



# ARCHITECTURE 324

## Structures II

Recitation 03  
Sections 04&05

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GSI  
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Jan 31, 2025

# Office Hours

## → Office Hours

→ Day: Fridays, 12:00 PM - 1:00 PM

→ Location Options:

- In-person meetings: [2223B]
- Virtual meetings via Zoom

Please make sure to sign up at least 24 hours in advance to allow for proper scheduling via this link:

<https://docs.google.com/forms/d/e/1FAIpQLSdOb4gAc6SoCdsMAZP4zKrn3ecPyGt6dwVahVcOD3EqXGG-oA/viewform?usp=dialog>

If the slots are fully booked or if you have a time conflict, please email me directly to find an alternative time ([arfazel@umich.edu](mailto:arfazel@umich.edu))

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  - Final Tower Test

# Wood Column Analysis

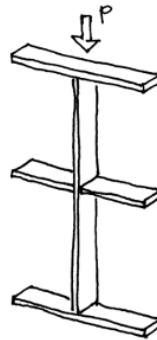
## Analysis of Wood Columns

### Data:

- Column – size, length
- Support conditions
- Material properties –  $F_c$ ,  $E_{min}$
- Load

### Required:

- Pass/Fail or margin of safety
1. Calculate slenderness ratio  $l_e/d$   
largest ratio governs. Must be  $< 50$
  2. Find adjustment factors  
 $C_D C_M C_t C_F C_i$
  3. Calculate  $C_P$
  4. Determine allowable  $F'_c$  by multiplying the  
tabulated  $F_c$  by all the above factors
  5. Calculate the actual stress:  $f_c = P/A$
  6. Compare Allowable and Actual stress.  
 $F'_c > f_c$  passes



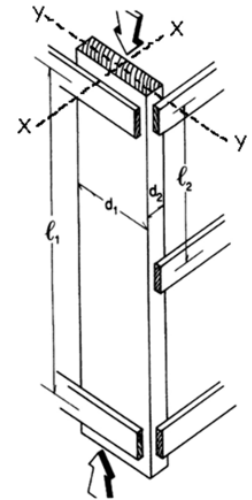
## Capacity Analysis of Columns

### Data:

- Column – size, length
- Support conditions
- Material properties –  $F_c$ ,  $E_{min}$

### Required:

- Maximum Load Capacity,  $P_{max}$
1. Calculate slenderness ratio  $l_e/d$   
largest ratio governs. Must be  $< 50$
  2. Find adjustment factors  
 $C_D C_M C_t C_F C_i$
  3. Calculate  $C_P$
  4. Determine  $F'_c$  by multiplying the tabulated  $F_c$   
by all the above factors
  5. Set actual stress = allowable,  $f_c = F'_c$
  6. Find the maximum allowable load  
 $P_{max} = F'_c A$





# Wood Column Design

## Timber Column Design

### Given:

- Lumber species, grade
- Conditions of use
- Load

### Required:

- column size
1. Find adjustment factors (all except  $C_P$ )  
 $C_D C_M C_t C_F C_i$
  2. Guess  $C_P$
  3. Estimate Area and  $d$  (based on bracing)
  4. Calculate slenderness ratio  $l_e/d$   
largest ratio governs. Must be  $< 50$
  5. Calculate  $C_P$
  6. Determine  $F'_c$  by multiplying the tabulated  $F_c$  by all the above factors
  7. Revise Area:  $A = P/F'_c$
  8. Revise  $C_P$
  9. Repeat until  $F'_c > P/A$



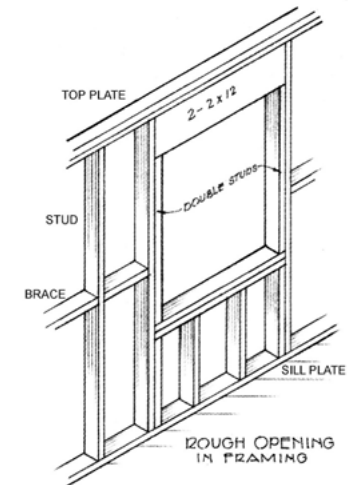
## Stud Wall Design

### Given:

- Lumber species, grade and size
- Conditions of use
- Load

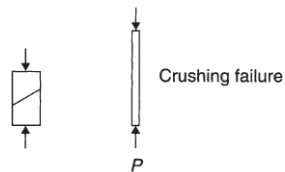
### Required:

- Stud spacing
1. Calculate slenderness ratio  $l_e/d$   
largest ratio governs. Must be  $< 50$
  2. Find adjustment factors (all except  $C_P$ )  
 $C_D C_M C_t C_F C_i$
  3. Calculate  $C_P$
  4. Determine  $F'_c$  by multiplying the tabulated  $F_c$  by all the above factors
  5. Set actual stress = allowable:  $f_c = F'_c$
  6. Find the capacity of one stud:  $P_{max} = F'_c A$
  7. Find allowable spacing (12", 16" or 24" o.c.)
  8. Check bearing.

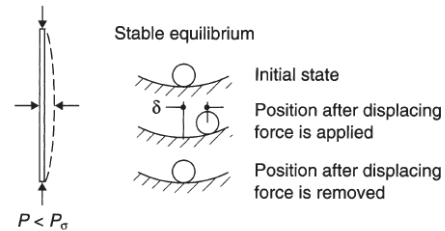


# Quick refresher

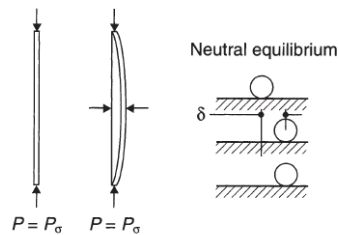
## Columns (General principles)



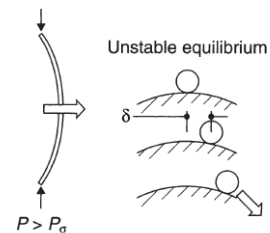
- (a) Short column: The member fails by a crushing action. Its strength depends on the cross-sectional area and crushing strength of the material used.



- (b) Long column (load less than the buckling load): The member is in a state of stable equilibrium. If the member is displaced slightly at midheight, it will spring back to its original configuration.

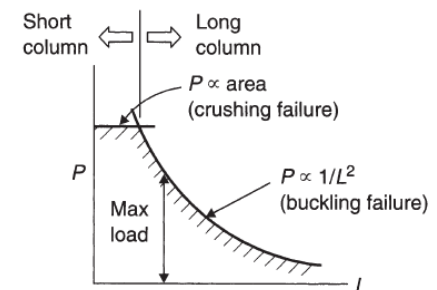


- (c) Long column (load = buckling load): When the load on the column reaches the critical buckling load, the member changes into a state of neutral equilibrium. If the member is displaced from its original linear configuration, it will remain in its new position and not spring back. The buckling load is the maximum load that the column can carry.



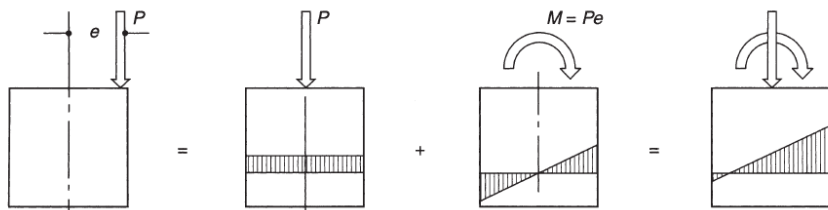
- (d) Long column (load greater than the buckling load): If the load on the column is increased beyond the critical buckling load, the member changes into a state of unstable equilibrium. The member will continue to deform under a constant load level until it finally snaps.

- (e) General relationship between member length and buckling load. Short members tend to crush while longer ones tend to buckle. The longer the member, the lower its load-carrying capacity (the buckling load of a long member varies inversely with the square of its length).

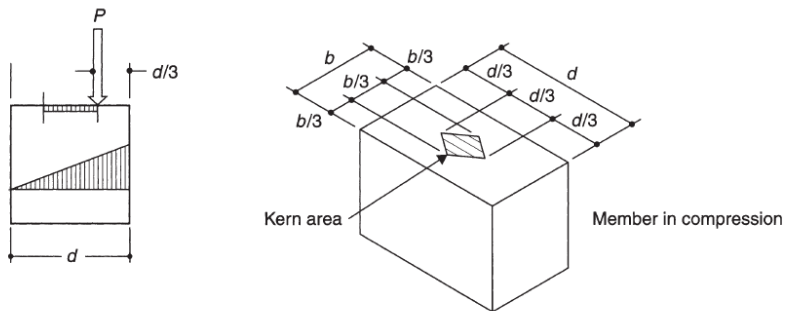


# Quick refresher

## Columns (General principles)



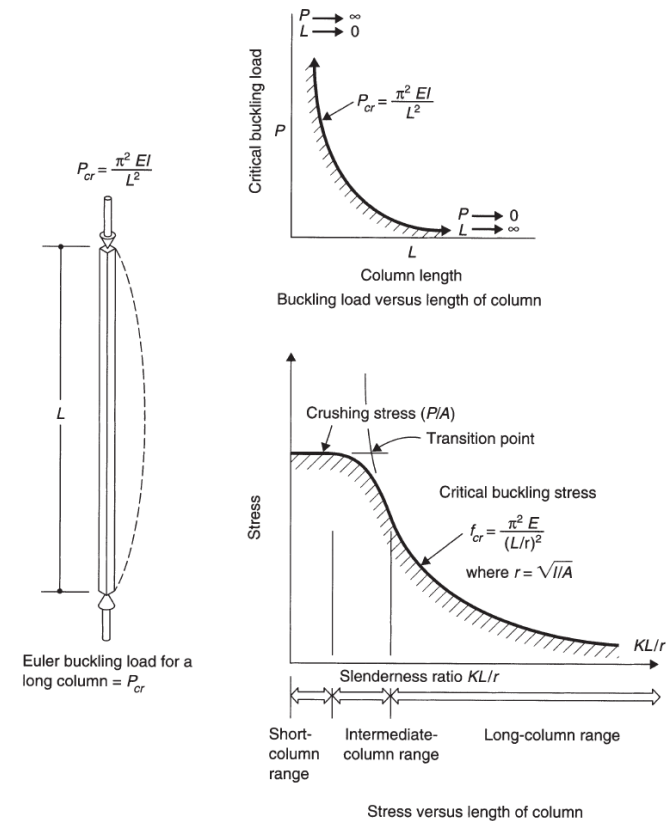
(a) Stress distributions under eccentric loads



(b) Placing a load anywhere within the middle one-third of the section produces only compressive stresses in the cross section. Placing a load outside the middle one-third leads to the development of tensile stresses on one face of the member.

FIGURE 7.2 Eccentrically loaded columns.

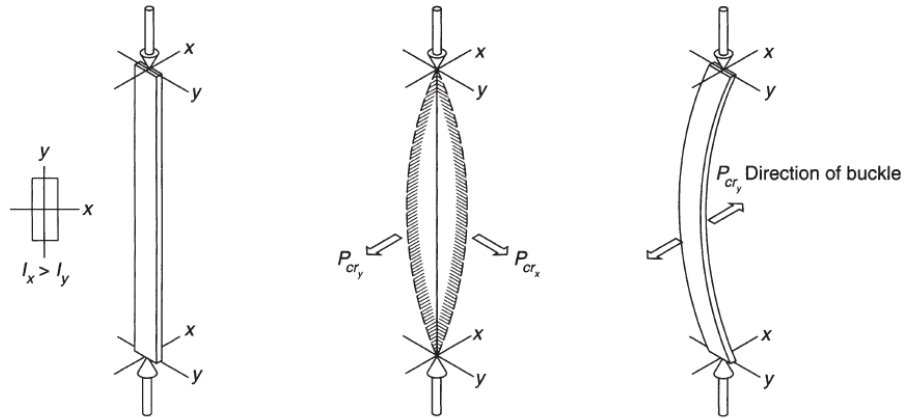
FIGURE 7.3 Euler buckling in long columns.



# Quick refresher

## Columns (General principles)

FIGURE 7.4 Buckling of asymmetric cross sections.



- (a) The moment of inertia about one axis is greater than that about the other.
- (b) The member can potentially fail by buckling about either axis. The load required to cause it to buckle about the stronger axis, however, exceeds the load that will cause buckling about the weaker axis.
- (c) Consequently, the member will buckle at  $P_{cr_y} = \pi^2 EI_y / L^2$  in the mode shown.



# Quick refresher

## Columns (General principles)

FIGURE 7.5 Effects of end conditions on critical buckling loads.

