Architecture 324: Structures II

LRFD Reinforced Concrete Beam Analysis — Guided Worksheet

Determine Factored Bending Resistance and Factored Design Moment (ACI 318)

Given Data: **Span:** 33 FT **b:** 21 IN **h:** 30 IN **Max. Agg.:** 0.75 IN **Bar #:** 11 **# Bars:** 3 **Stirrup #:** 4 **Cover:** 1.5 IN **f'_c:** 5,500 PSI **f_y:** 60,000 PSI

Standard Rebar Properties (look up your bar # and stirrup # here):

Bar #	3	4	5	6	7	8	9	10	11	14	18
Diameter, d_b (in)	0.375	0.500	0.625	0.750	0.875	1.000	1.128	1.270	1.410	1.693	2.257
Area, A_b (in^2)	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56	2.25	4.00

Part 1: Section Geometry

Q1 Flexural Steel Bar Diameter (d_b) [IN]

*Using the rebar table above, find the row for your **bar size number** and read off the diameter.*

Answer d_b: ________________ IN

Q2 Stirrup Bar Diameter [IN]

*Same table, but look up your **stirrup bar size number** this time.*

Answer: ________________ IN

Q3 Distance from Bottom Edge to Steel Center (d_c) [IN]

*The steel bars sit inside the beam, protected by concrete cover and stirrups. Measure from the **bottom face** of the beam up to the **centroid** of the flexural bars. The layers from bottom up are: concrete cover, then the stirrup bar, then half the flexural bar diameter to reach bar center.*

$$d_c = \text{cover} + d_{\text{stirrup}} + \frac{d_b}{2}$$

1. **Cover:** _______ in
2. **+ Stirrup diameter:** _______ in
3. **+ Half of flexural bar:** _______ / 2 = _______ in
4. **Sum:** _______ + _______ + _______

Answer d_c: ________________ IN

Q4 Effective Depth (d) [IN]

*The effective depth is the distance from the **top of the beam** (compression face) to the **centroid of the tension steel**. It is the lever arm that matters for bending — not the total height h.*

$$d = h - d_c$$

1. **Total height h:** _______ $- d_c$ **from Q3:** _______

Answer d: ________________ IN

Part 2: Steel Area Checks

Q5 Minimum Required Steel Area ($A_{s,\min}$) [IN^2]

*ACI requires a minimum amount of steel so that the beam does not fail in a sudden, brittle manner. Compute **both** criteria below (using f'_c and f_y in **PSI**), then take the **larger** value.*

$$\text{(a)}\ A_{s,\min} = \frac{3\sqrt{f'_c}}{f_y}\, b\, d \qquad \text{(b)}\ A_{s,\min} = \frac{200}{f_y}\, b\, d$$

1. **Criterion (a):**
   $3\sqrt{f'_c} = 3 \times \sqrt{\_\_\_\_\_\_\_\_}$ = __________ then divide by f_y: __________ ÷ ________ = ________
   Multiply: ________ $\times\, b$ = ______ $\times\, d$ = ______ = __________ in^2
2. **Criterion (b):**
   200 ÷ f_y = 200 ÷ ________ = ________ then: ________ × ______ × ______ = __________ in^2
3. **Which is larger?** Circle: (a) / (b)

Answer $A_{s,\min}$: ________________ IN^2

Q6 Actual Area of Flexural Steel (A_s) [IN^2]

*Look up the **area per bar** (A_b) from the rebar table for your bar size, then multiply by the number of bars. Confirm that A_s exceeds $A_{s,\min}$ from Q5 — otherwise the beam is under-reinforced by code.*

$$A_s = (\text{number of bars}) \times A_b$$

1. **Calculate:** ______ bars × _______ in^2/bar = __________ in^2
2. **Check:** Is $A_s \geq A_{s,\min}$? ☐ Yes (OK — proceed) ☐ No (beam is under-reinforced per ACI code; if the concrete cracks, the steel cannot carry the released tension force and the beam fails suddenly without warning. The design would need more bars. For this problem, continue the analysis with the given A_s.)

