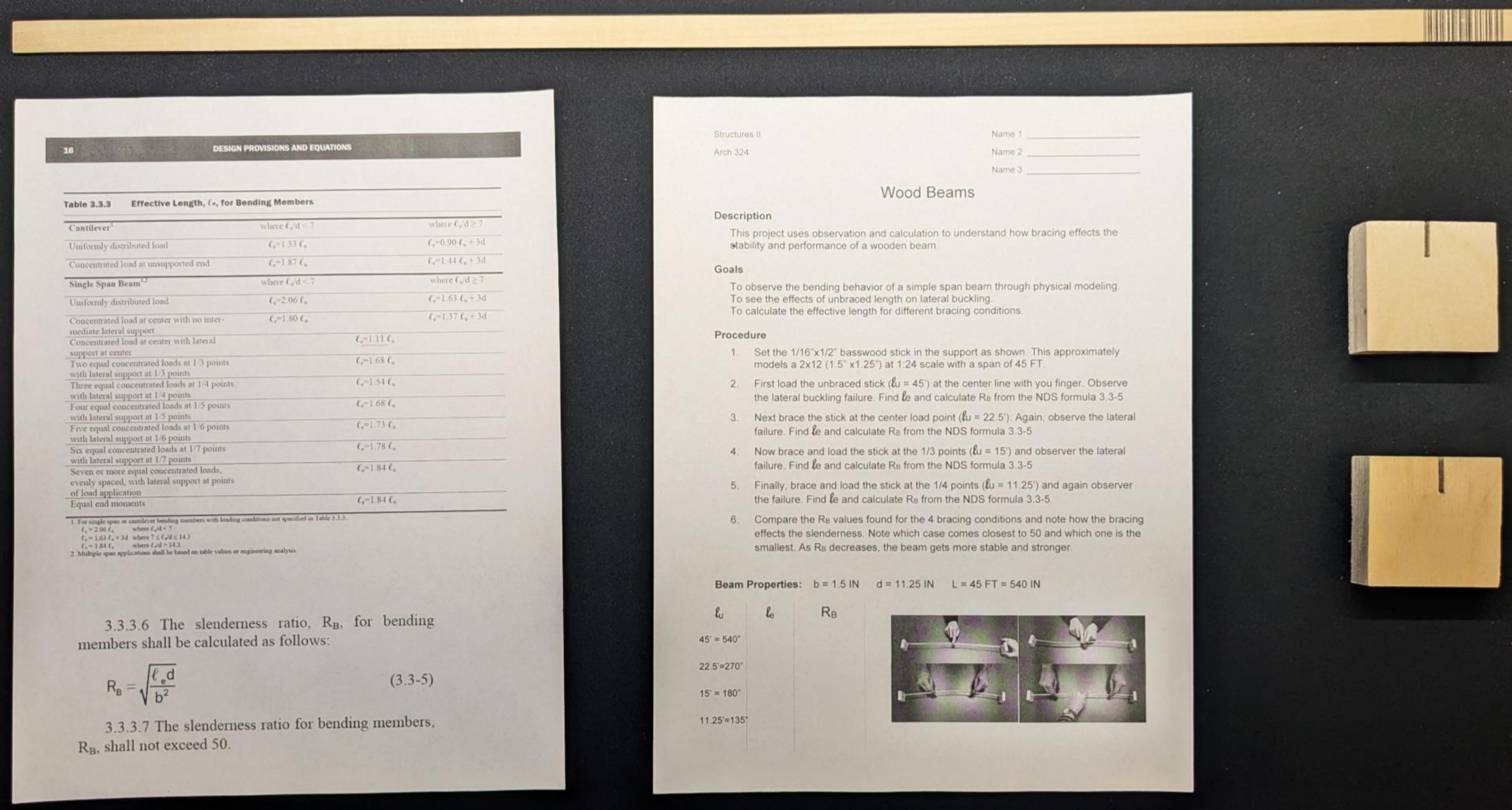
$f_v = VQ / I\,b = 1.5\ V/A$

Can use V at d from support as maximum

Shear at Supports



Lab01



16 DESIGN PROVISIONS AND EQUATIONS

Table 3.3.3 Effective Length, ℓ_e, for Bending Members

Cantilever[1]	where $\ell_u/d < 7$		where $\ell_u/d \geq 7$
Uniformly distributed load	ℓ_e=1.33 ℓ_u		ℓ_e=0.90 ℓ_u + 3d
Concentrated load at unsupported end	ℓ_e=1.87 ℓ_u		ℓ_e=1.44 ℓ_u + 3d
Single Span Beam[1,2]	where $\ell_u/d < 7$		where $\ell_u/d \geq 7$
Uniformly distributed load	ℓ_e=2.06 ℓ_u		ℓ_e=1.63 ℓ_u + 3d
Concentrated load at center with no intermediate lateral support	ℓ_e=1.80 ℓ_u		ℓ_e=1.37 ℓ_u + 3d
Concentrated load at center with lateral support at center		ℓ_e=1.11 ℓ_u	
Two equal concentrated loads at 1/3 points with lateral support at 1/3 points		ℓ_e=1.68 ℓ_u	
Three equal concentrated loads at 1/4 points with lateral support at 1/4 points		ℓ_e=1.54 ℓ_u	
Four equal concentrated loads at 1/5 points with lateral support at 1/5 points		ℓ_e=1.68 ℓ_u	
Five equal concentrated loads at 1/6 points with lateral support at 1/6 points		ℓ_e=1.73 ℓ_u	
Six equal concentrated loads at 1/7 points with lateral support at 1/7 points		ℓ_e=1.78 ℓ_u	
Seven or more equal concentrated loads, evenly spaced, with lateral support at points of load application		ℓ_e=1.84 ℓ_u	
Equal end moments		ℓ_e=1.84 ℓ_u	

1. For single span or cantilever bending members with loading conditions not specified in Table 3.3.3:
ℓ_e = 2.06 ℓ_u where $\ell_u/d < 7$
ℓ_e = 1.63 ℓ_u + 3d where $7 \leq \ell_u/d \leq 14.3$
ℓ_e = 1.84 ℓ_u where $\ell_u/d > 14.3$
2. Multiple span applications shall be based on table values or engineering analysis.

3.3.3.6 The slenderness ratio, R_B, for bending members shall be calculated as follows:

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} \qquad (3.3\text{-}5)$$

3.3.3.7 The slenderness ratio for bending members, R_B, shall not exceed 50.

Structures II
Arch 324

Name 1 ______
Name 2 ______
Name 3 ______

Wood Beams

Description

This project uses observation and calculation to understand how bracing effects the stability and performance of a wooden beam.

Goals

