Arch 324
Structures II

Winter 2026 Recitation 004

Peter von Bülow
Amely Wackerbauer

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Recitation 004

Welcome to session 8!

- Quick Lecture Recap
- Homework #8 Concrete Beam Analysis
- Lab: Flexural Strain
- **Tower Testing POSTPONED - Monday March 30th**

Feel free to ask questions anytime

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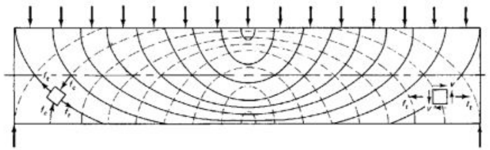
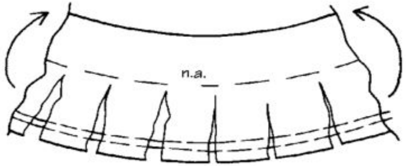
Lecture: Concrete Beam Analysis (3/16)

Flexure

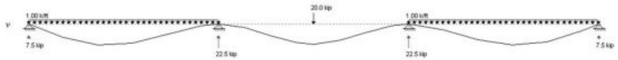


The stress trajectories in this simple beam, show principal tension as solid lines.

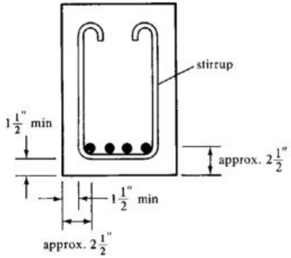


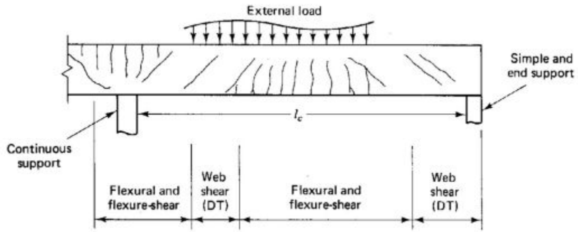
Reinforcement must be placed to resist these tensile forces

In beams continuous over supports, the stress reverses (negative moment).
In such areas, tensile steel is on top.



Shear reinforcement is provided by vertical or sloping stirrups.

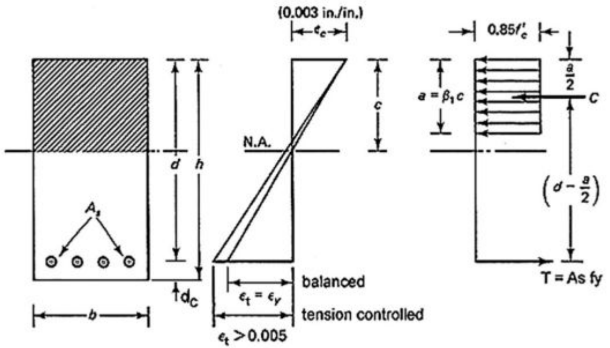




Lecture: Concrete Beam Analysis (3/16)

Flexure Equations

strain   ACI equivalent stress block



$$C = T$$

$$0.85 f_c' a b = A_s f_y$$

solving for a,

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{\rho f_y d}{0.85 f_c'}$$

$$\varepsilon_t = \frac{d - c}{c}(0.003)$$

$$\rho = \frac{A_s}{bd}$$

$$M_n = T\left(d - \frac{a}{2}\right) = A_s f_y \left(d - \frac{a}{2}\right)$$

$$M_u = \phi M_n$$

$$M_u = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

$$M_u = \phi A_s f_y d \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$$

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Lecture: Concrete Beam Analysis (3/16)

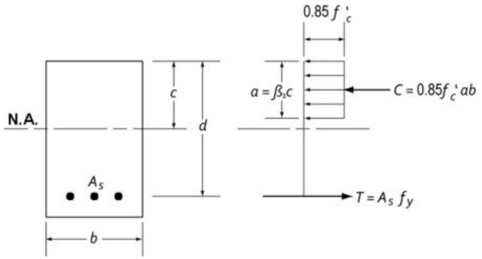
Rectangular Beam Analysis

Data:

- Section dimensions – b, h, (span)
- Steel area - As
- Material properties – f'c, fy

Required:

- Nominal Strength (of beam) Moment - Mn
- Required (by load) Design Moment – Mu
- Load capacity



As_{min}:
greater of (a) and (b)

(a) $\frac{3\sqrt{f_c'}}{f_y} b_w d$

(b) $\frac{200}{f_y} b_w d$

$$c = \frac{a}{\beta_1}$$

$$\varepsilon_t = \frac{d-c}{c} 0.003 \geq 0.005$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$\varphi M_n \geq M_u$$

$$M_u = \frac{(1.2 w_{DL} + 1.6 w_{LL}) l^2}{8}$$

$$1.6 w_{LL} = \frac{M_u 8}{l^2} - 1.2 w_{DL}$$

1. Calculate d
2. Check As min
3. Calculate a
4. Determine c
5. Check that $\varepsilon_t \geq 0.005$ (tension controlled)
6. Find nominal moment, Mn
7. Calculate required moment, ϕ Mn ≥ Mu

   (if $\varepsilon_t \geq 0.005$ then $\phi = 0.9$)
8. Determine max. loading (or span)

Lecture: Concrete Beam Design (3/18)

Details of Reinforcement

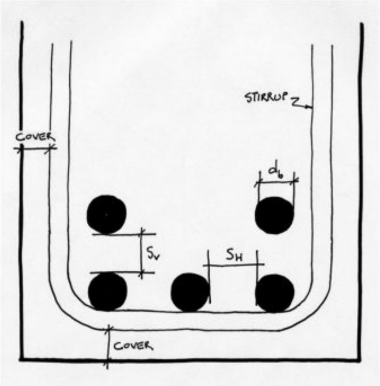
ACI 318 Chapter 25.2
Placement of Reinforcement

- Cover (ACI 20.6.1)
- Horizontal spacing in beams, s_h (ACI 25.2.1)
  - 1 inch
  - d_b
  - 4/3 $d_{agg,max}$
- Vertical spacing in beams (ACI 25.2.2)
  - Min 1 inch

https://www.constructioncost.co/honeycombing-in-concrete.html

Table 20.6.1.3.1—Specified concrete cover for cast-in-place nonprestressed concrete members

Concrete exposure	Member	Reinforcement	Specified cover, in.
Cast against and permanently in contact with ground	All	All	3
Exposed to weather or in contact with ground	All	No. 6 through No. 18 bars	2
		No. 5 bar, W31 or D31 wire, and smaller	1-1/2
Not exposed to weather or in contact with ground	Slabs, joists, and walls	No. 14 and No. 18 bars	1-1/2
		No. 11 bar and smaller	3/4
	Beams, columns, pedestals, and tension ties	Primary reinforcement, stirrups, ties, spirals, and hoops	1-1/2