Answer A_s: ________________ IN^2

Part 3: Stress Block & Neutral Axis

Q7 Depth of Concrete Stress Block (a) [IN]

At ultimate strength, the concrete compression zone is modeled as a rectangular stress block (Whitney block) with uniform stress $0.85f'_c$. Setting the compression force C equal to the tension force $T = A_s f_y$ (equilibrium) and solving for the block depth a:

$$a = \frac{A_s\, f_y}{0.85\, f'_c\, b}$$

Note: *Use consistent units — f_y in KSI and f'_c in KSI, or both in PSI.*

1. **Numerator** ($A_s \times f_y$): _______ × _______ = __________
2. **Denominator** ($0.85 \times f'_c \times b$): 0.85× _______ × _______ = __________
3. **Divide:** __________ ÷ __________

Answer a: ________________ IN

Q8 Factor β_1 [dimensionless]

*β_1 converts the neutral axis depth c to the equivalent stress block depth a. It depends on concrete strength f'_c **(in PSI)**. For $f'_c \leq 4000$ PSI, $\beta_1 = 0.85$. For higher strengths it decreases by 0.05 for each 1000 PSI above 4000, but never below 0.65.*

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \qquad \text{bounded: } 0.65 \leq \beta_1 \leq 0.85$$

1. **Calculate:** $0.85 - 0.05 \times \dfrac{\_\_\_\_\_\_ - 4000}{1000}$ = 0.85− ______ = _______
2. **Check bounds:** Is the result between 0.65 and 0.85? If below 0.65, use 0.65. If $f'_c \leq 4000$, use 0.85.

Answer β_1: ________________

Q9 Neutral Axis Depth (c) [IN]

The neutral axis is the boundary between compression and tension in the beam cross-section. It is located at depth c from the top. Since $a = \beta_1 c$, we can find c from a and β_1.

$$c = \frac{a}{\beta_1}$$

1. **Divide:** _______ ÷ _______

Answer c: ________________ IN

Part 4: Strain Check & Strength Reduction Factor

Q10 Strain in Flexural Steel (ε_t) [in/in]

ACI assumes the concrete crushes at a strain of $\varepsilon_{cu} = 0.003$. Using similar triangles on the linear strain diagram across the beam depth, the steel strain at the level of the tension bars is:

$$\varepsilon_t = \frac{d - c}{c} \times 0.003$$

1. **Calculate:** $\dfrac{\_\_\_\_\_\_ - \_\_\_\_\_\_}{\_\_\_\_\_\_}$ × 0.003 = __________

Answer ε_t: ________________

Decision Point — What does ε_t tell you?

☐ $\varepsilon_t \geq 0.005$ ⇒ **Tension Controlled:** Steel yields well before concrete crushes. Ductile, desirable failure. Use $\phi = 0.90$.

☐ $0.002 \leq \varepsilon_t < 0.005$ ⇒ **Transition Zone:** ϕ is interpolated between 0.65 and 0.90.

☐ $\varepsilon_t < 0.002$ ⇒ **Compression Controlled:** Concrete crushes before steel yields. Brittle, dangerous. $\phi = 0.65$.

Q11 Strength Reduction Factor (ϕ) [dimensionless]

Based on your ε_t from Q10, assign ϕ using the decision box above. For most properly designed beams, $\varepsilon_t \geq 0.005$ and $\phi = 0.90$.

Answer ϕ: ________________

Part 5: Moment Capacity

Q12 Tensile Force in Steel (T) [KIPS]

*At ultimate strength, the steel is assumed to have yielded (since $\varepsilon_t > \varepsilon_y$). The total tension force equals the steel area times the yield stress. Use f_y in **KSI** to get answer in kips.*

$$T = A_s \times f_y$$

1. **Calculate:** _______ in^2 × _______ KSI = __________ K

Answer T: ________________ KIPS

Q13 Nominal Bending Moment (M_n) [K-IN]

The nominal moment is the internal couple: the tension force T times the distance between T (at the steel) and C (at the centroid of the stress block, which is $a/2$ from the top). This distance is $(d - a/2)$.

$$M_n = T \times \left(d - \frac{a}{2}\right)$$

1. **Moment arm:** $d - a/2$ = _______ − _______/2 = __________ IN
2. **Multiply:** _______ K × _______ IN = __________ K-IN

Answer M_n: ________________ K-IN

Q14 Factored Bending Resistance (ϕM_n) [K-IN]

*The **design strength** is the nominal moment reduced by ϕ to account for uncertainties in material properties and construction. This is the maximum moment the beam can be relied upon to carry.*

$$\phi M_n = \phi \times M_n$$

1. **Calculate:** _______ × __________ K-IN = __________ K-IN

Answer ϕM_n: ________________ K-IN

Q15 Factored Design Moment (M_u) [K-FT]

Convert from kip-inches to kip-feet by dividing by 12. In design, the factored load demand M_u must not exceed ϕM_n. Here we are finding the beam's capacity expressed in K-FT.

$$M_u = \frac{\phi M_n}{12}$$

1. **Convert:** __________ K-IN ÷ 12 = __________ K-FT

Answer M_u: ________________ K-FT