To observe the bending behavior of a simple span beam through physical modeling.
To see the effects of unbraced length on lateral buckling.
To calculate the effective length for different bracing conditions.

Procedure

1. Set the 1/16"x1/2" basswood stick in the support as shown. This approximately models a 2x12 (1.5" x1.25") at 1:24 scale with a span of 45 FT.
2. First load the unbraced stick (ℓ_u = 45') at the center line with you finger. Observe the lateral buckling failure. Find ℓ_e and calculate R_B from the NDS formula 3.3-5
3. Next brace the stick at the center load point (ℓ_u = 22.5'). Again, observe the lateral failure. Find ℓ_e and calculate R_B from the NDS formula 3.3-5
4. Now brace and load the stick at the 1/3 points (ℓ_u = 15') and observer the lateral failure. Find ℓ_e and calculate R_B from the NDS formula 3.3-5
5. Finally, brace and load the stick at the 1/4 points (ℓ_u = 11.25') and again observer the failure. Find ℓ_e and calculate R_B from the NDS formula 3.3-5
6. Compare the R_B values found for the 4 bracing conditions and note how the bracing effects the slenderness. Note which case comes closest to 50 and which one is the smallest. As R_B decreases, the beam gets more stable and stronger.

Beam Properties: b = 1.5 IN d = 11.25 IN L = 45 FT = 540 IN

ℓ_u	ℓ_e	R_B
45' = 540"		
22.5'=270"		
15' = 180"		
11.25'=135"		

Lab01

Structures II — Name 1 ____________________

Arch 324 — Name 2 ____________________

Name 3 ____________________

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4. Now brace and load the stick at the 1/3 points (ℓ_u = 15') and observer the lateral failure. Find ℓ_e and calculate R_B from the NDS formula 3.3-5
5. Finally, brace and load the stick at the 1/4 points (ℓ_u = 11.25') and again observer the failure. Find ℓ_e and calculate R_B from the NDS formula 3.3-5
6. Compare the R_B values found for the 4 bracing conditions and note how the bracing effects the slenderness. Note which case comes closest to 50 and which one is the smallest. As R_B decreases, the beam gets more stable and stronger.

Beam Properties: b = 1.5 IN d = 11.25 IN L = 45 FT = 540 IN

ℓ_u	ℓ_e	R_B
45' = 540"		
22.5'=270"		
15' = 180"		
11.25'=135"		