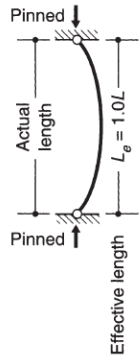
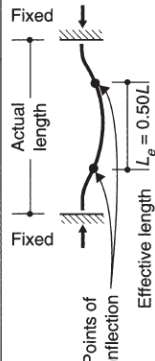
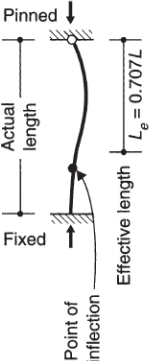
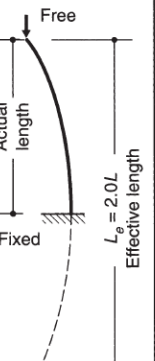
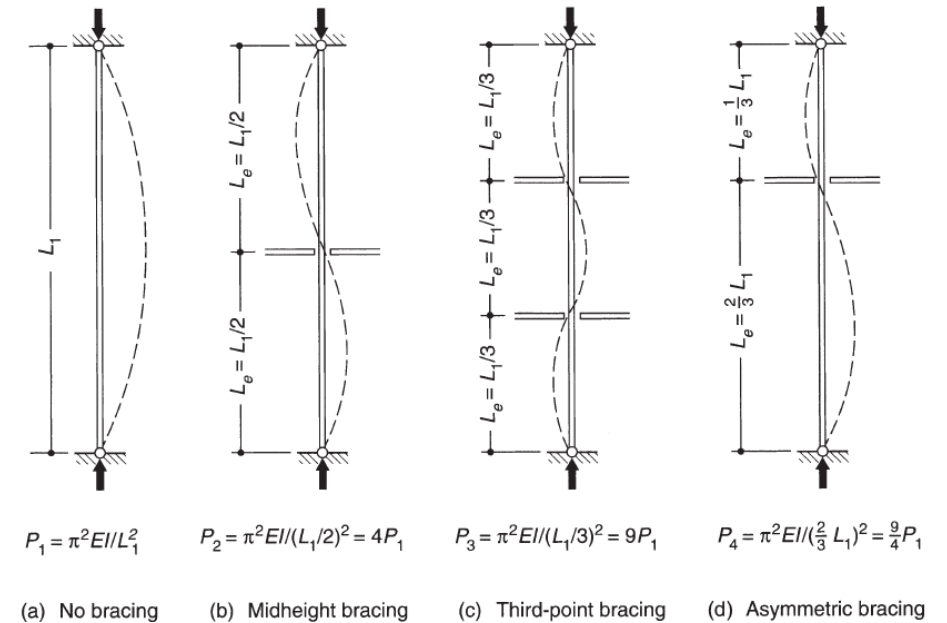
Both ends pinned	Both ends fixed	One end pinned and one end fixed	One end free and one end fixed
 <p>Actual length <math>L</math> Effective length <math>L_e = 1.0L</math></p>	 <p>Actual length <math>L</math> Effective length <math>L_e = 0.50L</math> Points of inflection</p>	 <p>Actual length <math>L</math> Effective length <math>L_e = 0.707L</math> Point of inflection</p>	 <p>Actual length <math>L</math> Effective length <math>L_e = 2.0L</math></p>
$k = 1.0$	$k = 0.50$	$k = 0.7$	$k = 2.0$
$P = \pi^2 EI / L^2$	$P = \pi^2 EI / (\frac{1}{2}L)^2 = 4\pi^2 EI / L^2$	$P = \pi^2 EI / (0.7L)^2 = 2\pi^2 EI / L^2$	$P = \pi^2 EI / (2L)^2 = \frac{1}{4}\pi^2 EI / L^2$
A	B	C	D

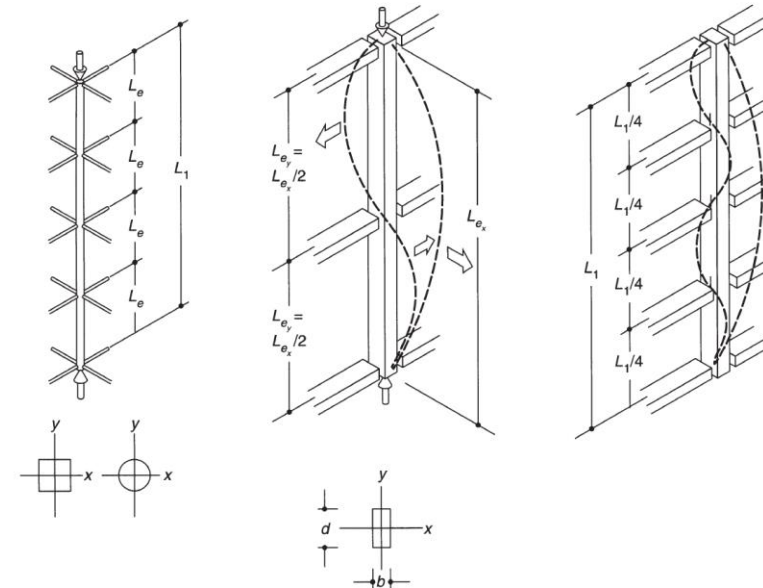
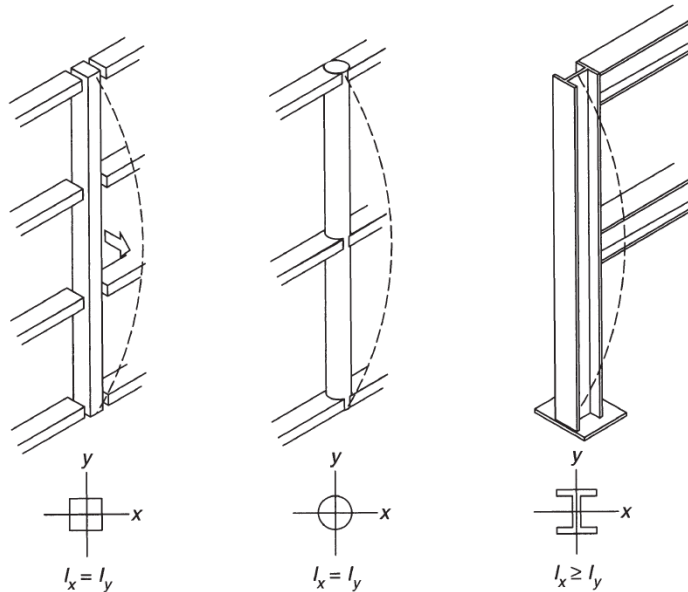
FIGURE 7.6 Effects of lateral bracing on column buckling. Bracing a column changes its buckling mode and, consequently, its effective length. The more a column is braced, the shorter its effective length becomes and the greater is the load required to cause buckling. If bracing is used, it is usually more effective when placed symmetrically.



# Quick refresher

## Columns (General principles)

**FIGURE 7.9** Ineffective use of bracing. Columns always buckle in the mode associated with the highest slenderness ratio ( $L/r$ ). The columns shown will buckle as illustrated. The corresponding buckling loads relate to an unbraced or unsupported column length of  $L$  and are the same as if the columns were not braced at all. The bracing patterns shown are ineffective in increasing the load-carrying capability of the columns.



**FIGURE 7.10** Effective use of bracing.

(a) The load-carrying capacity of a symmetrical column can be increased to the maximum extent possible by putting in a sufficient number of brace points such that the effective length of the column is reduced to the point where short-column behavior obtains. Consequently, no portion of the column would be susceptible to buckling. The maximum load would be associated with crushing rather than buckling.

(b) When the plane of the bracing possible is fixed, the load-carrying capacity of a column of a given cross-sectional area can be increased by varying the relative proportions of the cross section with the objective of making the column equally likely to buckle in any of its possible modes (i.e., making the slenderness ratios of the column similar in all directions). This occurs when the following ratios obtain:

$$I_x/I_y = (L_{e_y}/L_{e_x})^2 \text{ or } r_x/r_y = L_{e_y}/L_{e_x}$$

# Problem Set 03

## 3. Wood Column Analysis

For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. Moisture Content = 15%.  $C_t = C_i = 1.0$ . Assume pinned end conditions ( $K=1$ ).

DATASET: 1

-2-

-3-

Wood Species

SPRUCE-  
PINE-FIR

Wood Grade

No.1/No.2

Strong Axis Length,  $L_1$

11 FT

Weak Axis Length,  $L_2$

3.66666667 FT

Narrow Width,  $d_2$

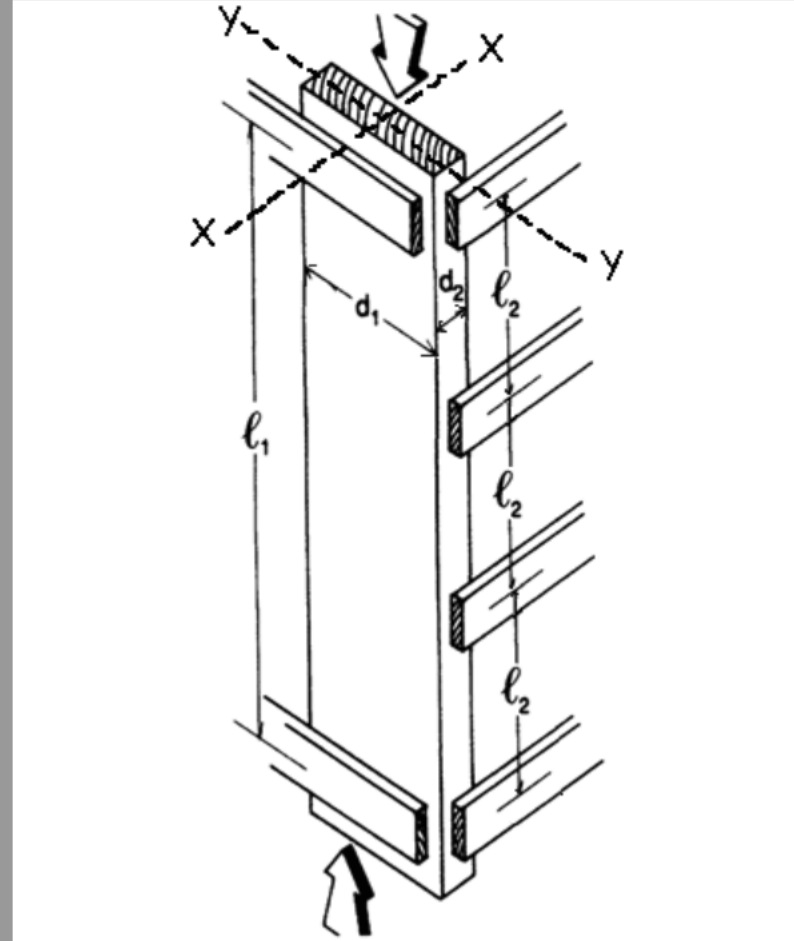
2 IN

Wide Width,  $d_1$

8 IN

LoadType

Live Load



# Problem Set 03

#Q1: Tabulated Allow. Compressive Stress,

#Q2: Tabulated Minimum Modulus of Elasticity, Emin

## 3. Wood Column Analysis

For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. Moisture Content = 15%. Ct = Ci = 1.0. Assume pinned end conditions (K=1).

