

# Architecture 324: Structures II

## LRFD Reinforced Concrete Beam Analysis

Problem 8: Concrete Beam Analysis [ANSWERS]

**Given Data:** Span: 33 FT  $b$ : 21 IN  $h$ : 30 IN Max. Agg.: 0.75 IN  
**Bar #:** 11 **Number of Bars:** 3 **Stirrup #:** 4 **Cover:** 1.5 IN  $f'_c$ : 5,500 PSI  $f_y$ : 60,000 PSI  
**Rebar Properties:** #11:  $d_b = 1.410$  in,  $A_{bar} = 1.56$  in<sup>2</sup> #4:  $d_{stirrup} = 0.500$  in

### Part 1: Section Geometry

#### Q1 Flexural Steel Bar Diameter ( $d_b$ )

From rebar table, Bar #11:

Final  $d_b$ : **1.410** IN

#### Q2 Stirrup Bar Diameter

From rebar table, Bar #4:

Final  $d_{stirrup}$ : **0.500** IN

#### Q3 Distance from Bottom Edge to Steel Center ( $d_c$ )

$$d_c = \text{cover} + d_{stirrup} + \frac{d_b}{2}$$

1. **Calculate:**  $1.500 + 0.500 + 1.410/2 = 1.500 + 0.500 + 0.705$

Final  $d_c$ : **2.705** IN

#### Q4 Effective Depth ( $d$ )

$$d = h - d_c$$

1. **Calculate:**  $30.000 - 2.705$

Final  $d$ : **27.295** IN

### Part 2: Steel Area Checks

#### Q5 Minimum Required Steel Area ( $A_{s,min}$ )

Greater of criteria (a) and (b), with  $f'_c$  and  $f_y$  in PSI:

1. **Criterion (a):**  $\frac{3\sqrt{f'_c}}{f_y} b d = \frac{3\sqrt{5500}}{60,000} \times 21 \times 27.295$

$$- 3\sqrt{5500} = 3 \times 74.162 = 222.49$$

$$- 222.49/60,000 = 0.003708$$

$$- 0.003708 \times 21 \times 27.295 = \mathbf{2.124} \text{ in}^2 \leftarrow \text{controls}$$

2. **Criterion (b):**  $\frac{200}{f_y} b d = \frac{200}{60,000} \times 21 \times 27.295$

$$- 200/60,000 = 0.003333$$

$$- 0.003333 \times 21 \times 27.295 = 1.911 \text{ in}^2$$

Final  $A_{s,min}$ : **2.124** IN<sup>2</sup>

#### Q6 Actual Area of Flexural Steel ( $A_s$ )

$$A_s = 3 \times 1.56 = 4.68 \text{ in}^2$$

1. **Check:**  $4.68 > 2.124$  ✓ OK

Final  $A_s$ : **4.68** IN<sup>2</sup>

### Part 3: Stress Block & Neutral Axis

#### Q7 Depth of Concrete Stress Block ( $a$ )

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

Note: use  $f_y$  in KSI = 60 and  $f'_c$  in KSI = 5.5 for unit consistency, or keep all in PSI/LB.

1. **Numerator:**  $4.68 \times 60 = 280.80$  K

2. **Denominator:**  $0.85 \times 5.5 \times 21 = 98.175$  K/in

3. **Divide:**  $280.80/98.175$

Final  $a$ : **2.861** IN

#### Q8 Factor $\beta_1$

$$0.85 \geq 0.85 - 0.05 \left( \frac{f'_c - 4000}{1000} \right) \geq 0.65$$

1. **Calculate:**  $0.85 - 0.05 \times \frac{5500 - 4000}{1000} = 0.85 - 0.05 \times 1.5 = 0.85 - 0.075$

2. **Result:**  $0.775$  (within bounds  $0.65 \leq 0.775 \leq 0.85$  ✓)

Final  $\beta_1$ : **0.775**

#### Q9 Neutral Axis Depth ( $c$ )

$$c = \frac{a}{\beta_1} = \frac{2.861}{0.775}$$

Final  $c$ : **3.692** IN

### Part 4: Strain Check & Strength Reduction

#### Q10 Strain in Flexural Steel ( $\epsilon_t$ )

$$\epsilon_t = \frac{d - c}{c} \times 0.003 = \frac{27.295 - 3.692}{3.692} \times 0.003$$

1. **Numerator:**  $27.295 - 3.692 = 23.603$

2. **Ratio:**  $23.603/3.692 = 6.393$

3. **Multiply:**  $6.393 \times 0.003$

Final  $\epsilon_t$ : **0.01918** ( $\gg 0.005 \Rightarrow$  **Tension Controlled**)

#### Q11 Strength Reduction Factor ( $\phi$ )

Since  $\epsilon_t = 0.01918 \geq 0.005$ , the section is tension-controlled:

Final  $\phi$ : **0.90**

### Part 5: Moment Capacity

#### Q12 Tensile Force in Steel ( $T$ )

$$T = A_s \times f_y = 4.68 \times 60$$

Final  $T$ : **280.8** KIPS

#### Q13 Nominal Bending Moment ( $M_n$ )

$$M_n = T \left( d - \frac{a}{2} \right)$$

1. **Moment arm:**  $27.295 - 2.861/2 = 27.295 - 1.4305 = 25.865$  IN

2. **Multiply:**  $280.8 \times 25.865$

Final  $M_n$ : **7,262.9** K-IN

#### Q14 Factored Bending Resistance ( $\phi M_n$ )

$$\phi M_n = 0.90 \times 7,262.9$$

Final  $\phi M_n$ : **6,536.6** K-IN

#### Q15 Factored Design Moment ( $M_u$ )

$$M_u = \frac{\phi M_n}{12} = \frac{6,536.6}{12}$$

Final  $M_u$ : **544.7** K-FT

#	Question	Answer	Units
1	Flexural steel bar diameter, $d_b$	<b>1.410</b>	IN
2	Stirrup bar diameter	<b>0.500</b>	IN
3	Distance to steel center from bottom, $d_c$	<b>2.705</b>	IN
4	Effective depth, $d$	<b>27.295</b>	IN
5	Minimum steel area, $A_{s,min}$	<b>2.124</b>	IN <sup>2</sup>
6	Actual steel area, $A_s$	<b>4.68</b>	IN <sup>2</sup>
7	Stress block depth, $a$	<b>2.861</b>	IN
8	Factor $\beta_1$	<b>0.775</b>	—
9	Neutral axis depth, $c$	<b>3.692</b>	IN
10	Steel strain, $\epsilon_t$	<b>0.01918</b>	—
11	Strength reduction factor, $\phi$	<b>0.90</b>	—
12	Tensile force, $T$	<b>280.8</b>	K
13	Nominal moment, $M_n$	<b>7,262.9</b>	K-IN
14	Factored resistance, $\phi M_n$	<b>6,536.6</b>	K-IN
15	Factored design moment, $M_u$	<b>544.7</b>	K-FT