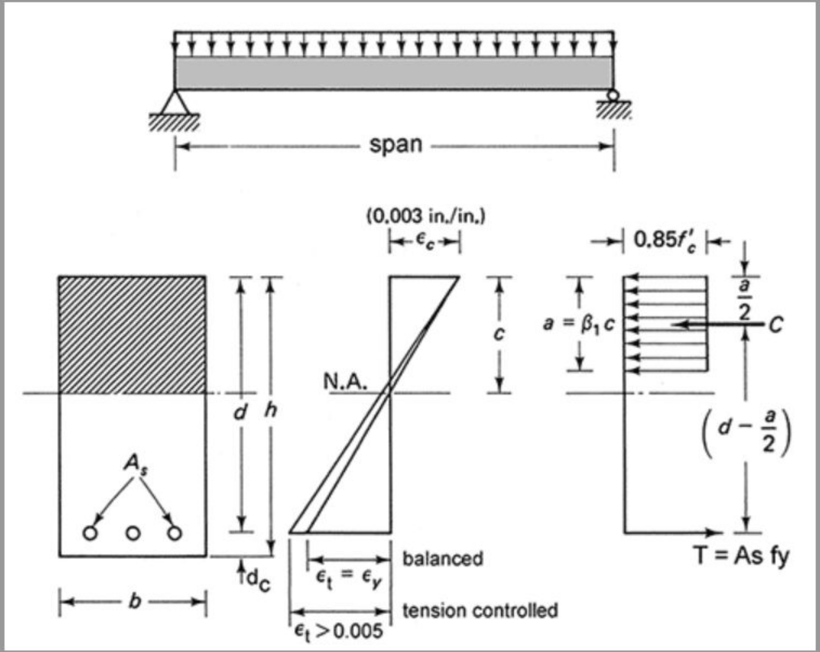
HW #8: Concrete Beam Analysis

8. Concrete Beam Analysis

Analyze the given composite floor system. Using a transformed section, determine peak stress values in both concrete and steel.

DATASET: 1 -2- -3-

simple span	17 FT
section width, b	14 IN
section height, h	25 IN
max. aggrigate size	0.75 IN
bar size number	5
the number of bars	6
stirrup bar size number	3
concrete cover	1.5 IN
concrete ultimate strength, f'c	6500 PSI
steel yield strength, fy	60000 PSI



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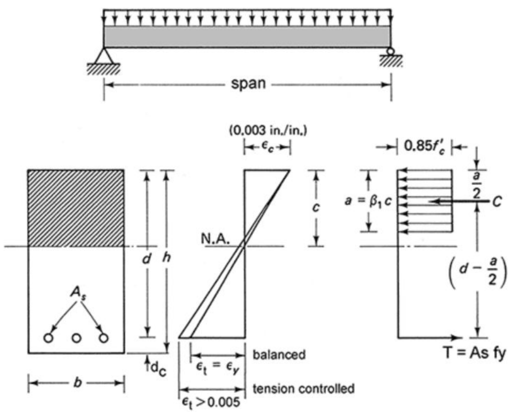

APPENDIX A—STEEL REINFORCEMENT INFORMATION

As an aid to users of the ACI Building Code, information on sizes, areas, and weights of various steel reinforcement is presented.

ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.65
18	2.257	4.00	13.60

Q1

Q2

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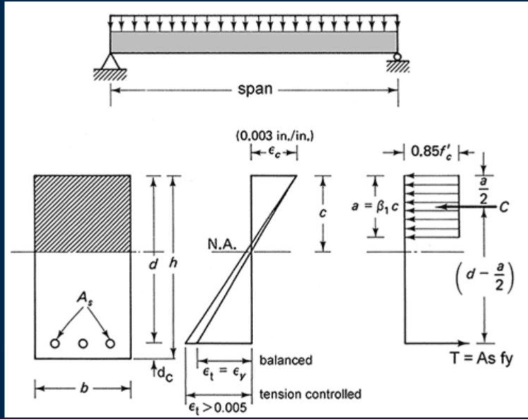
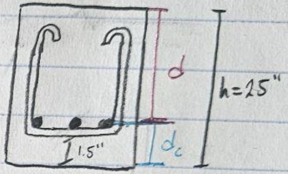


3. Distance from lower beam edge to center of flexural steel

$d_c = 1.5 + d_{stirrup} + 0.5(d_{steel\,bar})$ (1.5 = cover; $d_{stirrup}$ = #2; $d_{steel\,bar}$ = #1)

$= 1.5 + 0.375 + 0.5(0.625)$

$= \boxed{2.1875\text{ in}}$ ← Answer to #3



4. Distance from top beam edge to center of flexural steel

$d = h - d_c$ (h = given; d_c = #3)