DATASET: 1

-2-

-3-

Wood Species	SPRUCE-PINE-FIR
Wood Grade	No. 1/No. 2

According to the table, for “Spruce-Pine\_Fir grade, No. 1 and No. 2, we have the following:

$F_c = 1150 \text{ PSI}$  $E_{min} = 510,000$

Table 4A (Cont.) Reference Design Values for Visually Graded Dimension Lumber (2" - 4" thick)<sup>1,2,3</sup>

(All species except Southern Pine— see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

USE WITH TABLE 4A ADJUSTMENT FACTORS											
Species and commercial grade	Size classification	Design values in pounds per square inch (psi)						Modulus of Elasticity	Specific Gravity <sup>4</sup> G	Grading Rules Agency	
		Bending F <sub>b</sub>	Tension parallel to grain F <sub>t</sub>	Shear parallel to grain F <sub>v</sub>	Compression perpendicular to grain F <sub>c⊥</sub>	Compression parallel to grain F <sub>c</sub>	E				E <sub>min</sub>
RED OAK											
Select Structural	2" & wider	1,150	675	170	820	1,000	1,400,000	510,000	0.67	NELMA	
No. 1		825	500	170	820	825	1,300,000	470,000			
No. 2		800	475	170	820	625	1,200,000	440,000			
No. 3	2" & wider	475	275	170	820	375	1,100,000	400,000			
Stud		625	375	170	820	400	1,100,000	400,000			
Construction		925	550	170	820	850	1,200,000	440,000			
Standard	2" - 4" wide	525	300	170	820	650	1,100,000	400,000			
Utility		250	150	170	820	425	1,000,000	370,000			
REDWOOD											
Select Structural	2" & wider	1,100	625	160	425	1,100	1,100,000	400,000	0.37	RIS	
No. 1		775	450	160	425	900	1,100,000	400,000			
No. 2		725	425	160	425	700	1,000,000	370,000			
No. 3	2" & wider	425	250	160	425	400	900,000	330,000			
Stud		575	325	160	425	450	900,000	330,000			
Construction		825	475	160	425	925	900,000	330,000			
Standard	2" - 4" wide	450	275	160	425	725	900,000	330,000			
Utility		225	125	160	425	475	800,000	290,000			
SPRUCE-PINE-FIR											
Select Structural	2" & wider	1,250	700	135	425	1,400	1,500,000	550,000	0.42	NLGA	
No. 1/No. 2		875	450	135	425	1,150	1,400,000	510,000			
No. 3		500	250	135	425	650	1,200,000	440,000			
Stud	2" & wider	675	350	135	425	725	1,200,000	440,000			
Construction		1,000	500	135	425	1,400	1,300,000	470,000			
Standard		550	275	135	425	1,150	1,200,000	440,000			
Utility	2" - 4" wide	275	125	135	425	750	1,100,000	400,000			
SPRUCE-PINE-FIR (SOUTH)											
Select Structural	2" & wider	1,300	575	135	335	1,200	1,300,000	470,000	0.36	NELMA WCLIB WWPA	
No. 1		875	400	135	335	1,050	1,200,000	440,000			
No. 2		775	350	135	335	1,000	1,100,000	400,000			
No. 3	2" & wider	450	200	135	335	575	1,000,000	370,000			
Stud		600	275	135	335	625	1,000,000	370,000			
Construction		875	400	135	335	1,200	1,000,000	370,000			
Standard	2" - 4" wide	500	225	135	335	1,000	900,000	330,000			
Utility		225	100	135	335	675	900,000	330,000			
WESTERN CEDARS											
Select Structural	2" & wider	1,000	600	155	425	1,000	1,100,000	400,000	0.36	WCLIB WWPA	
No. 1		725	425	155	425	825	1,000,000	370,000			
No. 2		700	425	155	425	650	1,000,000	370,000			
No. 3	2" & wider	400	250	155	425	375	900,000	330,000			
Stud		550	325	155	425	400	900,000	330,000			
Construction		800	475	155	425	850	900,000	330,000			
Standard	2" - 4" wide	450	275	155	425	650	800,000	290,000			
Utility		225	125	155	425	425	800,000	290,000			
WESTERN WOODS											
Select Structural	2" & wider	900	400	135	335	1,050	1,200,000	440,000	0.36	WCLIB WWPA	
No. 1		675	300	135	335	950	1,100,000	400,000			
No. 2		675	300	135	335	900	1,000,000	370,000			
No. 3	2" & wider	375	175	135	335	525	900,000	330,000			
Stud		525	225	135	335	575	900,000	330,000			
Construction		775	350	135	335	1,100	1,000,000	370,000			
Standard	2" - 4" wide	425	200	135	335	925	900,000	330,000			
Utility		200	100	135	335	600	800,000	290,000			

# Problem Set 03

## #Q3: Load Duration Factor, CD

### 3. Wood Column Analysis

For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. Moisture Content = 15%. Ct = Ci = 1.0. Assume pinned end conditions (K=1).

DATASET: 1   -2-   -3-

Wood Species	SPRUCE- PINE-FIR
Wood Grade	No.1/No.2
Strong Axis Length, L1	11 FT
Weak Axis Length, L2	3.666666667 FT
Narrow Width, d2	2 IN
Wide Width, d1	8 IN
LoadType	Live Load

According to the table, for “Live load”, we have the following:

|  $C_D = 1$

#### 2.1 General

##### 2.1.1 General Requirement

Each wood structural member or connection shall be of sufficient size and capacity to carry the applied loads without exceeding the adjusted design values specified herein.

2.1.1.1 For ASD, calculation of adjusted design values shall be determined using applicable ASD adjustment factors specified herein.

2.1.1.2 For LRFD, calculation of adjusted design values shall be determined using applicable LRFD adjustment factors specified herein.

##### 2.1.2 Responsibility of Designer to Adjust for Conditions of Use

Adjusted design values for wood members and connections in particular end uses, shall be appropriate for the conditions under which the wood products are used, taking into account conditions such as the differences in wood strength properties with different moisture contents, load durations, and types of treatment. Common end use conditions are addressed in this Specification. It shall be the final responsibility of the designer to relate design assumptions with design values, and to make design value adjustments appropriate to the end use conditions.

#### 2.2 Reference Design Values

Reference design values and design value adjustments for wood products in 1.1.1.1 are based on methods specified in each of the wood product chapters. Chapters 4 through 10 contain design provisions for sawn lumber, glued laminated timber, poles and piles, prefabricated wood I-joists, structural composite lumber, wood structural panels, and cross-laminated timber, respectively. Chapters 11 through 14 contain design provisions for connections. Reference design values are for normal load duration under the moisture service conditions specified.

ber, wood structural panels, and cross-laminated timber, respectively. Chapters 11 through 14 contain design provisions for connections. Reference design values are for normal load duration under the moisture service conditions specified.

#### 2.3 Adjustment of Reference Design Values

##### 2.3.1 Applicability of Adjustment Factors

Reference design values shall be multiplied by all applicable adjustment factors to determine adjusted design values. The applicability of adjustment factors to sawn lumber, structural glued laminated timber, poles and piles, prefabricated wood I-joists, structural composite lumber, wood structural panels, cross-laminated timber, and connection design values is defined in 4.3, 5.3, 6.3, 7.3, 8.3, 9.3, 10.3, and 11.3, respectively.

reference design values except modulus of elasticity, E, modulus of elasticity for beam and column stability,  $E_{min}$ , and compression perpendicular to grain,  $F_{ci}$ , based on a deformation limit (see 4.2.6) shall be multiplied by the appropriate load duration factor,  $C_D$ , from Table 2.3.2 or Figure B1 (see Appendix B) to take into account the change in strength of wood with changes in load duration.

##### 2.3.2 Load Duration Factor, $C_D$ (ASD Only)

2.3.2.1 Wood has the property of carrying substantially greater maximum loads for short durations than for long durations of loading. Reference design values apply to normal load duration. Normal load duration represents a load that fully stresses a member to its allowable design value by the application of the full design load for a cumulative duration of approximately ten years. When the cumulative duration of the full maximum load does not exceed the specified time period, all

2.3.2.2 The load duration factor,  $C_D$ , for the shortest duration load in a combination of loads shall apply for that load combination. All applicable load combinations shall be evaluated to determine the critical load combination. Design of structural members and connections shall be based on the critical load combination (see Appendix B.2).

2.3.2.3 The load duration factors,  $C_D$ , in Table 2.3.2 and Appendix B are independent of load combination factors, and both shall be permitted to be used in design calculations (see 1.4.4 and Appendix B.4).

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Table 2.3.2 Frequently Used Load Duration Factors,  $C_D$ <sup>1</sup>

Load Duration	$C_D$	Typical Design Loads
Permanent	0.9	Dead Load
Ten years	1.0	Occupancy Live Load
Two months	1.15	Snow Load
Seven days	1.25	Construction Load
Ten minutes	1.6	Wind/Earthquake Load
Impact <sup>2</sup>	2.0	Impact Load

1. Load duration factors shall not apply to reference modulus of elasticity, E, reference modulus of elasticity for beam and column stability,  $E_{min}$ , nor to reference compression perpendicular to grain design values,  $F_{ci}$ , based on a deformation limit.  
2. Load duration factors greater than 1.6 shall not be used in the design of structural members pressure-treated with water-borne preservatives (see Reference 30), or fire retardant chemicals. Load duration factors greater than 1.6 shall not be used in the design of connections or wood structural panels.

##### 2.3.3 Temperature Factor, $C_t$

Reference design values shall be multiplied by the temperature factors,  $C_t$ , in Table 2.3.3 for structural members that will experience sustained exposure to elevated temperatures up to 150°F (see Appendix C).

##### 2.3.4 Fire Retardant Treatment

The effects of fire retardant chemical treatment on strength shall be accounted for in the design. Adjusted design values, including adjusted connection design values, for lumber and structural glued laminated timber pressure-treated with fire retardant chemicals shall be obtained from the company providing the treatment and redrying service. Load duration factors greater than 1.6 shall not apply to structural members pressure-treated with fire retardant chemicals (see Table 2.3.2).

Table 2.3.3 Temperature Factor,  $C_t$

Reference Design Values	In-Service Moisture Conditions <sup>1</sup>	$C_t$		
		≤100°F	100°F < T ≤ 125°F	125°F < T ≤ 150°F
$F_b$ , E, $E_{min}$	Wet or Dry	1.0	0.9	0.9
$F_v$ , $F_c$ , and $F_{ci}$	Dry	1.0	0.8	0.7
	Wet	1.0	0.7	0.5

1. Wet and dry service conditions for sawn lumber, structural glued laminated timber, prefabricated wood I-joists, structural composite lumber, wood structural panels and cross-laminated timber are specified in 4.1.4, 5.1.4, 7.1.4, 8.1.4, 9.3.3, and 10.1.5 respectively.

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##### 2.3.5 Format Conversion Factor, $K_F$ (LRFD Only)

For LRFD, reference design values shall be multiplied by the format conversion factor,  $K_F$ , specified in Table 2.3.5. The format conversion factor,  $K_F$ , shall not apply for designs in accordance with ASD methods specified herein.

##### 2.3.6 Resistance Factor, $\phi$ (LRFD Only)

For LRFD, reference design values shall be multiplied by the resistance factor,  $\phi$ , specified in Table 2.3.6. The resistance factor,  $\phi$ , shall not apply for designs in accordance with ASD methods specified herein.

##### 2.3.7 Time Effect Factor, $\lambda$ (LRFD Only)

For LRFD, reference design values shall be multiplied by the time effect factor,  $\lambda$ , specified in Appendix N.3.3. The time effect factor,  $\lambda$ , shall not apply for designs in accordance with ASD methods specified herein.



# Problem Set 03

## #Q4: Size Factor, CF

### 3. Wood Column Analysis

For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. Moisture Content = 15%. Ct = Ci = 1.0. Assume pinned end conditions (K=1).

DATASET: 1 -2- -3-

Wood Species	SPRUCE- PINE-FIR
Wood Grade	No.1/No.2
Strong Axis Length, L1	11 FT
Weak Axis Length, L2	3.666666667 FT
Narrow Width, d2	2 IN
Wide Width, d1	8 IN
LoadType	Live Load

According to the table, for 2×8” lumber, we have the following:

|  $C_F = 1.05$

Table 4A Adjustment Factors

#### Repetitive Member Factor, Cr

Bending design values, Fb, for dimension lumber 2" to 4" thick shall be multiplied by the repetitive member factor, Cr = 1.15, when such members are used as joists, truss chords, rafters, studs, planks, decking, or similar members which are in contact or spaced not more than 24" on center, are not less than 3 in number and are joined by floor, roof, or other load distributing elements adequate to support the design load.

#### Wet Service Factor, CM

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors, CM					
Fb	Fi	Fv	FcL	Fc	E and Emin
0.85*	1.0	0.97	0.67	0.8**	0.9

\* when (Fb)(Cp) ≤ 1,150 psi, CM = 1.0

\*\* when (Fb)(Cp) ≤ 750 psi, CM = 1.0

#### Flat Use Factor, Cfu

Bending design values adjusted by size factors are based on edgewise use (load applied to narrow face). When dimension lumber is used flatwise (load applied to wide face), the bending design value, Fb, shall also be permitted to be multiplied by the following flat use factors:

Flat Use Factors, Cfu		
Width (depth)	Thickness (breadth)	
	2" & 3"	4"
2" & 3"	1.0	—
4"	1.1	1.0
5"	1.1	1.05
6"	1.15	1.05
8"	1.15	1.05
10" & wider	1.2	1.1

#### NOTE

To facilitate the use of Table 4A, shading has been employed to distinguish design values based on a 4" nominal width (Construction, Standard, and Utility grades) or a 6" nominal width (Stud grade) from design values based on a 12" nominal width (Select Structural, No.1 & Btr, No.1, No.2, and No.3 grades).

