Architecture 324: Structures II

LRFD Reinforced Concrete Beam Design — Guided Worksheet

Determine Required Steel, Factored Bending Resistance, and Factored Design Moment (ACI 318)

Given Data: **Span of Slab:** 12 FT **Span of Beam:** 22 FT **Slab Thickness:** 8 IN **b:** 11 IN **h:** 17 IN **Max. Agg.:** 0.75 IN
Bar #: 9 **Stirrup #:** 4 **Cover:** 1.5 IN **f'_c:** 6,500 PSI **f_y:** 60,000 PSI **Floor Live Load:** 55 PSF **Concrete Unit Wt. (γ_c):** 150 PCF

Standard Rebar Properties:

Bar #	3	4	5	6	7	8	9	10	11	14	18
Diameter, d_b (in)	0.375	0.500	0.625	0.750	0.875	1.000	1.128	1.270	1.410	1.693	2.257
Area, A_b (in^2)	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56	2.25	4.00

β_1 Table:

f'_c (PSI)	≤4000	5000	6000	6500	7000	8000	≥8000
β_1	0.85	0.80	0.75	0.725	0.70	0.65	0.65

Formula: $\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right)$, $0.65 \le \beta_1 \le 0.85$

Part 1: Loads

Q1 Unfactored Dead Load on Beam from Slab [PLF]

*The slab spans between beams. The **tributary width** is the width of slab that loads this beam. With beams on both sides, each beam carries half the slab span: tributary = span/2 = 6 ft. Multiply the slab weight per square foot by the tributary width.*

$$w_{\text{slab}} = \frac{t_{\text{slab}}}{12} \times \gamma_c \times \text{tributary width}$$

1. **Slab weight per SF:** $\frac{\_\_\_\_}{12}$ × ______ = _________ PSF
2. **Tributary width:** ______/2 = ______ FT
3. **Multiply:** _________ × ______

Answer DL$_{\text{slab}}$: __________________ PLF

Q2 Unfactored Dead Load from Beam Self-Weight [PLF]

The beam's own weight per linear foot is its cross-sectional area (in ft^2) times the unit weight of concrete.

$$w_{\text{beam}} = \frac{b}{12} \times \frac{h}{12} \times \gamma_c$$

1. **Calculate:** $\frac{\_\_\_\_}{12}$ × $\frac{\_\_\_\_}{12}$ × ______ =

Answer DL$_{\text{beam}}$: __________________ PLF

Q3 Unfactored Live Load on Beam [PLF]

Same tributary width as the slab dead load. Multiply the floor live load (PSF) by the tributary width.

$$w_{LL} = \text{LL} \times \text{tributary width}$$

1. **Calculate:** ______ PSF × ______ FT =

Answer LL: __________________ PLF

Q4 Total Factored Beam Load (w_u) [PLF]

ACI load combination: $w_u = 1.2 \times DL + 1.6 \times LL$. Add slab dead load and beam self-weight for total DL.

$$w_u = 1.2\,(w_{\text{slab}} + w_{\text{beam}}) + 1.6\,w_{LL}$$

1. **Total DL:** _______ + _______ = ___________ PLF
2. **Factored:** 1.2× _________ +1.6× _________ =

Answer w_u: __________________ PLF

Q5 Factored Design Moment from Loads (M_u) [FT-K]

For a simply supported beam with uniform load, the maximum moment at midspan is:

$$M_u = \frac{w_u\,L^2}{8}$$

1. **Calculate:** $\frac{\_\_\_\_\_ \times \_\_\_\_\_^2}{8}$ = ___________ FT-LB
2. **Convert to FT-K:** ÷ 1000 =

Answer M_u: __________________ FT-K

Part 2: Section Geometry

Q6 Effective Depth (d) [IN]

*Measure from the **top** of the beam (compression face) to the **centroid** of the tension steel. Layers from bottom up: cover, stirrup, half the flexural bar.*

$$d = h - \text{cover} - d_{\text{stirrup}} - \frac{d_b}{2}$$

1. **Look up d_b:** _______ in d_{stirrup}: _______ in
2. **d_c:** ______ + ______ + ______/2 = _________ in
3. **d:** ______ − _________ =

Answer d: __________________ IN

Part 3: Steel Design (Iterative Method 1)

Q7 Required Area of Steel ($A_{s,\text{req}}$) [IN^2]

Use the iterative procedure from lecture. First estimate the moment arm $z \approx 0.9d$, then iterate until A_s converges (within 2%).

$$A_s = \frac{M_u}{\phi\,f_y\left(d - \frac{a}{2}\right)} \qquad a = \frac{A_s\,f_y}{0.85\,f'_c\,b}$$

