Arch 324
Structures II

Winter 2026 Recitation 004

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TAUBMAN COLLEGE

Recitation 004

Welcome to session 10!

- Quick Lecture Recap
- Homework #9 Concrete Beam Design
- Lab: None
- Tower Final Report

Feel free to ask questions anytime

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Lecture:

Rectangular Beam Design

Two approaches:

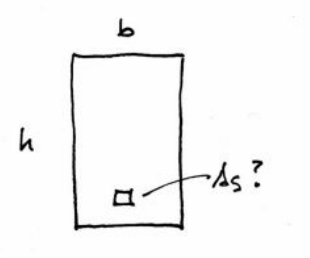
Method 1:

Data:

- Load and Span
- Material properties – f'_c, f_y
- All section dimensions: h and b

Required:

- Steel area – A_s



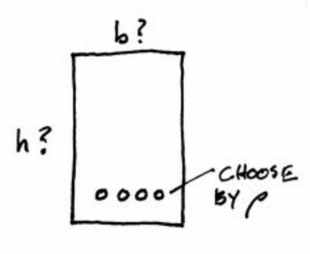
Method 2:

Data:

- Load and Span
- Some section dimensions – h or b
- Material properties – f'_c, f_y
- Choose ρ

Required:

- Steel area – A_s
- Beam dimensions – b or h



Lecture:

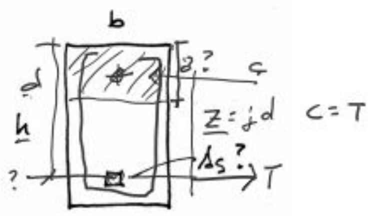
Rectangular Beam Design – **Method 1**

Data:

- Load and Span
- Material properties – f'_c, f_y
- All section dimensions – b and h

Required:

- Steel area - A_s



1. Calculate the factored load and find factored required moment, M_u
2. Find d = h – cover – stirrup – $d_b/2$
3. Estimate moment arm z = jd, for beams j ≈ 0.9 for slabs j ≈ 0.95
4. Estimate A_s based on estimate of jd.
5. Use As to find a
6. Use a to find A_s (repeat…until **2%** accuracy)
7. Choose bars for A_s and check A_s max & min
8. Check that $\varepsilon_t \geq 0.005$
9. Check $M_u \leq \phi M_n$ (final condition)

10. Design shear reinforcement (stirrups)
11. Check deflection, crack control, rebar development length

$$M_u = \frac{(\gamma_{DL} w_{DL} + \gamma_{LL} w_{LL}) l^2}{8}$$

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

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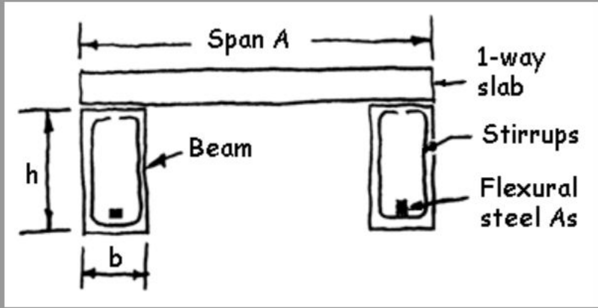
HW #9: Concrete Beam Design

Using the Ultimate Strength Method, analyze the given section to determine its safe moment capacity, Mu, based on the given parameters. Check that the section is tension controlled (epsilon_t > 0.005), and that the amount of steel, As is more than the minimum, As_min.

DATASET: 1 -2- -3-

Span of slab	19 FT
Span of beam	30 FT
Thickness of slab	12 IN
section width, b	18 IN
section height, h	39 IN
max. aggrigate size	0.75 IN
bar size number	9
stirrup bar size number	3
concrete cover	1.5 IN
concrete ultimate strength, f'c	5500 PSI
steel yield strength, fy	60000 PSI
Floor Live Load	45 PSF



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HW #9: Concrete Beam Design

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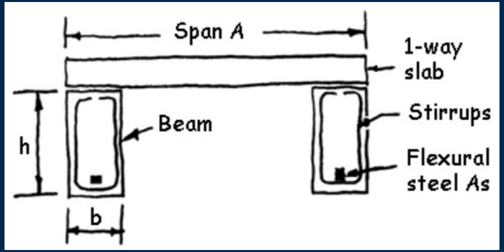