#### Size Factor, CF

Tabulated bending, tension, and compression parallel to grain design values for dimension lumber 2" to 4" thick shall be multiplied by the following size factors:

Size Factors, C <sub>F</sub>					
Grades	Width (depth)	F <sub>b</sub>		F <sub>t</sub>	F <sub>c</sub>
		Thickness (breadth)			
		2" & 3"	4"		
Select Structural, No.1 & Btr, No.1, No.2, No.3	2", 3", & 4"	1.5	1.5	1.5	1.15
	5"	1.4	1.4	1.4	1.1
	6"	1.3	1.3	1.3	1.1
	8"	1.2	1.3	1.2	1.05
	10"	1.1	1.2	1.1	1.0
	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
Stud	2", 3", & 4"	1.1	1.1	1.1	1.05
	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3 Grade tabulated design values and size factors			
Construction, Standard	2", 3", & 4"	1.0	1.0	1.0	1.0
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	—	0.4	0.6

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# Problem Set 03

## #Q5: Factored Allow. Modulus of Elasticity, E'min

### 3. Wood Column Analysis

For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. **Moisture Content = 15%.  $C_t = C_i = 1.0$ . Assume pinned end conditions ( $K=1$ ).**

$$\begin{aligned}
 E'_{min} &= E_{min} (C_m \cdot C_t \cdot C_i \cdot C_T) \\
 &= 510000(1 \cdot 1 \cdot 1 \cdot 1) \\
 &= 510000
 \end{aligned}$$

#### 4.4 Special Design Considerations

##### 4.4.1 Stability of Bending Members

4.4.1.1 Sawn lumber bending members shall be designed in accordance with the lateral stability calculations in 3.3.3 or shall meet the lateral support requirements in 4.4.1.2 and 4.4.1.3.

4.4.1.2 As an alternative to 4.4.1.1, rectangular sawn lumber beams, rafters, joists, or other bending members, shall be designed in accordance with the following provisions to provide restraint against rotation or lateral displacement. If the depth to breadth,  $d/b$ , based on nominal dimensions is:

- $d/b \leq 2$ ; no lateral support shall be required.
- $2 < d/b \leq 4$ ; the ends shall be held in position, as by full depth solid blocking, bridging, hangers, nailing, or bolting to other framing members, or other acceptable means.
- $4 < d/b \leq 5$ ; the compression edge of the member shall be held in line for its entire length to prevent lateral displacement, as by adequate sheathing or subflooring, and ends at point of bearing shall be held in position to prevent rotation and/or lateral displacement.
- $5 < d/b \leq 6$ ; bridging, full depth solid blocking or diagonal cross bracing shall be installed at intervals not exceeding 8 feet, the compression edge of the member shall be held in line as by adequate sheathing or subflooring, and the ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.
- $6 < d/b \leq 7$ ; both edges of the member shall be held in line for their entire length and ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

4.4.1.3 If a bending member is subjected to both flexure and axial compression, the depth to breadth ratio shall be no more than 5 to 1 if one edge is firmly held in line. If under all combinations of load, the unbraced edge of the member is in tension, the depth to breadth ratio shall be no more than 6 to 1.

##### 4.4.2 Wood Trusses

4.4.2.1 Increased chord stiffness relative to axial loads where a 2" x 4" or smaller sawn lumber truss compression chord is subjected to combined flexure and axial compression under dry service condition and has 3/8" or thicker wood structural panel sheathing nailed to the narrow face of the chord in accordance with code required roof sheathing fastener schedules (see References 32, 33, and 34), shall be permitted to be accounted for by multiplying the reference modulus of elasticity design value for beam and column stability,  $E_{min}$ , by the buckling stiffness factor,  $C_T$ , in column stability calculations (see 3.7 and Appendix H). When  $\ell_e \leq 96"$ ,  $C_T$  shall be calculated as follows:

$$C_T = 1 + \frac{K_{tr} \ell_e^2}{10E}$$

where:

$\ell_e$  = effective column length of truss compression chord (see 3.7), in.

$K_{tr}$  = 2300 for wood seasoned to 19% moisture content or less at the time of wood structural panel sheathing attachment.

= 1200 for unseasoned or partially seasoned wood at the time of wood structural panel sheathing attachment.

$K_{tr} = 1 - 1.645(COV_k)$

= 0.59 for visually graded lumber

= 0.75 for machine evaluated lumber (MEL)

= 0.82 for products with  $COV_k \leq 0.11$  (see Appendix F.2)

When  $\ell_e > 96"$ ,  $C_T$  shall be calculated based on  $\ell_e$ .

4.4.2.2 For additional information concerning metal plate connected wood trusses see Reference 9.

4

SAWN LUMBER

Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

	ASD only	ASD and LRFD										LRFD only		
		Load Duration Factor	Wet Service Factor	Temperature Factor	Beam Stability Factor	Size Factor	Flat Use Factor	Incising Factor	Repetitive Member Factor	Column Stability Factor	Buckling Stiffness Factor	Bearing Area Factor	Format Conversion Factor	Resistance Factor
													$K_F$	$\phi$
$F_b' = F_b$	X	$C_D$	$C_M$	$C_t$	$C_L$	$C_F$	$C_{Fu}$	$C_i$	$C_T$	-	-	-	2.54	0.85
$F_t' = F_t$	X	$C_D$	$C_M$	$C_t$	-	$C_F$	-	$C_i$	-	-	-	-	2.70	0.80
$F_v' = F_v$	X	$C_D$	$C_M$	$C_t$	-	-	-	$C_i$	-	-	-	-	2.88	0.75
$F_c' = F_c$	X	$C_D$	$C_M$	$C_t$	-	$C_F$	-	$C_i$	-	$C_P$	-	-	2.40	0.90
$F_{cL}' = F_{cL}$	X	-	$C_M$	$C_t$	-	-	-	$C_i$	-	-	-	$C_b$	1.67	0.90
$E' = E$	X	-	$C_M$	$C_t$	-	-	-	$C_i$	-	-	-	-	-	-
$E_{min}' = E_{min}$	X	-	$C_M$	$C_t$	-	-	-	$C_i$	-	-	$C_T$	-	1.76	0.85

#### Wet Service Factor, $C_M$

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors, $C_M$					
$F_b$	$F_t$	$F_v$	$F_{cL}$	$F_c$	$E$ and $E_{min}$
0.85*	1.0	0.97	0.67	0.8**	0.9

\* when  $(F_b)(C_t) \leq 1,150$  psi,  $C_M = 1.0$

\*\* when  $(F_c)(C_t) \leq 750$  psi,  $C_M = 1.0$

(depth)	2" & 3"	4"
2" & 3"	1.0	—
4"	1.1	1.0
5"	1.1	1.05
6"	1.15	1.05
8"	1.15	1.05
10" & wider	1.2	1.1

#### NOTE

To facilitate the use of Table 4A, shading has been employed to distinguish design values based on a 4" nominal width (Construction, Standard, and Utility grades) or a 6" nominal width (Stud grade) from design values based on a 12" nominal width (Select Structural, No.1 & Btr, No.1, No.2, and No.3 grades).

# Problem Set 03

#Q6: Strong Axis (x-x) Slenderness Ratio,  $l_{ex}/d_1$

#Q7: Weak Axis (y-y) Slenderness Ratio,  $l_{ey}/d_2$

#Q8: Controlling Slenderness Ratio,  $l_e/d$

Strong Axis Length,  $L_1$  11 FT

Weak Axis Length,  $L_2$  3.666666667 FT

Narrow Width,  $d_2$  2 IN

Wide Width,  $d_1$  8 IN

$$x-x \text{ axis} \left| \frac{L_1}{d_1} = \frac{11 \text{ FT} \times \frac{12 \text{ IN}}{1 \text{ FT}}}{7.25 \text{ IN}} = 16.5 \right.$$

$$y-y \text{ axis} \left| \frac{L_2}{d_2} = \frac{3.66 \text{ FT} \times \frac{12 \text{ IN}}{1 \text{ FT}}}{1.5 \text{ IN}} = 29.28 < 50 \right.$$

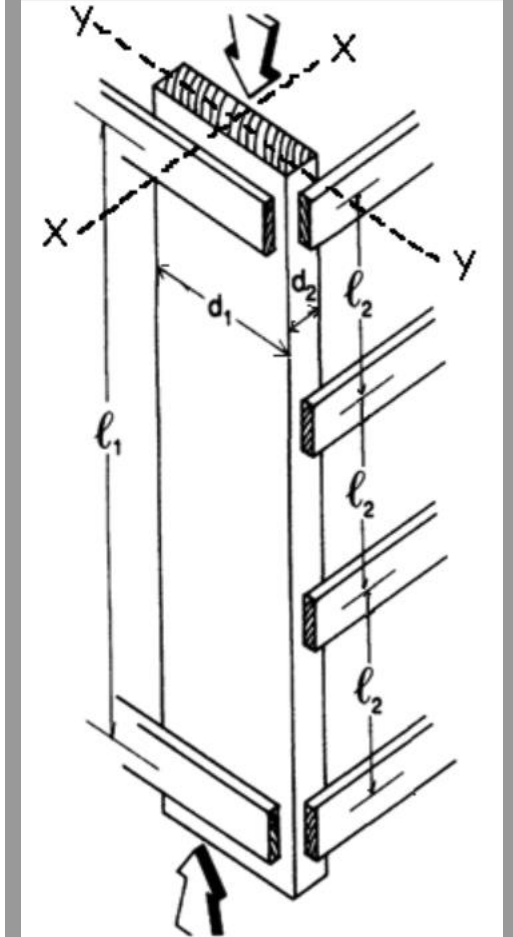
$$L_{ey} > L_{ex} \Rightarrow L_e = L_2 = 3.66 \text{ FT}$$