$= 25 - 2.188$

$= \boxed{22.81\text{ in}}$ ← Answer to #4

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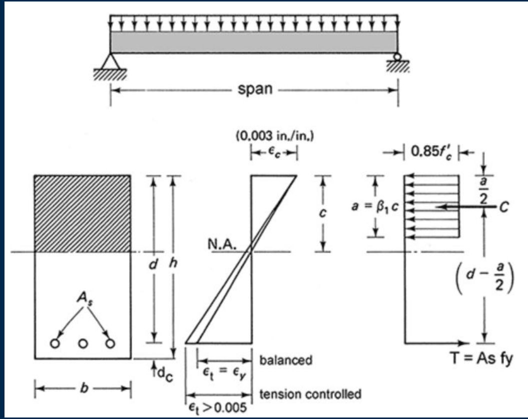

5. Minimum required area of steel

a. $\frac{3\sqrt{f'_c}}{f_y} b_w d = \frac{3\sqrt{6500}}{60{,}000}(14)(22.81) = \boxed{1.2873 \text{ in}^2}$ ←

b. $\frac{200}{f_y} b_w d = \frac{200}{60{,}000}(14)(22.81) = 1.0645$

6. Actual area of flexural steel

A_s = # of bars (←given) × bar area

= 6 (0.31) ↖ table we used for #1-2

= $\boxed{1.86 \text{ in}^2}$ ← Answer to #6

7. Depth of concrete stress block

$a = \frac{A_s f_y}{0.85 f'_c b}$ (#6) $= \frac{1.86(60{,}000)}{0.85(6{,}500)(14)} = \boxed{1.44 \text{ in}}$ ← Answer to #7

As,min:

greater of a and b

(a) $\frac{3\sqrt{f'_c}}{f_y} b_w d$

(b) $\frac{200}{f_y} b_w d$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

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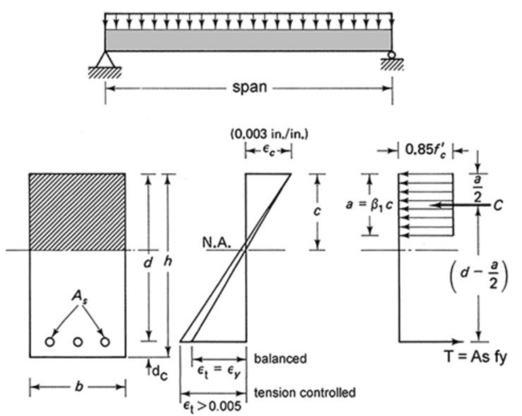

psi

f'_c	β_1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65

8. Use table
   f'c = given = 6,500psi

   Halfway between 6000 and 7000 is the value I am looking for (6,500psi)
   So…
   Halfway between 0.75 and 0.7 = **0.725** Q8

9. Distance to Neutral Axis from top of beam

$$c = \frac{a^{\#7}}{\beta 1_{\#8}} = \frac{1.44}{0.725} = \boxed{1.986\text{in}} \leftarrow \text{Answer to \#9}$$

$$c = \frac{a}{\beta_1}$$

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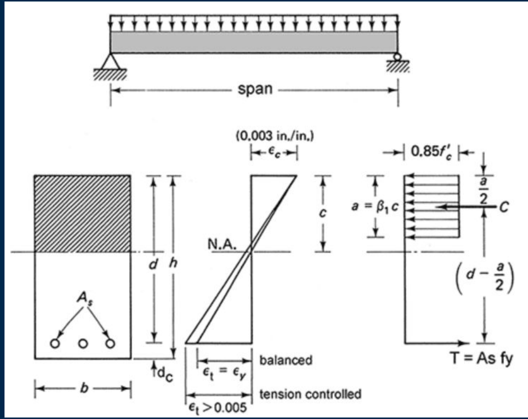