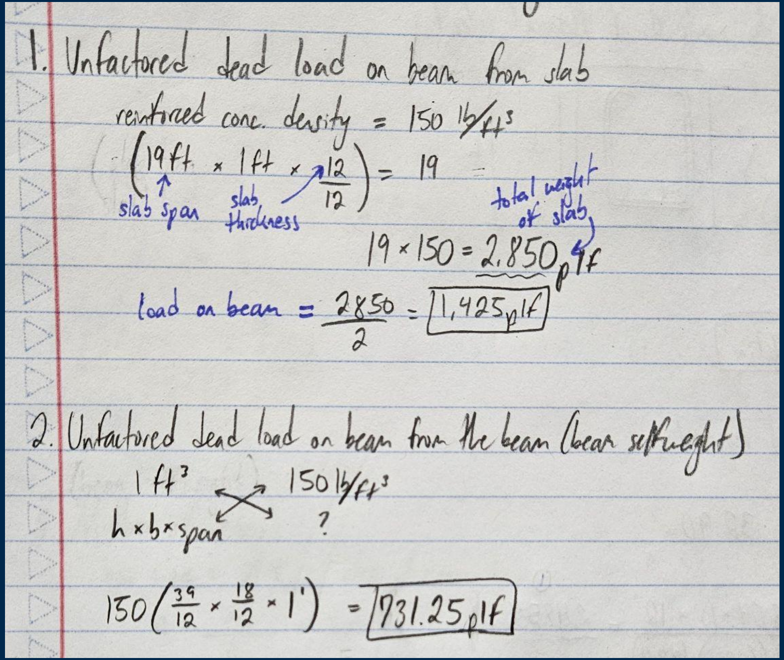

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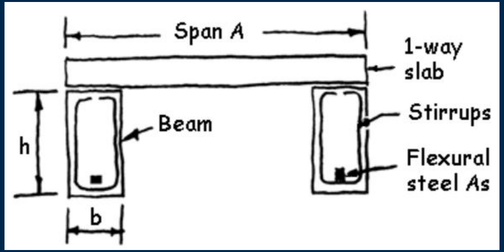

3. Unfactored live load on the beam

live load × $\frac{\text{slab span}}{2}$ × per linear ft of beam

$$45\left(\frac{19}{2}\right) \times 1 = \boxed{427.5 \text{ plf}}$$

4. Total factored beam load

$$1.2D + 1.6L$$

$$1.2(1{,}425 + 731.25) + 1.6(427.5)$$

slab (#1) beam (#2) LL (#3)

$$= \boxed{3{,}271.5 \text{ plf}}$$

5. Factored design moment from the loads

$$M_u = \frac{wL^2}{8} = \frac{3{,}271.5(30^2)}{8} = 368{,}043.75 \times \frac{1}{1000}$$

$$= \boxed{368.04 \text{ ft-k}}$$

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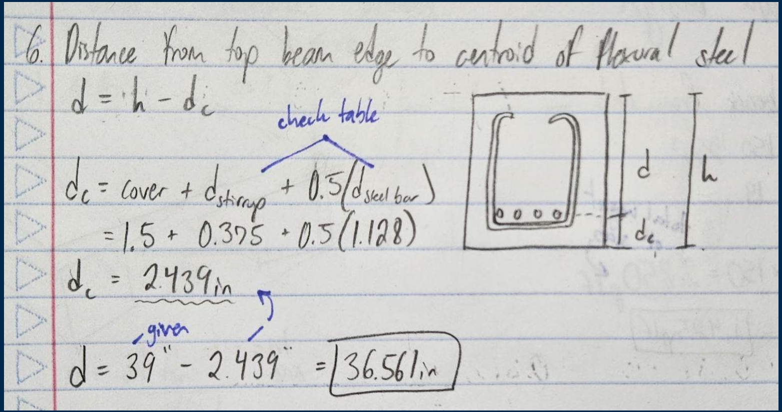
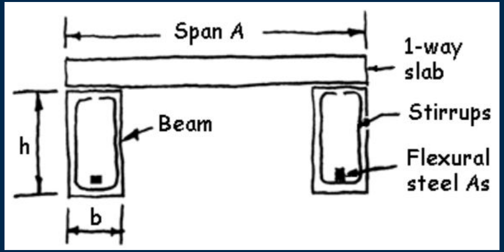


APPENDIX A—STEEL REINFORCEMENT INFORMATION

As an aid to users of the ACI Building Code, information on sizes, areas, and weights of various steel reinforcement is presented.

ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.65
18	2.257	4.00	13.60



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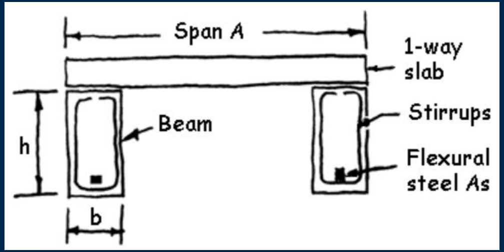

7. Final calculated area of steel required

* estimate moment arm *

$$z = jd = 0.9(36.561) = 32.90$$

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{368.04 \times 10^3 \times 12}{0.9(60{,}000)(32.9)} = 2.4859 \quad ①$$

Use A_s to find a → a to find A_s

$$a = \frac{A_s f_y}{0.85 f'_c (b)} = \frac{2.4859(60{,}000)}{0.85(5500)(18)} = 1.7725 \text{ in}$$

$$d - \frac{a}{2} = 36.561 - \frac{1.7725}{2} = 35.6748$$

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{368.04 \times 10^3 \times 12}{0.9(60{,}000)(35.6748)} =$$

New A_s: 2.2926 in^2 ②

Check!

$$\frac{2.4859 - 2.2926}{2.4859} = 0.0778 \times 100 = 7.78\%$$

☹ Need within 2% Iterate again!

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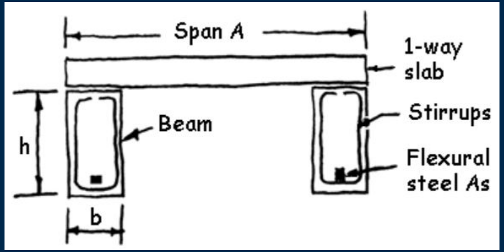

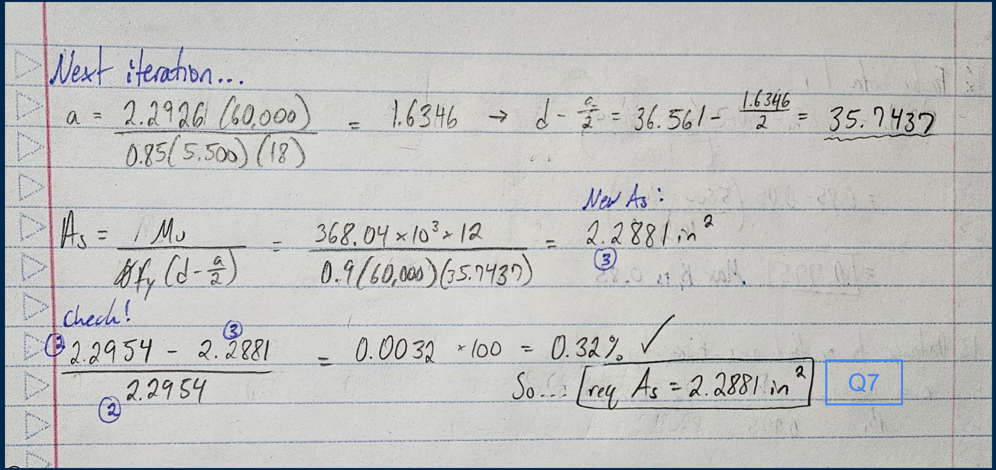

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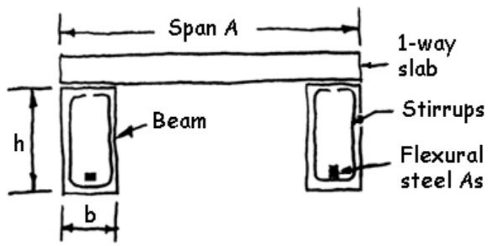

8. Number of rebars

bar size = #9 area of each bar = 1.00 in² ← Table A.2

$n = \frac{A_s}{A_{\#9}} = \frac{2.2882}{1.00} = 2.2882$ round up → [3]

9. Actual A_s used

# of rebar × Area per bar

$3(1.0) = 3$

10. Minimum req. area of steel

$\frac{3\sqrt{f'_c}(bd)}{f_y} = \frac{3\sqrt{5,500}(18)(36.561)}{60,000} = \boxed{2.44 \text{ in}^2}$ ←