for buckling calculations, we select bigger  $L_e$  number

Table 1B Section Properties of Standard Dressed (S4S) Sawn Lumber

Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A in. <sup>2</sup>	X-X AXIS		Y-Y AXIS		Approximate weight in pounds per linear foot (lbs/ft) of piece when density of wood equals:							
			Section Modulus S <sub>x</sub> in. <sup>3</sup>	Moment of Inertia I <sub>x</sub> in. <sup>4</sup>	Section Modulus S <sub>y</sub> in. <sup>3</sup>	Moment of Inertia I <sub>y</sub> in. <sup>4</sup>	25 lbs/ft <sup>3</sup>	30 lbs/ft <sup>3</sup>	35 lbs/ft <sup>3</sup>	40 lbs/ft <sup>3</sup>	45 lbs/ft <sup>3</sup>	50 lbs/ft <sup>3</sup>		
Boards														
1 x 3	3/4 x 2-1/2	1.875	0.781	0.977	0.234	0.088	0.328	0.391	0.456	0.521	0.586	0.651		
1 x 4	3/4 x 3-1/2	2.625	1.531	2.680	0.328	0.123	0.456	0.547	0.638	0.729	0.820	0.911		
1 x 6	3/4 x 5-1/2	4.125	3.781	10.40	0.516	0.193	0.716	0.859	1.003	1.146	1.289	1.432		
1 x 8	3/4 x 7-1/4	5.438	6.570	23.82	0.680	0.255	0.944	1.133	1.322	1.510	1.699	1.888		
1 x 10	3/4 x 9-1/4	6.938	10.70	49.47	0.867	0.325	1.204	1.445	1.686	1.927	2.168	2.409		
1 x 12	3/4 x 11-1/4	8.438	15.82	88.99	1.055	0.396	1.465	1.758	2.051	2.344	2.637	2.930		
Dimension Lumber (see NDS 4.1.3.2) and Decking (see NDS 4.1.3.5)														
2 x 3	1-1/2 x 2-1/2	3.750	1.56	1.953	0.938	0.703	0.651	0.781	0.911	1.042	1.172	1.302		
2 x 4	1-1/2 x 3-1/2	5.250	3.06	5.359	1.313	0.984	0.911	1.094	1.276	1.458	1.641	1.823		
2 x 5	1-1/2 x 4-1/2	6.750	5.06	11.39	1.888	1.266	1.172	1.406	1.641	1.875	2.109	2.344		
2 x 6	1-1/2 x 5-1/2	8.250	7.56	20.80	2.063	1.547	1.432	1.719	2.005	2.292	2.578	2.865		
2 x 8	1-1/2 x 7-1/4	10.88	13.14	47.63	2.719	2.039	1.888	2.266	2.643	3.021	3.398	3.776		
2 x 10	1-1/2 x 9-1/4	13.88	21.39	98.93	3.469	2.602	2.409	2.891	3.372	3.854	4.336	4.818		
2 x 12	1-1/2 x 11-1/4	16.88	31.64	178.0	4.219	3.164	2.930	3.516	4.102	4.688	5.273	5.859		
2 x 14	1-1/2 x 13-1/4	19.88	43.89	290.8	4.969	3.727	3.451	4.141	4.831	5.521	6.211	6.901		
3 x 4	2-1/2 x 3-1/2	8.75	5.10	8.932	3.646	4.557	1.593	2.833	2.124	2.431	2.734	3.038		
3 x 5	2-1/2 x 4-1/2	11.25	8.44	18.98	4.688	5.859	1.959	3.242	2.734	3.125	3.516	3.906		
3 x 6	2-1/2 x 5-1/2	13.75	12.60	34.66	5.720	7.161	2.387	3.865	3.342	3.819	4.297	4.774		
3 x 8	2-1/2 x 7-1/4	18.13	21.90	79.39	7.552	9.440	3.147	3.776	4.405	5.035	5.664	6.293		
3 x 10	2-1/2 x 9-1/4	23.13	35.65	164.9	9.635	12.04	4.015	4.818	5.621	6.424	7.227	8.030		
3 x 12	2-1/2 x 11-1/4	28.13	52.73	296.6	11.72	14.65	4.883	5.859	6.836	7.813	8.789	9.766		
3 x 14	2-1/2 x 13-1/4	33.13	73.15	484.6	13.80	17.25	5.751	6.901	8.051	9.201	10.35	11.50		
3 x 16	2-1/2 x 15-1/4	38.13	96.90	738.9	15.89	19.66	6.619	7.942	9.266	10.59	11.91	13.24		
4 x 4	3-1/2 x 3-1/2	12.25	7.15	12.51	1.466	1.023	1.212	2.552	2.977	3.403	3.828	4.253		
4 x 5	3-1/2 x 4-1/2	15.75	11.81	26.58	1.888	1.266	1.519	3.381	3.828	4.274	4.721	5.168		
4 x 6	3-1/2 x 5-1/2	19.25	17.65	48.53	2.266	1.547	1.819	4.010	4.679	5.347	6.016	6.684		
4 x 8	3-1/2 x 7-1/4	25.38	30.66	111.1	2.719	1.953	2.405	5.286	6.168	7.049	7.930	8.811		
4 x 10	3-1/2 x 9-1/4	32.38	49.91	230.8	3.469	2.602	3.021	6.745	7.889	8.993	10.12	11.24		
4 x 12	3-1/2 x 11-1/4	39.38	73.83	415.3	4.219	3.164	3.620	8.203	9.570	10.94	12.30	13.67		
4 x 14	3-1/2 x 13-1/4	46.38	102.41	678.5	4.969	3.727	4.341	9.661	11.27	12.88	14.49	16.10		
4 x 16	3-1/2 x 15-1/4	53.38	135.66	1034	5.719	4.49	5.498	10.26	11.12	12.97	14.83	16.68		
Timbers (5" x 5" and larger)														
Post and Timber (see NDS 4.1.3.4 and NDS 4.1.5.3)														
5 x 5	4-1/2 x 4-1/2	20.25	15.19	34.17	15.19	34.17	3.516	4.219	4.922	5.625	6.328	7.031		
6 x 6	5-1/2 x 5-1/2	30.25	27.73	76.26	27.73	76.26	5.252	6.302	7.352	8.403	9.453	10.50		
6 x 8	5-1/2 x 7-1/2	41.25	51.56	193.4	37.81	104.0	7.161	8.594	10.03	11.46	12.89	14.32		
8 x 8	7-1/2 x 7-1/2	58.25	70.31	263.7	53.81	147.2	9.768	11.72	13.67	15.63	17.58	19.53		
8 x 10	7-1/2 x 9-1/2	71.25	112.8	335.9	69.06	193.4	12.37	14.84	17.32	19.79	22.27	24.74		
10 x 10	9-1/2 x 9-1/2	90.25	142.9	478.8	102.9	283.7	15.87	18.80	21.74	24.67	27.60	30.53		
10 x 12	9-1/2 x 11-1/2	109.3	209.4	620.4	123.0	341.7	18.97	22.76	26.55	30.35	34.14	37.93		
12 x 12	11-1/2 x 11-1/2	132.3	253.5	815.8	145.8	401.7	22.96	27.55	32.14	36.74	41.33	45.92		
12 x 14	11-1/2 x 13-1/2	155.3	349.3	1035.8	171.1	468.8	26.95	32.34	37.73	43.13	48.52	53.91		
14 x 14	13-1/2 x 13-1/2	182.3	410.1	1268	201.1	558.8	31.94	37.97	44.00	50.03	56.06	62.09		
14 x 16	13-1/2 x 15-1/2	209.3	540.6	1489	226.1	625.9	36.93	43.59	50.25	56.91	63.57	70.23		
16 x 16	15-1/2 x 15-1/2	240.3	620.6	1810	260.6	725.0	41.92	49.68	57.44	65.20	72.96	80.72		
16 x 18	15-1/2 x 17-1/2	271.3	791.1	2131	285.6	794.1	46.91	55.85	64.79	73.73	82.67	91.61		
18 x 18	17-1/2 x 17-1/2	306.3	893.2	2516	320.6	893.2	51.91	61.84	71.77	81.70	91.63	101.56		
18 x 20	17-1/2 x 19-1/2	341.3	1109	2901	345.6	978.3	56.89	67.82	77.74	87.67	97.60	107.53		
20 x 20	19-1/2 x 19-1/2	380.3	1236	3246	380.3	1063	61.88	72.82	82.74	92.67	102.60	112.53		
20 x 22	19-1/2 x 21-1/2	419.3	1502	3631	405.3	1148	66.87	79.79	89.71	99.63	109.56	119.49		
22 x 22	21-1/2 x 21-1/2	462.3	1656	4026	440.3	1233	71.86	84.78	94.69	104.61	114.53	124.45		
22 x 24	21-1/2 x 23-1/2	505.3	1979	4511	485.3	1318	76.85	90.77	100.68	110.59	120.51	130.43		
24 x 24	23-1/2 x 23-1/2	552.3	2163	5006	530.3	1403	81.84	95.76	105.67	115.58	125.49	135.41		

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# Problem Set 03

## #Q9: Critical Buckling Design Value for Compression, $F_{cE}$

$$F_{cE} = \frac{0.822 E'_{min}}{\left(\frac{l_e}{d}\right)^2}$$

$$E'_{min} = 510,000, \quad \frac{l_e}{d} = 29.28 \Rightarrow$$

$$F_{cE} = \frac{0.822 (510000)}{(29.28)^2} = 488.98 \text{ PSI}$$

### 3.6.6 Column Bracing

Column bracing shall be installed where necessary to resist wind or other lateral forces (see Appendix A).

### 3.6.7 Lateral Support of Arches, Studs, and Compression Chords of Trusses

Guidelines for providing lateral support and determining  $\ell_e/d$  in arches, studs, and compression chords of trusses are specified in Appendix A.11.

### 3.7 Solid Columns

#### 3.7.1 Column Stability Factor, $C_p$

3.7.1.1 When a compression member is supported throughout its length to prevent lateral displacement in all directions,  $C_p = 1.0$ .

3.7.1.2 The effective column length,  $\ell_e$ , for a solid column shall be determined in accordance with principles of engineering mechanics. One method for determining effective column length, when end-fixity conditions are known, is to multiply actual column length by the appropriate effective length factor specified in Appendix G,  $\ell_e = (K_e)(\ell)$ .

3.7.1.3 For solid columns with rectangular cross section, the slenderness ratio,  $\ell_e/d$ , shall be taken as the larger of the ratios  $\ell_{e1}/d_1$  or  $\ell_{e2}/d_2$  (see Figure 3F) where each ratio has been adjusted by the appropriate buckling length coefficient,  $K_e$ , from Appendix G.

3.7.1.4 The slenderness ratio for solid columns,  $\ell_e/d$ , shall not exceed 50, except that during construction  $\ell_e/d$  shall not exceed 75.

3.7.1.5 The column stability factor shall be calculated as follows:

$$C_p = \frac{1 + (F_{cE}/F_c^*)}{2c} - \sqrt{\left[ \frac{1 + (F_{cE}/F_c^*)}{2c} \right]^2 - \frac{F_{cE}/F_c^*}{c}} \quad (3.7-1)$$

where:

$F_c^*$  = reference compression design value parallel to grain multiplied by all applicable adjustment factors except  $C_p$  (see 2.3), psi

$$F_{cE} = \frac{0.822 E'_{min}}{(\ell_e/d)^2}$$

$c = 0.8$  for sawn lumber

$c = 0.85$  for round timber poles and piles

$c = 0.9$  for structural glued laminated timber, structural composite lumber, and cross-laminated timber

3.7.1.6 For especially severe service conditions and/or extraordinary hazard, use of lower adjusted design values may be necessary. See Appendix H for background information concerning column stability calculations and Appendix F for information concerning coefficient of variation in modulus of elasticity ( $COV_E$ ).

#### 3.7.2 Tapered Columns

For design of a column with rectangular cross section, tapered at one or both ends, the representative dimension,  $d$ , for each face of the column shall be derived as follows:

$$d = d_{min} + (d_{max} - d_{min}) \left[ a - 0.15 \left( 1 - \frac{d_{min}}{d_{max}} \right) \right] \quad (3.7-2)$$

where:

$d$  = representative dimension for tapered column, in.

$d_{min}$  = the minimum dimension for that face of the column, in.

$d_{max}$  = the maximum dimension for that face of the column, in.

#### Support Conditions

Large end fixed, small end unsupported or simply supported  $a = 0.70$

Small end fixed, large end unsupported or simply supported  $a = 0.30$

Both ends simply supported: Tapered toward one end  $a = 0.50$

Tapered toward both ends  $a = 0.70$

For all other support conditions:

$$d = d_{min} + (d_{max} - d_{min})(1/3) \quad (3.7-3)$$

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# Problem Set 03

## #Q10: Reference Compression Design Value, $F_c^*$

### 3. Wood Column Analysis

For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. Moisture Content = 15%,  $C_t = C_i = 1.0$ . Assume pinned end conditions ( $K=1$ ).

$F_c^*$  = reference compression design value parallel to grain multiplied by all applicable adjustments **except**  $C_p$

$$F_c^* = F_c (C_D \cdot C_M \cdot C_t \cdot C_F \cdot C_i)$$

we calculated the following before:

$$C_F = 1.05, C_D = 1, C_M = 1 \Rightarrow$$

$$F_c^* = 1150(1 \times 1 \times 1 \times 1.05 \times 1) = 1207.5 \text{ PSI}$$

Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

			ASD only	ASD and LRFD										LRFD only		
				Load Duration Factor	Wet Service Factor	Temperature Factor	Beam Stability Factor	Size Factor	Flat Use Factor	Incising Factor	Repetitive Member Factor	Column Stability Factor	Buckling Stiffness factor	Bearing Area Factor	Format Conversion Factor $K_F$	Resistance Factor $\phi$
$F_b' = F_b$	X	$C_D$	$C_M$	$C_t$	$C_L$	$C_F$	$C_{Fu}$	$C_i$	$C_t$	-	-	-	2.54	0.85	$\lambda$	
$F_t' = F_t$	X	$C_D$	$C_M$	$C_t$	-	$C_F$	-	$C_i$	-	-	-	-	2.70	0.80	$\lambda$	
$F_v' = F_v$	X	$C_D$	$C_M$	$C_t$	-	-	-	$C_i$	-	-	-	-	2.88	0.75	$\lambda$	
$F_c' = F_c$	X	$C_D$	$C_M$	$C_t$	-	$C_F$	-	$C_i$	-	$C_F$	-	-	2.40	0.90	$\lambda$	
$F_{cL} = F_{cL}$	X	-	$C_M$	$C_t$	-	-	-	$C_i$	-	-	-	$C_b$	1.67	0.90	-	
$E' = E$	X	-	$C_M$	$C_t$	-	-	-	$C_i$	-	-	-	-	-	-	-	
$E_{min}' = E_{min}$	X	-	$C_M$	$C_t$	-	-	-	$C_i$	-	-	$C_T$	-	1.76	0.85	-	

4

SAWN LUMBER

### 3.7 Solid Columns

#### 3.7.1 Column Stability Factor, $C_p$

3.7.1.1 When a compression member is supported throughout its length to prevent lateral displacement in all directions,  $C_p = 1.0$ .

3.7.1.2 The effective column length,  $\ell_e$ , for a solid column shall be determined in accordance with principles of engineering mechanics. One method for determining effective column length, when end-fixity conditions are known, is to multiply actual column length by the appropriate effective length factor specified in Appendix G,  $\ell_e = (K_e)(\ell)$ .