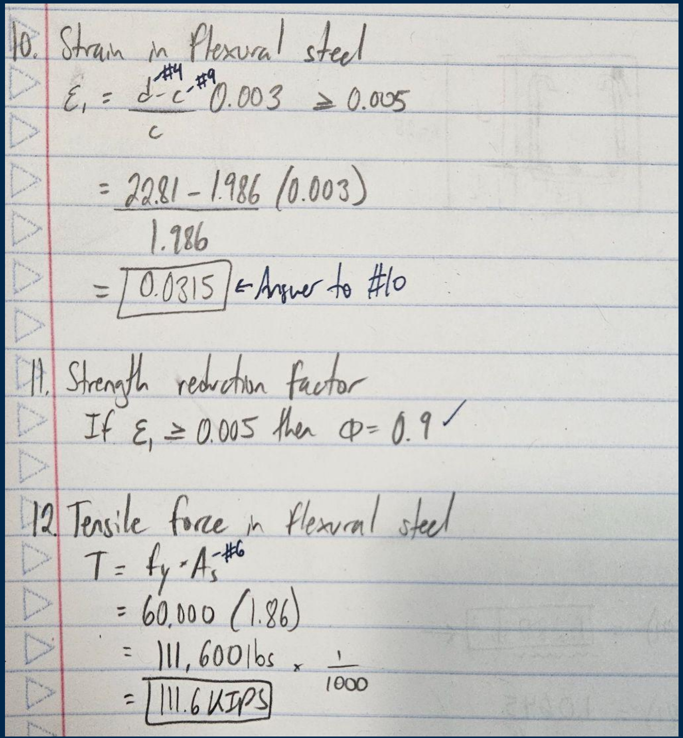

10. Strain in Flexural steel

$\varepsilon_1 = \frac{d - c}{c} 0.003 \geq 0.005$ (#4, #9)

$= \frac{22.81 - 1.986}{1.986} (0.003)$

$= \boxed{0.0315}$ ← Answer to #10

11. Strength reduction factor

If $\varepsilon_1 \geq 0.005$ then $\Phi = 0.9$ ✓

12. Tensile force in flexural steel

$T = f_y \cdot A_s$ (#6)

$= 60,000 (1.86)$

$= 111,600 \text{ lbs} \times \frac{1}{1000}$

$= \boxed{111.6 \text{ KIPS}}$

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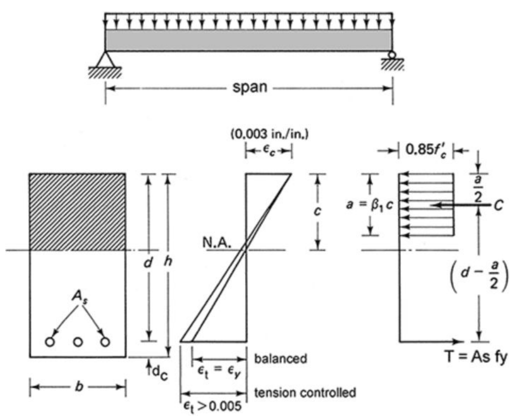

13. Nominal bending moment

$$M_n = T\left(d - \frac{a}{2}\right)$$ [#12] [#4] [#7]

$$= 111.6\left(22.81 - \frac{1.44}{2}\right)$$

$$= 2{,}465.24 \text{ KIP-IN}$$

14. Factored bending resistance

$$\Phi M_n = 0.9\,(2{,}465.24)$$ [#11] [#13]