$\frac{200(bd)}{f_y} = \frac{200(18)(36.561)}{60,000} = 2.19 \text{ in}^2$

} Greater of the two values

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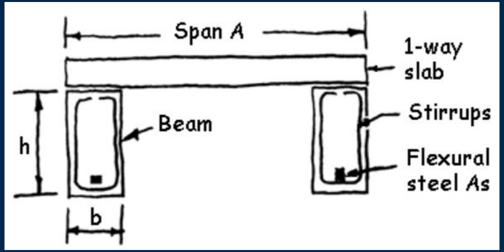

11. Depth of concrete stress block

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{{}^{\#9}3.0(60{,}000)}{0.85(5500)(18)} = \boxed{2.14\text{ in}}$$

12. Factor beta 1

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right)$$

$$= 0.85 - 0.05\left(\frac{5500 - 4000}{1000}\right)$$

$$= \boxed{0.775}$$ Max β_1 is 0.85

13. Distance to neutral axis from top of beam

$$c = \frac{a}{\beta_1} = \frac{2.14}{0.775} = \boxed{2.76\text{ in}}$$

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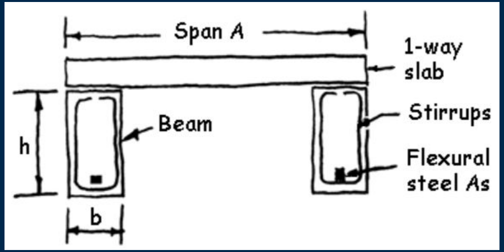


14. Strain in flexural steel

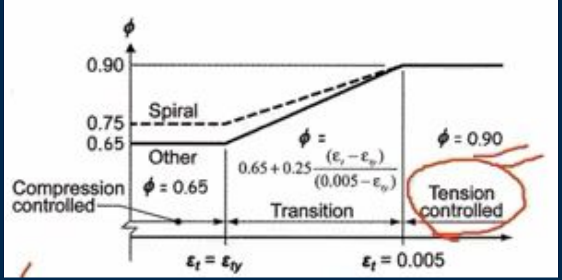
$$\varepsilon_t = \frac{d-c}{c}(0.003)$$

$$= \frac{36.56-2.76}{2.76}(0.003)$$

$$= 0.0367 \quad > 0.005$$

tension-controlled

so $\phi = 0.9$ (Q#15)



16. Tensile force in the flexural steel

$T = A_s f_y = 3(60{,}000)$ (#9)

$= 180{,}000 \times \frac{1}{1000}$

$= 180\text{ k}$

17. Nominal bending moment

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$= 3(60{,}000)\left(36.56 - \frac{2.14}{2}\right)$$

$$= 6{,}388{,}200 \times \frac{1}{1000}$$

$$= 6{,}388.2\text{ k-in}$$

18. Factored bending resistance

$$\phi M_n = 0.9(6{,}388.2)$$

$$= 5{,}749.38 \times \frac{1}{12}$$

$$= 479.12\text{ k-ft}$$

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Using the Ultimate Strength Method, analyze the given section to determine its safe moment capacity, Mu, based on the given parameters. Check that the section is tension controlled (epsilon_t > 0.005), and that the amount of steel, As is more than the minimum, As_min.

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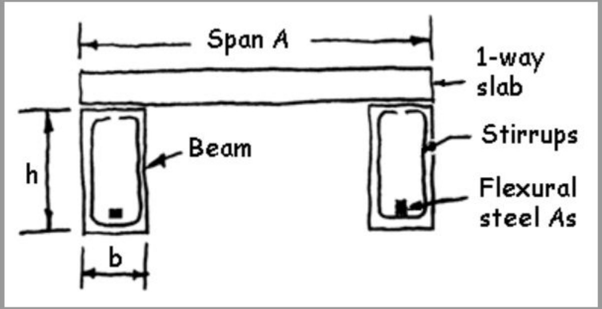

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Tower Final Report

- Due April 19
- Tower Testing results are being uploaded
  - Use video to screenshot frames and show failure
- See Structures website for guidelines and **score sheet**
  - Score sheet includes exactly what we are looking for in your report
  - I may review this next week :)
- Report sample

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