3.7.1.3 For solid columns with rectangular cross section, the slenderness ratio,  $\ell_e/d$ , shall be taken as the larger of the ratios  $\ell_{ex}/d_x$  or  $\ell_{ey}/d_y$  (see Figure 3F) where each ratio has been adjusted by the appropriate buckling length coefficient,  $K_e$ , from Appendix G.

3.7.1.4 The slenderness ratio for solid columns,  $\ell_e/d$ , shall not exceed 50, except that during construction  $\ell_e/d$  shall not exceed 75.

3.7.1.5 The column stability factor shall be calculated as follows:

$$C_p = \frac{1 + (F_{ce}/F_c^*)}{2c} - \sqrt{\left[ \frac{1 + (F_{ce}/F_c^*)}{2c} \right]^2 - \frac{F_{ce}/F_c^*}{c}} \quad (3.7-1)$$

where:

$F_c^*$  = reference compression design value parallel to grain multiplied by all applicable adjustment factors except  $C_p$  (see 2.3), psi

$$F_{ce} = \frac{0.822 E_{min}}{(\ell_e / d)^2}$$

$c = 0.8$  for sawn lumber

$c = 0.85$  for round timber poles and piles

$c = 0.9$  for structural glued laminated timber, structural composite lumber, and cross-laminated timber

3.7.1.6 For especially severe service conditions and/or extraordinary hazard, use of lower adjusted design values may be necessary. See Appendix H for background information concerning column stability calculations and Appendix F for information concerning coefficient of variation in modulus of elasticity ( $COV_E$ ).

#### 3.7.2 Tapered Columns

For design of a column with rectangular cross section, tapered at one or both ends, the representative dimension,  $d$ , for each face of the column shall be derived as follows:

$$d = d_{min} + (d_{max} - d_{min}) \left[ a - 0.15 \left( 1 - \frac{d_{min}}{d_{max}} \right) \right] \quad (3.7-2)$$

where:

$d$  = representative dimension for tapered column, in.

$d_{min}$  = the minimum dimension for that face of the column, in.

$d_{max}$  = the maximum dimension for that face of the column, in.

#### Support Conditions

Large end fixed, small end unsupported or simply supported  $a = 0.70$

Small end fixed, large end unsupported or simply supported  $a = 0.30$

Both ends simply supported:

Tapered toward one end  $a = 0.50$

Tapered toward both ends  $a = 0.70$

For all other support conditions:

$$d = d_{min} + (d_{max} - d_{min})(1/3) \quad (3.7-3)$$

3

DESIGN PROVISIONS AND EQUATIONS



# Problem Set 03

#Q11: Constant for Sawn Lumber,  $c$

#Q12: Column Stability Factor,  $C_P$

$C = 0.8$  (for sawn lumber)

$$C_P = \frac{1 + \left(\frac{F_{cE}}{F_C^*}\right)}{2c} - \sqrt{\left[\frac{1 + \left(\frac{F_{cE}}{F_C^*}\right)}{2c}\right]^2 - \frac{F_{cE}}{F_C^*}}$$

$C_P = 0.363$

## 3.7 Solid Columns

### 3.7.1 Column Stability Factor, $C_P$

3.7.1.1 When a compression member is supported throughout its length to prevent lateral displacement in all directions,  $C_P = 1.0$ .

3.7.1.2 The effective column length,  $\ell_e$ , for a solid column shall be determined in accordance with principles of engineering mechanics. One method for determining effective column length, when end-fixity conditions are known, is to multiply actual column length by the appropriate effective length factor specified in Appendix G,  $\ell_e = (K_e)(\ell)$ .

3.7.1.3 For solid columns with rectangular cross section, the slenderness ratio,  $\ell_e/d$ , shall be taken as the larger of the ratios  $\ell_{e1}/d_1$  or  $\ell_{e2}/d_2$  (see Figure 3F) where each ratio has been adjusted by the appropriate buckling length coefficient,  $K_e$ , from Appendix G.

3.7.1.4 The slenderness ratio for solid columns,  $\ell_e/d$ , shall not exceed 50, except that during construction  $\ell_e/d$  shall not exceed 75.

3.7.1.5 The column stability factor shall be calculated as follows:

$$C_P = \frac{1 + (F_{cE}/F_C^*)}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_C^*)}{2c}\right]^2 - \frac{F_{cE}/F_C^*}{c}} \quad (3.7-1)$$

where:

$F_C^*$  = reference compression design value parallel to grain multiplied by all applicable adjustment factors except  $C_P$  (see 2.3), psi

$$F_{cE} = \frac{0.822 E_{\min}'}{(\ell_e/d)^2}$$

$c = 0.8$  for sawn lumber

$c = 0.85$  for round timber poles and piles

$c = 0.9$  for structural glued laminated timber, structural composite lumber, and cross-laminated timber

3.7.1.6 For especially severe service conditions and/or extraordinary hazard, use of lower adjusted design values may be necessary. See Appendix H for background information concerning column stability calculations and Appendix F for information concerning coefficient of variation in modulus of elasticity ( $COV_E$ ).

### 3.7.2 Tapered Columns

For design of a column with rectangular cross section, tapered at one or both ends, the representative dimension,  $d$ , for each face of the column shall be derived as follows:

$$d = d_{\min} + (d_{\max} - d_{\min}) \left[ a - 0.15 \left( 1 - \frac{d_{\min}}{d_{\max}} \right) \right] \quad (3.7-2)$$

where:

$d$  = representative dimension for tapered column, in.

$d_{\min}$  = the minimum dimension for that face of the column, in.

$d_{\max}$  = the maximum dimension for that face of the column, in.

#### Support Conditions

Large end fixed, small end unsupported or simply supported  $a = 0.70$

Small end fixed, large end unsupported or simply supported  $a = 0.30$

Both ends simply supported: Tapered toward one end  $a = 0.50$

Tapered toward both ends  $a = 0.70$

For all other support conditions:

$$d = d_{\min} + (d_{\max} - d_{\min})(1/3) \quad (3.7-3)$$

# Problem Set 03

## #Q13: Factored Allow. Compressive Stress, $F'_c$

$$\begin{aligned}
 F'_c &= F_c (C_D \cdot C_M \cdot C_t \cdot C_F \cdot C_i \cdot \mathbf{C_P}) \\
 &= F_c^* \times C_P \\
 &= 1207.5 \times 0.363 = \mathbf{438.32 \text{ PSI}}
 \end{aligned}$$

**Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber**

		ASD only	ASD and LRFD										LRFD only		
		Load Duration Factor	Wet Service Factor	Temperature Factor	Beam Stability Factor	Size Factor	Flat Use Factor	Incising Factor	Repetitive Member Factor	Column Stability Factor	Buckling Stiffness Factor	Bearing Area Factor	Format Conversion Factor	Resistance Factor	Time Effect Factor
													$K_F$	$\phi$	
$F'_b = F_b$	x	$C_D$	$C_M$	$C_t$	$C_L$	$C_F$	$C_{fu}$	$C_i$	$C_r$	-	-	-	2.54	0.85	$\lambda$
$F'_t = F_t$	x	$C_D$	$C_M$	$C_t$	-	$C_F$	-	$C_i$	-	-	-	-	2.70	0.80	$\lambda$
$F'_v = F_v$	x	$C_D$	$C_M$	$C_t$	-	-	-	$C_i$	-	-	-	-	2.88	0.75	$\lambda$
$F'_c = F_c$	x	$C_D$	$C_M$	$C_t$	-	$C_F$	-	$C_i$	-	$C_P$	-	-	2.40	0.90	$\lambda$
$F'_{c\perp} = F_{c\perp}$	x	-	$C_M$	$C_t$	-	-	-	$C_i$	-	-	-	$C_b$	1.67	0.90	-
$E' = E$	x	-	$C_M$	$C_t$	-	-	-	$C_i$	-	-	-	-	-	-	-
$E'_{min} = E_{min}$	x	-	$C_M$	$C_t$	-	-	-	$C_i$	-	-	$C_T$	-	1.76	0.85	-