1. **Convert M_u to IN-K:** _________ FT-K ×12 = ___________ IN-K
2. **Trial 1:** $z = 0.9 \times d = 0.9\times$ ________ = _________ in

   $A_s = \frac{M_u}{\phi\,f_y\,z} = \frac{\_\_\_\_\_\_\_\_}{0.9 \times 60 \times \_\_\_\_\_\_\_\_}$ = _________ in^2
3. **Trial 2:** Find a, then refine A_s:

   $a = \frac{A_s \times 60}{0.85 \times 6.5 \times 11}$ = _________ in

   $A_s = \frac{M_u}{0.9 \times 60 \times (d - a/2)}$ = _________ in^2
4. **Check convergence:** Change < 2%? ☐ Yes ☐ No (repeat)

Answer $A_{s,\text{req}}$: __________________ IN^2

Q8 Number of Rebars [count]

*Divide $A_{s,req}$ by the area of one bar (A_b) and **round up** to the next whole number.*

$$n = \left\lceil \frac{A_{s,\text{req}}}{A_b} \right\rceil$$

1. **Calculate:** ________ ÷ ________ = ________ → round up =

Answer: __________________ bars

Q9 Actual Area of Flexural Steel Used ($A_{s,\text{used}}$) [IN^2]

$$A_{s,\text{used}} = n \times A_b$$

1. **Calculate:** ______ × ______ =

Answer $A_{s,\text{used}}$: __________________ IN^2

Q10 Minimum Steel Area ($A_{s,\min}$) [IN^2]

*ACI 318-14 §9.6.1.2 for beams. Compute both, take the **larger**.*

$$\text{(a)}\ \frac{3\sqrt{f'_c}}{f_y}\,b\,d \qquad \text{(b)}\ \frac{200}{f_y}\,b\,d$$

1. **(a):** $\frac{3\sqrt{\_\_\_\_}}{\_\_\_\_}$ × _____ × ______ = _________ in^2
2. **(b):** $\frac{200}{\_\_\_\_}$ × _____ × ______ = _________ in^2
3. **Which is larger?** Circle: (a) / (b)
4. **Check:** $A_{s,\text{used}} \ge A_{s,\min}$? ☐ Yes ☐ No

Answer $A_{s,\min}$: __________________ IN^2

Part 4: Stress Block, Neutral Axis, & Strain Check

Q11 Depth of Stress Block (a) [IN]

Recalculate a using $A_{s,used}$ (not $A_{s,req}$).

$$a = \frac{A_{s,\text{used}} \times f_y}{0.85\,f'_c\,b}$$

Answer a: __________________ IN

Q12 Factor β_1 [dimensionless]

Use the β_1 table or formula above for $f'_c = 6{,}500$ PSI.

Answer β_1: __________________

Q13 Neutral Axis Depth (c) [IN]

$$c = \frac{a}{\beta_1}$$

Answer c: __________________ IN

Q14 Strain in Flexural Steel (ε_t) [in/in]

$$\varepsilon_t = \frac{d - c}{c} \times 0.003$$

Answer ε_t: __________________ Check: $\varepsilon_t \ge 0.005$? ☐ Yes (tension controlled) ☐ No

Q15 Strength Reduction Factor (ϕ) [dimensionless]

$\varepsilon_t \ge 0.005 \Rightarrow \phi = 0.90$ (tension controlled) $0.002 \le \varepsilon_t < 0.005 \Rightarrow$ interpolate $\varepsilon_t < 0.002 \Rightarrow \phi = 0.65$

Answer ϕ: __________________

Part 5: Moment Capacity

Q16 Tensile Force in Steel (T) [K]

$$T = A_{s,\text{used}} \times f_y \quad (\text{use } f_y \text{ in KSI})$$

Answer T: __________________ K

Q17 Nominal Bending Moment (M_n) [K-IN]

$$M_n = T \times \left(d - \frac{a}{2}\right)$$

1. **Moment arm:** $d - a/2$ = ________ − ________ = _________ IN
2. **Multiply:** ________ K × ________ IN =

Answer M_n: __________________ K-IN

Q18 Factored Bending Resistance (ϕM_n) [K-FT]

*Apply ϕ, then convert to K-FT. This is the beam's **capacity**. Verify $\phi M_n \ge M_u$ (from Q5).*

$$\phi M_n = \frac{\phi \times M_n}{12}$$

1. **Calculate:** $\frac{\_\_\_\_\_ \times \_\_\_\_\_\_\_\_}{12}$ = ___________ K-FT

Answer ϕM_n: __________________ K-FT