$$= 2{,}218.72 \text{ KIP-IN}$$

15. Factored design moment

$$M_u = \Phi M_n$$

$$= 2{,}218.72 \times \frac{1}{12}$$

$$= 184.89 \text{ KIP-FT}$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

$$\varphi M_n \geq M_u$$

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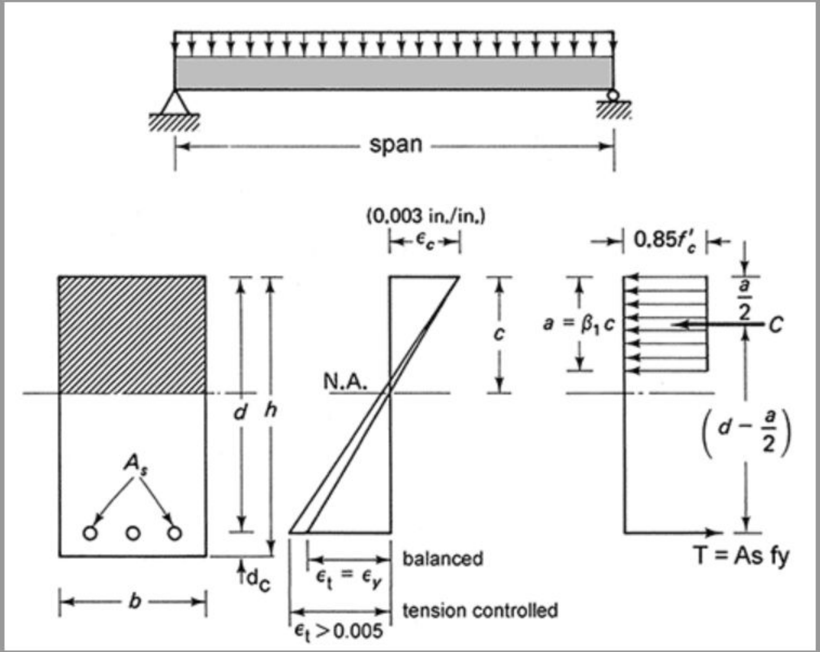

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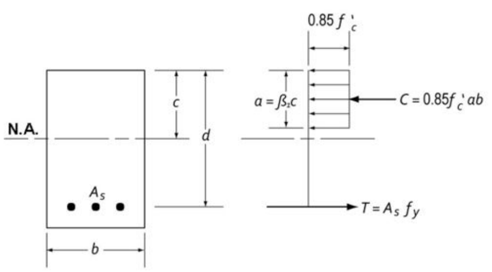


$$c = \frac{a}{\beta_1}$$

$$\varepsilon_t = \frac{d-c}{c} 0.003 \geq 0.005$$

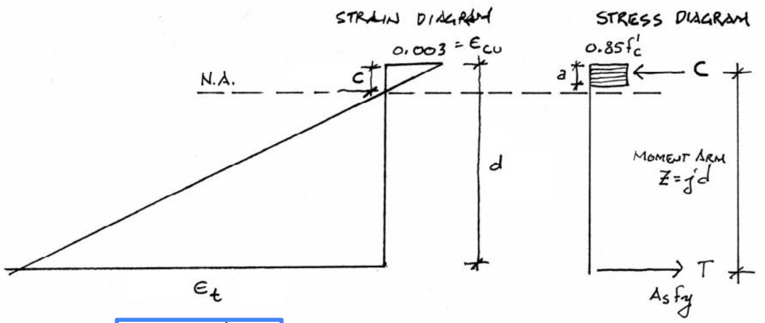
$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$\varphi M_n \geq M_u$$

$$M_u = \frac{(1.2w_{DL} + 1.6w_{LL})l^2}{8}$$

$$1.6w_{LL} = \frac{M_u 8}{l^2} - 1.2w_{DL}$$



$$\frac{\epsilon_t}{\epsilon_{cu}} : \frac{d-c}{c}$$

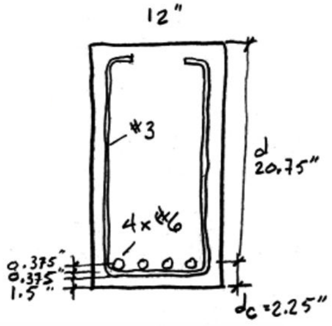
Lab: Flexural Strain

C = 0.85(f'c)ab

T = $A_s(f_y)$

Z = d - (a/2)

Z = jd so j = Z/d



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