# Problem Set 03

#Q14: Column Area, A

#Q15: Maximum Allowable Axial Load Capacity, Pmax

$$\text{Column area} = 10.88 \text{ IN}^2$$

$$F'_C = \frac{P}{A} \Rightarrow$$

$$P = F'_C \times A$$

$$= 438.32 \text{ PSI} \times 10.88 \text{ IN}^2 = 4,768.92 \text{ LBS}$$

Table 1B Section Properties of Standard Dressed (S4S) Sawn Lumber

Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A in. <sup>2</sup>	Section Modulus S <sub>x</sub> in. <sup>3</sup>	X-X AXIS Moment of Inertia I <sub>xx</sub> in. <sup>4</sup>	Y-Y AXIS Moment of Inertia I <sub>yy</sub> in. <sup>4</sup>	Approximate weight in pounds per linear foot (lbs/ft) of piece when density of wood equals:					
						25 lbs/ft <sup>3</sup>	30 lbs/ft <sup>3</sup>	35 lbs/ft <sup>3</sup>	40 lbs/ft <sup>3</sup>	45 lbs/ft <sup>3</sup>	50 lbs/ft <sup>3</sup>
Boards <sup>1</sup>											
1 x 3	3/4 x 2-1/2	1.875	0.781	0.977	0.234	0.088	0.326	0.391	0.456	0.521	0.586
1 x 4	3/4 x 3-1/2	2.625	1.531	2.680	0.328	0.123	0.456	0.547	0.638	0.729	0.820
1 x 6	3/4 x 5-1/2	4.125	3.781	10.40	0.516	0.193	0.716	0.859	1.003	1.146	1.289
1 x 8	3/4 x 7-1/4	5.438	6.570	23.82	0.680	0.255	0.944	1.133	1.322	1.510	1.699
1 x 10	3/4 x 9-1/4	6.938	10.70	49.47	0.867	0.325	1.204	1.445	1.686	1.927	2.168
1 x 12	3/4 x 11-1/4	8.438	15.82	88.99	1.055	0.396	1.465	1.768	2.051	2.344	2.637
Dimension Lumber (see NDS 4.1.3.2) and Decking (see NDS 4.1.3.5)											
2 x 3	1-1/2 x 2-1/2	3.750	1.56	1.953	0.938	0.703	0.651	0.781	0.911	1.042	1.172
2 x 4	1-1/2 x 3-1/2	5.250	3.06	5.359	1.313	0.984	0.911	1.094	1.276	1.458	1.641
2 x 5	1-1/2 x 4-1/2	6.750	5.06	11.39	1.688	1.266	1.172	1.406	1.641	1.875	2.109
2 x 6	1-1/2 x 5-1/2	8.250	7.56	20.80	2.063	1.547	1.432	1.719	2.005	2.292	2.578
2 x 8	1-1/2 x 7-1/4	10.88	13.14	47.63	2.719	2.039	1.888	2.266	2.643	3.021	3.398
2 x 10	1-1/2 x 9-1/4	13.88	21.39	98.93	3.469	2.602	2.409	2.891	3.372	3.854	4.336
2 x 12	1-1/2 x 11-1/4	16.88	31.64	178.0	4.219	3.164	2.930	3.516	4.102	4.688	5.273
2 x 14	1-1/2 x 13-1/4	19.88	43.89	290.8	4.969	3.727	3.451	4.141	4.831	5.521	6.211
3 x 4	2-1/2 x 3-1/2	8.75	5.10	8.932	3.646	4.557	1.519	1.823	2.127	2.431	2.734
3 x 5	2-1/2 x 4-1/2	11.25	8.44	18.88	4.958	5.859	1.953	2.344	2.734	3.125	3.516
3 x 6	2-1/2 x 5-1/2	13.75	12.60	34.66	5.729	7.161	2.387	2.865	3.342	3.819	4.297
3 x 8	2-1/2 x 7-1/4	18.13	21.90	79.39	7.552	9.440	3.147	3.776	4.405	5.035	5.664
3 x 10	2-1/2 x 9-1/4	23.13	35.65	164.9	9.635	12.04	4.015	4.818	5.621	6.424	7.227
3 x 12	2-1/2 x 11-1/4	28.13	52.73	296.6	11.72	14.65	4.883	5.859	6.836	7.813	8.789
3 x 14	2-1/2 x 13-1/4	33.13	73.15	484.6	13.80	17.25	5.751	6.901	8.051	9.201	10.35
3 x 16	2-1/2 x 15-1/4	38.13	96.80	738.9	15.89	19.86	6.619	7.943	9.266	10.59	11.91
4 x 4	3-1/2 x 3-1/2	12.25	7.15	12.51	7.146	12.51	2.127	2.552	2.977	3.403	3.828
4 x 5	3-1/2 x 4-1/2	15.75	11.81	26.58	9.188	16.08	2.734	3.281	3.828	4.375	4.922
4 x 6	3-1/2 x 5-1/2	19.25	17.65	48.53	11.23	19.65	3.342	4.010	4.679	5.347	6.016
4 x 8	3-1/2 x 7-1/4	25.38	30.66	111.1	14.80	25.90	4.405	5.286	6.168	7.049	7.930
4 x 10	3-1/2 x 9-1/4	32.38	49.91	230.8	18.89	33.05	5.621	6.745	7.869	8.993	10.12
4 x 12	3-1/2 x 11-1/4	39.38	73.83	415.3	22.97	40.20	6.836	8.203	9.570	10.94	12.30
4 x 14	3-1/2 x 13-1/4	46.38	102.41	678.5	27.05	47.34	8.051	9.681	11.27	12.88	14.49
4 x 16	3-1/2 x 15-1/4	53.38	135.66	1034	31.14	54.49	9.266	11.12	12.97	14.83	16.68
Timbers (5" x 5" and larger) <sup>1</sup>											
Post and Timber (see NDS 4.1.3.4 and NDS 4.1.5.3)											
6 x 5	4-1/2 x 4-1/2	20.25	15.19	34.17	15.19	34.17	3.516	4.219	4.922	5.625	6.328
6 x 6	5-1/2 x 5-1/2	30.25	27.73	76.26	27.73	76.26	5.252	6.302	7.352	8.403	9.453
6 x 8	5-1/2 x 7-1/2	41.25	51.56	193.4	37.81	104.0	7.161	8.594	10.03	11.46	12.89
8 x 8	7-1/2 x 7-1/2	56.25	70.31	263.7	70.31	263.7	9.766	11.72	13.67	15.63	17.58
8 x 10	7-1/2 x 9-1/2	71.25	112.8	535.9	89.06	334.0	12.37	14.94	17.32	19.79	22.27
10 x 10	9-1/2 x 9-1/2	90.25	142.9	678.8	142.9	678.8	15.67	18.80	21.94	25.07	28.20
10 x 12	9-1/2 x 11-1/2	109.3	209.4	1204	173.0	821.7	18.97	22.76	26.55	30.35	34.14
12 x 12	11-1/2 x 11-1/2	132.3	253.6	1458	253.6	1458	22.96	27.55	32.14	36.74	41.33
12 x 14	11-1/2 x 13-1/2	155.3	349.3	2358	297.6	1711	26.95	32.34	37.73	43.13	48.52
14 x 14	13-1/2 x 13-1/2	182.3	410.1	2768	410.1	2768	31.64	37.97	44.30	50.63	56.95
14 x 16	13-1/2 x 15-1/2	209.3	540.6	4189	470.8	3178	36.33	43.59	50.85	58.13	65.39
16 x 16	15-1/2 x 15-1/2	240.3	620.6	4810	620.6	4810	41.71	50.05	58.39	66.74	75.08
16 x 18	15-1/2 x 17-1/2	271.3	771.1	5923	700.7	5431	47.09	56.51	65.93	75.35	84.77
18 x 18	17-1/2 x 17-1/2	306.3	893.2	7816	893.2	7816	53.17	63.80	74.44	85.07	95.70
18 x 20	17-1/2 x 19-1/2	341.3	1109	10813	995.3	8709	59.24	71.09	82.94	94.79	106.6
20 x 20	19-1/2 x 19-1/2	380.3	1236	12049	1236	12049	66.02	79.22	92.4	105.6	118.8
20 x 22	19-1/2 x 21-1/2	419.3	1502	16150	1363	13285	72.79	87.34	101.9	116.5	131.0
22 x 22	21-1/2 x 21-1/2	462.3	1656	17806	1656	17806	80.25	96.30	112.4	128.4	144.5
22 x 24	21-1/2 x 23-1/2	505.3	1979	23252	1810	19463	87.72	105.3	122.6	140.3	157.9
24 x 24	23-1/2 x 23-1/2	552.3	2163	25415	2163	25415	95.88	115.1	134.2	153.4	172.6

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# Lab 02



Structures II  
Arch 324

Name 1 \_\_\_\_\_  
Name 2 \_\_\_\_\_  
Name 3 \_\_\_\_\_

## Columns

### Description

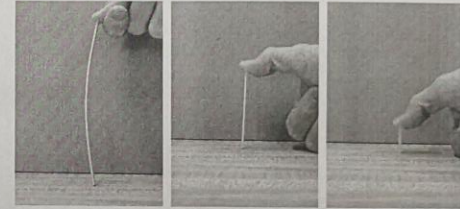
This project uses observation and calculation to understand the effect of slenderness on column capacity.

### Goals

- To observe the buckling behavior of columns through physical modeling.
- To find the controlling slenderness ratio.
- To calculate the critical buckling and crushing loads.

### Procedure

- For the 1/16"x1/4" basswood column provided, with L=6" calculate the controlling (weak axis) slenderness ratio and  $P_{cr}$  using the Euler equation. Use  $K=1.0$ .
- Find the actual critical buckling load approximating the load with your finger.
- Repeat the procedure for L=3" and L=1".
- Calculate the slenderness and  $P_{cr}$  for both of these lengths.
- Calculate the ultimate crushing load based on the max compressive stress,  $F_c$ .
- Approximately locate P for each length on the load vs. slenderness curve shown below



### Basswood Properties

$E_{min} = 1,650,000$  psi

$P_{cr} = (\text{Lesser of } P, P_{max})$

$F_c = 4745$  psi

Area =  $0.015625$  in<sup>2</sup>

$d_1 = 0.25$  in

$d_2 = 0.0625$  in

### Buckling Equation:

$$F_c E = \frac{0.822 E_{min}}{(L/d)^2}$$

$$P = F_c E \times A$$

### Crushing Equation:

$$P_{max} = F_c \times A$$

L = 6" L/d =

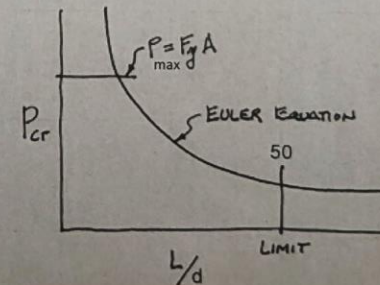
$P_{cr} =$

L = 3" L/d =

$P_{cr} =$

L = 1" L/d =

$P_{cr} =$





# Lab02

Structures II

Arch 324

Name 1 \_\_\_\_\_

Name 2 \_\_\_\_\_

Name 3 \_\_\_\_\_

## Columns

### Description

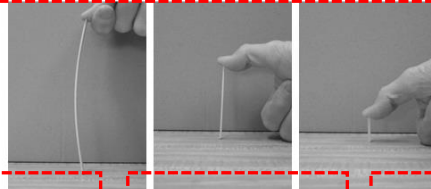
This project uses observation and calculation to understand the effect of slenderness on column capacity.

### Goals

- To observe the buckling behavior of columns through physical modeling.
- To find the controlling slenderness ratio.
- To calculate the critical buckling and crushing loads.

### Procedure

- For the 1/16"x1/4" basswood column provided, with L=6" calculate the controlling (weak axis) slenderness ratio and Pcr using the Euler equation. Use K=1.0.
- Find the actual critical buckling load approximating the load with your finger.
- Repeat the procedure for L=3" and L=1".
- Calculate the slenderness and Pcr for both of these lengths.
- Calculate the ultimate crushing load based on the max compressive stress, Fc.
- Approximately locate P for each length on the load vs. slenderness curve shown below



### Basswood Properties

Emin = 1,650,000. psi

Fc = 4745 psi  
Area = 0.015625 in<sup>2</sup>  
d1 = 0.25 in  
d2 = 0.0625 in

L = 6" L/d =

P =

L = 3" L/d =

P =

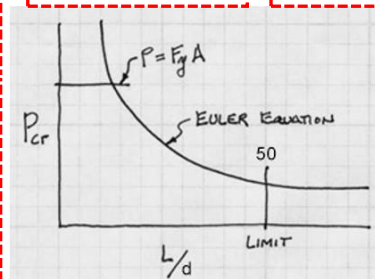
L = 1" L/d =

P =

### Equations:

$$F_c E = \frac{0.822 E'_{min}}{(l_e/d)^2}$$

$$P_{max} = F_c \times A$$



## Failure Modes – Stability

### Long Columns – fail by buckling

#### Traditional Euler

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

- E = Modulus of elasticity of the column material (psi)
- K = Stiffness (curvature mode) factor
- L = Column length between ends (inches)
- r = radius of gyration =  $\sqrt{I/A}$  (inches)



#### NDS Equation

$$F_{cE} = \frac{0.822 E'_{min}}{\left(\frac{l_e}{d}\right)^2}$$

- E'min = reduced E modulus (psi)
- le = Ke l (inches)
- d (inches)
- 0.822 =  $\pi^2/12$

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Structures II

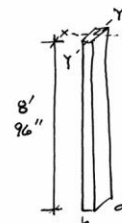
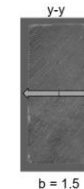
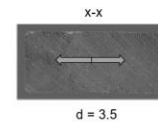
Slide 5 of 27

## Slenderness Ratio le/d

### Slenderness Ratios:

The larger ratio will fail first.  
Try to balance for efficiency.

Slenderness Limited to < 50



$$\begin{aligned} X-X \\ K_e &= 1.0 \\ l_e &= 1.0(96) \\ \frac{l_e}{d} &= \frac{96}{3.5} = 27.4 \end{aligned}$$

$$\begin{aligned} Y-Y \\ K_e &= 1.0 \\ l_e &= 1.0(96) \\ \frac{l_e}{b} &= \frac{96}{1.5} = 64 \end{aligned}$$

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Structures II

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## Analysis of Wood Columns

### Data:

- Column – size, length
- Support conditions
- Material properties – Fc, Emin
- Load

### Required:

- Pass/Fail or margin of safety

- Calculate slenderness ratio le/d largest ratio governs. Must be < 50
- Find adjustment factors C<sub>D</sub>, C<sub>M</sub>, C<sub>t</sub>, C<sub>F</sub>, C<sub>i</sub>
- Calculate C<sub>P</sub>
- Determine allowable F'<sub>c</sub> by multiplying the tabulated F<sub>c</sub> by all the above factors
- Calculate the actual stress: f<sub>c</sub> = P/A
- Compare Allowable and Actual stress. F'<sub>c</sub> > f<sub>c</sub> passes



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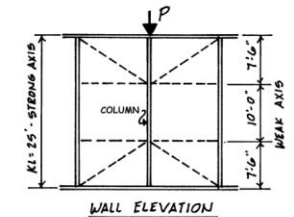
Structures II

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## Analysis Example: Pass/Fail

Data: section 4x8 (nominal)  
Douglas Fir-Larch No1  
M.C. 15%  
P = 7000 LBS (Snow Load)

Find: Pass/Fail



From NDS Supplement Table 4A

F<sub>c</sub> = 1500 psi  
E<sub>min</sub> = 620000 psi

C<sub>D</sub> = 1.15 (snow)

C<sub>M</sub> = 1.0

C<sub>t</sub> = 1.0

C<sub>F</sub> = 1.05 (4x8)

C<sub>i</sub> = 1.0

C<sub>P</sub> = ?

Species and commercial grade	Size classification	Compression perpendicular to grain	Compression parallel to grain	Modulus of Elasticity	
		F <sub>c⊥</sub>	F <sub>c</sub>	E	E <sub>min</sub>
DOUGLAS FIR-LARCH					
Select Structural		625	1,700	1,800,000	690,000
No. 1 & Btr		625	1,550	1,800,000	690,000
No. 1	2" & wider	625	1,500	1,700,000	620,000
No. 2		625	1,350	1,600,000	580,000
No. 3		625	775	1,400,000	510,000
Stud	2" & wider	625	850	1,400,000	510,000
Construction		625	1,680	1,800,000	590,000
Standard	2" - 4" wide	625	1,400	1,400,000	510,000
Utility		625	900	1,300,000	470,000

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Arch 544

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# Lab02

## → Group work instructions

Please form groups of 2 to 4 students.

Please do not forget to write **all group members' names** on both sheets.

Return the completed sheets to me at the end of the session.

Please ensure that you attend the recitation sessions.