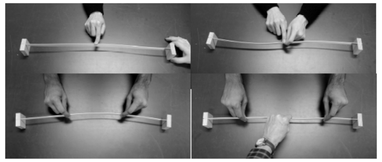


Table 3.3.3 Effective Length, ℓ_e, for Bending Members

Cantilever[1]	where $\ell_u/d < 7$		where $\ell_u/d \geq 7$
Uniformly distributed load	ℓ_e=1.33 ℓ_u		ℓ_e=0.90 ℓ_u + 3d
Concentrated load at unsupported end	ℓ_e=1.87 ℓ_u		ℓ_e=1.44 ℓ_u + 3d
Single Span Beam[1,2]	where $\ell_u/d < 7$		where $\ell_u/d \geq 7$
Uniformly distributed load	ℓ_e=2.06 ℓ_u		ℓ_e=1.63 ℓ_u + 3d
Concentrated load at center with no intermediate lateral support	ℓ_e=1.80 ℓ_u		ℓ_e=1.37 ℓ_u + 3d
Concentrated load at center with lateral support at center		ℓ_e=1.11 ℓ_u	
Two equal concentrated loads at 1/3 points with lateral support at 1/3 points		ℓ_e=1.68 ℓ_u	
Three equal concentrated loads at 1/4 points with lateral support at 1/4 points		ℓ_e=1.54 ℓ_u	
Four equal concentrated loads at 1/5 points with lateral support at 1/5 points		ℓ_e=1.68 ℓ_u	
Five equal concentrated loads at 1/6 points with lateral support at 1/6 points		ℓ_e=1.73 ℓ_u	
Six equal concentrated loads at 1/7 points with lateral support at 1/7 points		ℓ_e=1.78 ℓ_u	
Seven or more equal concentrated loads, evenly spaced, with lateral support at points of load application		ℓ_e=1.84 ℓ_u	
Equal end moments		ℓ_e=1.84 ℓ_u	

1. For single span or cantilever bending members with loading conditions not specified in Table 3.3.3:
ℓ_e = 2.06 ℓ_u where $\ell_u/d < 7$
ℓ_e = 1.63 ℓ_u + 3d where $7 \leq \ell_u/d \leq 14.3$
ℓ_e = 1.84 ℓ_u where $\ell_u/d > 14.3$
2. Multiple span applications shall be based on table values or engineering analysis.

3.3.3.6 The slenderness ratio, R_B, for bending members shall be calculated as follows:

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} \qquad (3.3\text{-}5)$$

3.3.3.7 The slenderness ratio for bending members, R_B, shall not exceed 50.

3.3.3.8 The beam stability factor shall be calculated as follows:

$$C_L = \frac{1+(F_{bE}/F_b^*)}{1.9} - \sqrt{\left[\frac{1+(F_{bE}/F_b^*)}{1.9}\right]^2 - \frac{F_{bE}/F_b^*}{0.95}} \qquad (3.3\text{-}6)$$

where:

F_b^* = reference bending design value multiplied by all applicable adjustment factors except C_{fu}, C_V (when $C_V \leq 1.0$), and C_L (see 2.3), psi

$$F_{bE} = \frac{1.20\, E_{min}'}{R_B^2}$$

3.3.3.9 See Appendix D for background information concerning beam stability calculations and Appendix F for information concerning coefficient of variation in modulus of elasticity (COV_E).

3.3.3.10 Members subjected to flexure about both principal axes (biaxial bending) shall be designed in accordance with 3.9.2.

3

DESIGN PROVISIONS AND EQUATIONS

3.4 Bending Members – Shear

3.4.1 Strength in Shear Parallel to Grain (Horizontal Shear)

3.4.1.1 The actual shear stress parallel to grain or shear force at any cross section of the bending member shall not exceed the adjusted shear design value. A check of the strength of wood bending members in shear perpendicular to grain is not required.

3.4.1.2 The shear design procedures specified herein for calculating f_v at or near points of vertical support are limited to solid flexural members such as sawn lumber, structural glued laminated timber, structural composite lumber, or mechanically laminated timber beams. Shear design at supports for built-up components containing load-bearing connections at or near points of support, such as between the web and chord of a truss, shall be based on test or other techniques.

3.4.2 Shear Design Equations

The actual shear stress parallel to grain induced in a sawn lumber, structural glued laminated timber, structural composite lumber, or timber pole or pile bending member shall be calculated as follows:

$$f_v = \frac{VQ}{Ib} \qquad (3.4\text{-}1)$$

For a rectangular bending member of breadth, b, and depth, d, this becomes:

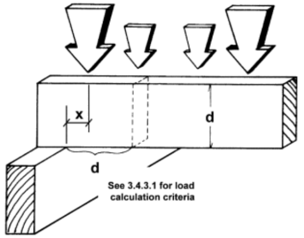
$$f_v = \frac{3V}{2bd} \qquad (3.4\text{-}2)$$

3.4.3 Shear Design

3.4.3.1 When calculating the shear force, V, in bending members:

(a) For beams supported by full bearing on one surface and loads applied to the opposite surface, uniformly distributed loads within a distance from supports equal to the depth of the bending member, d, shall be permitted to be ignored. For beams supported by full bearing on one surface and loads applied to the opposite surface, concentrated loads within a distance, d, from supports shall be permitted to be multiplied by x/d where x is the distance from the beam support face to the load (see Figure 3C).

Figure 3C Shear at Supports



Lab01

→ Group work instructions

Please form groups of 2 to 4 students.

Please do not forget to write all group members' names on both sheets.

Return the completed sheets to me at the end of the session.

Please ensure that you attend the recitation sessions.

If you are unable to attend a session, send me an email so that we can discuss how to proceed. *Email: arfazel@umich.edu*