Architecture 324: Structures II

LRFD Composite Steel Beam + Concrete Slab Analysis

Guided Worksheet: Case 1 (PNA Within Slab)

Given Data: W-section: W21×93 **Span A (beam span):** 67 FT **Span B (beam spacing):** 15 FT **Slab thickness,** t**:** 7 IN F_y**:** 50 KSI f'_c**:** 4 KSI γ_c**:** 150 PCF

W21×93 Properties (AISC): $A_s = 27.3$ in^2 $d = 21.6$ in $b_f = 8.42$ in $t_f = 0.930$ in Self-weight = 93 plf

Part 1: Effective Flange Width

Q1 Effective Flange Width (b_e) [IN]

The concrete slab acts as the compression flange of the composite beam, but only a limited width is effective. For a slab on ***both sides****,* b_e *is the* ***least*** *of the three criteria below. Each criterion limits how far the slab extends on each side of the beam web.*

1. **Criterion 1** (fraction of span):

$$b_e = \frac{SpanA}{4} = \frac{\_\_\_\_ \times 12}{4} = \_\_\_\_\_\_\_\_ \text{ IN}$$

2. **Criterion 2** (slab thickness controlled):
   Each overhang $\leq 8t$, plus the steel flange width:
   $b_e = 2(8t) + b_f(fromsteeltable) = 2 \times 8 \times$ ____ + _____ = _________ IN
3. **Criterion 3** (beam spacing controlled):
   b_e = Span B = _____ × 12 = _________ IN
4. **Which is the smallest?** Circle: (1) / (2) / (3)

Answer b_e**:** ______________ IN

Part 2: Stress Block and Case Check

Q2 Depth of Concrete Stress Block (a) [IN]

At ultimate strength, the concrete in the effective flange carries compression through a rectangular stress block of depth a *and uniform stress* $0.85f'_c$*. Setting the compression force* $C = 0.85f'_c\, a\, b_e$ *equal to the tension force* $T = A_sF_y$ *(horizontal equilibrium) and solving for* a*:*

$$a = \frac{A_s\ F_y}{0.85\ f'_c\ b_e(fromsteeltable)}$$

1. **Numerator** ($A_s \times F_y$): ______ × ______ = _________
2. **Denominator** ($0.85 \times f'_c \times b_e$): 0.85× ______ × ______ = _________
3. **Divide:** _________ ÷ _________

Answer a**:** ______________ IN

Q3 Is a Within the Slab? [1 = yes, 0 = no]

If $a \leq t$ *(slab thickness), the entire compression block fits within the concrete slab. This means the Plastic Neutral Axis (PNA) is within the slab and the* ***full steel section is in tension****. This is Case 1, the simpler and more common situation.*

Decision Point:

☐ $a \leq t$ ⇒ **Case 1:** PNA is in the slab. Full steel section is in tension. Proceed with this worksheet.

☐ $a > t$ ⇒ **Case 2:** PNA drops into the steel section. Part of the steel is in compression. Requires a different procedure (see lecture slides 15 – 18).

1. **Compare:** a = ______ in vs. t = ______ in

Answer: ______________

Part 3: Moment Capacity

Q4 Nominal Bending Moment (M_n) [K·IN]

The nominal moment is the internal couple: the tension force $T = A_sF_y$ *acts at mid-depth of the steel section (d/2 from the beam's top flange). The compression resultant* C *acts at* $a/2$ *from the top of the slab. The moment arm* z *between them is:*

$$z = \frac{d(fromsteeltable)}{2} + t - \frac{a}{2}$$

$$M_n = A_s(fromsteeltable)\, F_y \times z = A_s\ F_y\left(\frac{d}{2} + t - \frac{a}{2}\right)$$

1. **Moment arm:** $d/2$ = ______, t = ______, $a/2$ = ______
   z = ______ + ______ − ______ = _________ IN
2. **Tension force:** $T = A_sF_y$ = ______ × ______ = _________ K
3. **Multiply:** M_n = _________ × _________

Answer M_n**:** ______________ K·IN

Q5 Factored Bending Resistance (ϕM_n) [K·IN]

The design strength is the nominal moment reduced by the strength reduction factor $\phi = 0.90$ *for flexure (LRFD). This accounts for uncertainties in material properties, dimensions, and the analysis model.*

$$\phi M_n = 0.90 \times M_n$$

1. **Calculate:** 0.90× _________

Answer ϕM_n**:** ______________ K·IN

Q6 Factored Design Moment (M_u) [K·FT]

Convert ϕM_n *from kip-inches to kip-feet. This is the maximum factored moment the composite section can resist.*

$$M_u = \frac{\phi M_n}{12}$$

1. **Convert:** _________ K·IN ÷ 12 = _________ K·FT

Answer M_u**:** ______________ K·FT

Part 4: Load Capacity

Q7 Total Factored Load (w_u) [KLF]

For a simply supported beam with uniform load, $M_u = w_uL^2/8$*. Solve for* w_u*:*

$$w_u = \frac{8\, M_u}{SpanA^2}$$

Note: *Use* M_u *in K·FT and SpanA in FT to get* w_u *in KLF.*

1. **Calculate:** $\frac{8 \times \_\_\_\_\_\_}{\_\_\_\_\_^2} = \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$

Answer w_u**:** ______________ KLF

Q8 Selfweight of the Concrete Slab [PSF]

The slab weight per square foot depends on its thickness and the unit weight of concrete ($\gamma_c = 150$ *PCF).*

$$w_{\text{slab}} = \frac{t}{12} \times \gamma_c$$

1. **Calculate:** $\frac{\_\_\_\_\_}{12} \times 150$ = _________ PSF

Answer: ______________ PSF

Q9 Total Unfactored Dead Load (w_{DL}) [KLF]

The beam carries: (1) the slab weight over its tributary width (Span B), and (2) its own self-weight. Add these together as the total unfactored dead load in PLF, then convert to KLF.

$$w_{DL} = w_{\text{slab}} \times \text{Span B} + w_{\text{beam}}$$

1. **Slab load on beam:** ______ PSF × ______ FT = _________ PLF
2. **Beam self-weight (from steel table):** _________ PLF
3. **Total:** _________ + _________ = _________ PLF
4. **Convert:** ÷ 1000 = _________ KLF

Answer w_{DL}**:** ______________ KLF

Q10 Unfactored Beam Live Load Capacity (w_{LL}) [KLF]

The total factored load is composed of dead and live loads combined using LRFD load factors: $w_u = 1.2\, w_{DL} + 1.6\, w_{LL}$*. Solve for* w_{LL}*:*

$$w_{LL} = \frac{w_u - 1.2\, w_{DL}}{1.6}$$

1. **Factored DL:** 1.2× _______ = _________ KLF
2. **Subtract:** _______ − _______ = _________ KLF
3. **Divide by 1.6:** _________ ÷ 1.6 = _________ KLF

Answer w_{LL}**:** ______________ KLF

Q11 Floor Live Load Capacity [PSF]

The beam live load w_{LL} *(in PLF) is spread over the tributary width (Span B) to find the floor capacity in PSF.*

$$LL = \frac{w_{LL} \text{ (in PLF)}}{\text{Span B}}$$

1. **Convert w_{LL} to PLF:** _______ KLF × 1000 = _________ PLF
2. **Divide by Span B:** _________ ÷ ______

Answer LL: ______________ PSF