If you are unable to attend a session, send me an email so that we can discuss how to proceed. *Email: [arfazel@umich.edu](mailto:arfazel@umich.edu)*

# Tower project

Architecture 324  
Structures II

Prof. Peter von Buelow  
Winter 2025

## Tower Project

### Description

This project gives students the chance to apply concepts learned in column analysis to the design of a structural system that carries primarily a compression load – a tower. Work is to be done in groups of up to four people. The project is divided into 3 parts: 1) initial conceptual design, 2) design development and testing, 3) final analysis and documentation.

### Goals

- to explore design parameters of geometry and material under compression.
- to develop a design of a compression member to meet the criteria below.
- to make some rough hand calculation to estimate the expected performance.
- to test the compression member and record the results.
- to document the results in a well organized and clear report format.

### Criteria

- The tower is to be made of wood. Either linear wood (sticks) or wood panels (sheets) can be used. Glue can be used to connect the elements. Gussel plates at the joints are allowed and can also be glued. But **no steel pins** or fasteners may be used.
- Wood: **any species. maximum cross-sectional dimension = 1/4"**.
- NO** paper, mylar or plastic or string or dental floss.
- If a member is made by laminating multiple pieces together, the maximum cross-sectional dimension or thickness still cannot exceed 1/4".
- The height of the tower = 48"**
- The tower **must hold at least 50 lbs**.
- The entire tower **can weigh no more than 4 oz**.
- The top of the tower must be loadable. The weights will be stacked on top of the tower, but you may optionally use a loose piece of MDF or plywood as a tray under the weights. (It will not be counted in either weight or load)
- Towers will be graded on their low weight, high load-carrying capacity, and the load/weight ratio. The evaluation formula is:  
$$(4/\text{weight in OZ}) + (\text{load in LBS}/50) + (\text{load LBS}/\text{weight OZ}) \times 1.5$$
- The score will be normalized to a range of 50 to 100. It is used together with report scores to assess your project (a detailed evaluation form is given separately).

### Procedure

- Develop a structural concept for a tower meeting the above criteria.
- Analyze the design concept with **either** hand calculations or a computer program (e.g. Dr. Frame)
- Determine the capacity of the major members and of the overall tower (total capacity in LBS)
- Estimate your expected score using the formula above.
- Write the preliminary report.
- Construct the structural model.
- Test the model. 5-pound steel bars will be placed on top of the model, until the model fails. (bar size: 1 1/2" x 2" x 5/16")
- Produce final report documenting requirements and process. See also score sheet.

### Due Dates

See Course Schedule

### Scoring

Preliminary Report	40 pts
Testing	60 pts
Final Report	150 pts

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## Tower Project – Preliminary Report Requirements

**Explanation** – describe how the design was developed, the basis of the structural concept, and how the principles of column behavior influenced the design decisions.

**Illustration** – include diagrams/drawings that describe the structure in its entirety. **At least a horizontal cross-section and an elevation of the tower are required.** Dimensions are to be included and the member sizes labeled.

**Analysis** – the report should include the following:

- Choose wood type and stress properties.** Either use values below for typical model grade Basswood or use values in the NDS or find test values online. Indicate in the report which values you choose.
- Determine the cross-sectional area of each member.** Find the axial force P and the allowable stress F<sub>c</sub>. The force P can be determined either by a hand calculated truss analysis or as a second order analysis in Dr. Frame or STAAD Pro. The stress F<sub>c</sub> should be found using the NDS equations for C<sub>p</sub> and F<sub>c</sub>. Other NDS stress adjustment factors (C<sub>o</sub>, C<sub>e</sub>, C<sub>t</sub>, C<sub>r</sub> and C<sub>i</sub>) can be taken equal to 1.0. Size members based on the predicted load, P and the allowable stress F<sub>c</sub>. Target (or predict) some total capacity load for the tower. A minimum of 50 LBS is required. Then size the members based on the force in each member.
- Predict the total weight of the tower.** Provide a table with each member type showing, length, section and weight for each. Make an estimate of the weight added by glue joints and/or gussel plates. The total weight should be under 4 OZ.
- Predict Capacity.** Predict the ultimate capacity in pounds that the entire tower can carry based on the actual cross-sections chosen. Produce a utilization table to show for each member type (e.g. main vertical, horizontal tie, diagonal brace) the utilization ratio f<sub>c</sub>/F<sub>c</sub> based on the predicted total capacity load. This ratio should be below 1.0 for all members.
- Calculate the buckling capacity of the tower as a whole.** This is done by treating the tower as one column loaded at the top, made up in cross section of multiple columns. Show the moment of inertia of the tower cross-section, and use it to calculate the critical buckling load using the Euler equation. An example of this calculation is given in the slides from the class lecture. The ultimate capacity is the lower of the two capacities (critical member or tower as a whole).

**Note:** If an excel spreadsheet is used to make calculations, show the equations being used for each cell or column in the table. If STAAD Pro or Dr. Frame is used to do any of the above, include print-outs showing the applied loads and resulting member forces.

**Format** - Reports should be formatted for **8 1/2 X 11** paper. 11x17 format reports will not be accepted. Once returned to you graded, **save the original copy of the preliminary report** for submission together with the Final Report.

The report is a professional document. Text should be clear, grammatically correct, and language should be appropriate and professional. All calculations should be legible and clearly described – not just numbers or results, but with a clear description of what is being calculated included.

### Properties of Basswood: (like in the Media Center)

Density (oven dry)	29 pcf **
E (buckling)	1,650,000 psi **
F (Compression $\parallel$ to grain)	4745 psi *
F (Compression $\perp$ to grain)	377 psi *
F (Tension $\parallel$ to grain)	4500 psi (estimate)
F (Tension $\perp$ to grain)	348 psi *
F (Shear $\parallel$ to grain)	986 psi *
F (Flexure)	5900 psi *

\* from <http://www.matweb.com/>

\*\* tested by PvB (small pieces in compression)

Proceedings of the IASS-SLITE 2014 Symposium  
"Shells, Membranes and Spatial Structures: Footprints"  
15 to 19 September 2014, Brasilia, Brazil  
Reyolando M.L.R.F. BRASIL and Ray M.O. PAULETTI (eds.)

## Form Exploration and GA-Based Optimization of Lattice Towers Comparing with Shukhov Water Tower

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\* University of Michigan

### Abstract

The main objective of this research is to develop a form exploration technique based on parametric form generation using concepts of Formex algebra and evolutionary optimization based on ParaGen approach [1]. Accordingly, the design process of the lattice towers with polygon bases is studied. The focus of this research is to demonstrate the ability of a computational form-finding method in multi-objective design, and to offer arrays of comparable good solutions instead of a single "optimal" form. In addition, the Shukhov water tower is considered in order to draw a comparison between the results of the presented form finding technique and a well known successful design.

First, different tower configurations are described through geometrical concepts of Formex algebra [2]. Formex algebra is a mathematical system that allows the designer to define geometrical formulation of forms. The geometrical parameters used in formulations are the base shape of the towers, frequency of elements along the height of the tower, diameter of bottom base and the mesh patterns. The constant parameters, such as the height of the tower and diameter of the top are set to match the values of the Shukhov water tower.

Then, the ParaGen framework uses a non-destructive, dynamic population GA (NDGP GA) to fill a database with solutions linked to a variety of performance characteristics [1]. The database of solutions can then be explored for any single or multi-objective performance criteria. Because the solutions are linked to descriptive images, the exploration process takes place at both a visual qualitative level as well as a performance driven quantitative level. For the purpose of comparison, the properties of A-36 structural steel pipe sections are used. Using STAAD Pro (Bentley Systems) for the analysis, it was also possible to size all members using AISC steel code as well as collect additional performance parameters such as deflection and modal frequency.

At last, results are entered into a SQL database which is linked to visual images of the designs. This allows for the comparison of designs both visually and using quantitative data. Pareto front graphs were produced based on the computational study, and the best performing results are plotted on these graphs in which the Shukhov water tower is also located for comparison. In conclusion, the strengths and concerns of applying the proposed method are also discussed, and it is explained how designers can expand their design perspective and be provided with arrays of appropriate solutions. Instead of simply one best solution, using a dynamic process of form generation and optimization.

**Keywords:** Topology Optimization, Genetic Algorithm, Formex Configuration processing, Lattice Towers

### 1. Introduction

Designing a form is broadly considered as a creative and purposeful procedure. In structural design usually some concerns are defined in terms of parameters such as topology or geometry of the structural form, material properties and load cases. Then, it is tried to achieve the best solution(s) that can meet the requirements. Within this process of design and optimization two main challenges may be raised. First, there are multiple criteria through the procedure which make the decision making more complicated. Second, using a trial-and-error method to seek one single best solution demands a huge amount of time and effort, and at last it may decline the designer's tendency to explore all the possibilities. Hence, in order to expand the designer's perspective,

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See Course Schedule

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Testing	60 pts
Final Report	150 pts

Architecture 324

Winter 2025

## ARCHITECTURAL STRUCTURES II (3)

### Lecture and Assignment Schedule

DATE	TOPIC	Text Reading	PROBLEMS (due dates online)
JAN 8	Course Intro	Onouye, Schodek	
JAN 13	1 - Wood Properties	NDS	
JAN 15	2 - Wood Beam Analysis	Schodek 6.4.2	
JAN 17	Recitation [1-Wood Beams]	Topic Quiz 1	1. Wood Beam Analysis
JAN 20	<b>Martin Luther King Day **** No Class **** Martin Luther King Day **** No Class</b>		
JAN 22	3 - Wood Beam Design	Onouye 8	
JAN 24	Recitation	Topic Quiz 2	
JAN 27	4 - Wood Column Analysis	Onouye 9.1-9.2 & 9.4, Schodek 7.4.3	
JAN 29	5 - Wood Column Design	NDS	Tower Intro
JAN 31	Recitation [2-Wood Columns]	Topic Quiz 3	2. Wood Column Analysis
FEB 3	6 - Cross Laminated Timbers	CLT Handbook	
FEB 5	7 - Steel Properties	AISC, Onouye 8.7	
FEB 7	Recitation - Tower Project	Topic Quiz 4	
FEB 10	8 - Steel Beam Analysis	Schodek 6.4.3	
FEB 12	9 - Steel Beam Analysis	Schodek 6.4.3	
FEB 14	Recitation [3-Steel Beams]	Topic Quiz 5	<b>Prelim. Tower Report Due</b> 3 Steel Beam Analysis
FEB 17	10 - Steel Beam Design	Schodek 6.4.3	
FEB 19	11 - Steel Column Analysis	Onouye 9.3, Schodek 7.4.4	
FEB 21	Recitation [4-Steel Columns]	Topic Quiz 6	4. Steel Beam Design
FEB 24	12 - Steel Column Design	Onouye 9.3, Schodek 7.4.4	
FEB 26	"Skyscrapers" David Macaulay video		
FEB 28	Recitation	Topic Quiz 7	5. Steel Column Analysis
MAR 3	<b>WINTER RECESS **** NO CLASS **** WINTER RECESS **** NO CLASS ****</b>		
MAR 5	<b>WINTER RECESS **** NO CLASS **** WINTER RECESS **** NO CLASS ****</b>		
MAR 7	<b>WINTER RECESS **** NO CLASS **** WINTER RECESS **** NO CLASS ****</b>		
MAR 10	13 - Continuous Beams	I. Engel Ch. 17, Schodek 8	
MAR 12	14 - Gerber Beams	Schodek 8.4.4	
MAR 14	Recitation [5-Continuous Beams]	Topic Quiz 8	6. Three Moment Theorem
MAR 17	15 - Intro to Concrete - PCA video.		
MAR 19	16 - Concrete Beams	Schodek 6.4.4 - 6.4.6	
MAR 21	Recitation [6-Stress vs Strain]	Topic Quiz 9	
MAR 24	<b>Tower Testing **** Tower Testing **** Tower Testing **** Tower Testing ****</b>		
MAR 26	17 - Concrete Beams	I. Engel Ch. 15	
MAR 28	Recitation	Topic Quiz 10	7. Concrete Beam Analysis
MAR 31	18 - Concrete Beam Design	Schodek 7.4.5	
APR 2	19 - Concrete Columns		
APR 4	Recitation [7-Concrete Reinforcing]	Topic Quiz 11	8. Concrete Beam Design
APR 7	20 - Composite Sections		
APR 9	21 - Composite Sections		
APR 11	Recitation [8-Composite Sections]	Topic Quiz 12	9. Composite Sections
APR 14	22 - Masonry Intro.	TMS 402	
APR 16	23 - Masonry Walls	TMS 402	
APR 18	Recitation [9-Lateral Stability]		<b>Final Tower Report Due</b> 10. Masonry Walls
APR 21	24 - Masonry Walls	TMS 402	