Architecture 324: Structures II
LRFD Reinforced Concrete Beam Design
Problem 9: Concrete Beam Design [**ANSWERS**]

Given Data: **Span of Slab:** 12 FT **Span of Beam:** 22 FT **Slab Thickness:** 8 IN b: 11 IN h: 17 IN **Max. Agg.:** 0.75 IN
Bar #: 9 **Stirrup #:** 4 **Cover:** 1.5 IN f'_c: 6,500 PSI f_y: 60,000 PSI **Floor Live Load:** 55 PSF γ_c: 150 PCF
Rebar Properties: #9: $d_b = 1.128$ in, $A_b = 1.00$ in^2 #4: $d_{\text{stirrup}} = 0.500$ in **Tributary width** $= 12/2 = 6$ ft (slab on one side)

Part 1: Loads

Q1 Unfactored Dead Load on Beam from Slab

$$w_{\text{slab}} = \frac{t_{\text{slab}}}{12} \times \gamma_c \times \text{trib.} = \frac{8}{12} \times 150 \times 6$$

1. Slab weight per SF: $8/12 \times 150 = 100$ PSF
2. Tributary width: $12/2 = 6$ FT
3. 100×6

Final DL$_{\text{slab}}$: 600 PLF

Q2 Unfactored Dead Load from Beam Self-Weight

$$w_{\text{beam}} = \frac{b}{12} \times \frac{h}{12} \times \gamma_c = \frac{11}{12} \times \frac{17}{12} \times 150$$

1. $0.9167 \times 1.4167 \times 150$

Final DL$_{\text{beam}}$: 194.79 PLF

Q3 Unfactored Live Load on Beam

$$w_{LL} = 55 \times 6$$

Final LL: 330 PLF

Q4 Total Factored Beam Load (w_u)

$$w_u = 1.2(600 + 194.79) + 1.6(330)$$

1. Total DL: $600 + 194.79 = 794.79$ PLF
2. $1.2 \times 794.79 = 953.75$ PLF
3. $1.6 \times 330 = 528.00$ PLF
4. $953.75 + 528.00$

Final w_u: 1,481.75 PLF

Q5 Factored Design Moment from Loads (M_u)

$$M_u = \frac{w_u\,L^2}{8} = \frac{1{,}481.75 \times 22^2}{8} = \frac{1{,}481.75 \times 484}{8}$$

1. $1{,}481.75 \times 484 = 717{,}167$ FT-LB
2. $717{,}167/8 = 89{,}646$ FT-LB
3. $89{,}646/1{,}000$

Final M_u: 89.65 FT-K

Part 2: Section Geometry

Q6 Effective Depth (d)

$$d = h - \text{cover} - d_{\text{stirrup}} - \frac{d_b}{2}$$

1. Bar #9: $d_b = 1.128$ in; Stirrup #4: $d_{\text{stirrup}} = 0.500$ in
2. $d_c = 1.500 + 0.500 + 1.128/2 = 1.500 + 0.500 + 0.564 = 2.564$ in
3. $d = 17 - 2.564$

Final d: 14.436 IN (some rounding: 14.4375 using $d_b/2 = 0.5625$)

Part 3: Steel Design (Iterative Method 1)

Q7 Required Area of Steel ($A_{s,\text{req}}$)

Convert: $M_u = 89.65 \times 12 = 1{,}075.8$ IN-K. Use $\phi = 0.9$, $f_y = 60$ KSI.

1. **Trial 1:** $z = 0.9d = 0.9 \times 14.4375 = 12.994$ in
   $$A_s = \frac{1{,}075.8}{0.9 \times 60 \times 12.994} = \frac{1{,}075.8}{701.7} = 1.533 \text{ in}^2$$
2. **Trial 2:** $a = \dfrac{1.533 \times 60}{0.85 \times 6.5 \times 11} = \dfrac{91.98}{60.78} = 1.513$ in
   $$A_s = \frac{1{,}075.8}{0.9 \times 60 \times (14.4375 - 0.757)} = \frac{1{,}075.8}{54 \times 13.681} = \frac{1{,}075.8}{738.8} = 1.456 \text{ in}^2$$
3. **Trial 3:** $a = \dfrac{1.456 \times 60}{60.78} = 1.437$ in
   $$A_s = \frac{1{,}075.8}{54 \times (14.4375 - 0.719)} = \frac{1{,}075.8}{54 \times 13.719} = 1.452 \text{ in}^2$$
4. Change $< 2\%$ ✓ converged

Final $A_{s,\text{req}}$: 1.45 IN^2

Q8 Number of Rebars

$$n = \lceil 1.45/1.00 \rceil = 2$$

Check fit in $b = 11$ in: $2(1.5) + 2(0.5) + 2(1.128) + 1(1.128) = 7.38$ in < 11 in ✓

Final: 2 bars

Q9 Actual Area of Flexural Steel ($A_{s,\text{used}}$)

$$A_{s,\text{used}} = 2 \times 1.00$$

Final $A_{s,\text{used}}$: 2.00 IN^2

Q10 Minimum Steel Area ($A_{s,\min}$)

1. **(a):** $\dfrac{3\sqrt{6500}}{60{,}000} \times 11 \times 14.4375 = \dfrac{3 \times 80.623}{60{,}000} \times 158.81 = 0.004031 \times 158.81 = $ **0.640** in^2 $\leftarrow$ **controls**
2. **(b):** $\dfrac{200}{60{,}000} \times 11 \times 14.4375 = 0.003333 \times 158.81 = 0.529$ in^2
3. Check: $A_{s,\text{used}} = 2.00 > 0.640$ ✓

Final $A_{s,\min}$: 0.640 IN^2

Part 4: Stress Block, Neutral Axis, & Strain Check

Q11 Depth of Stress Block (a)

$$a = \frac{A_{s,\text{used}} \times f_y}{0.85\,f'_c\,b} = \frac{2.00 \times 60}{0.85 \times 6.5 \times 11} = \frac{120}{60.775}$$

Final a: 1.974 IN

Q12 Factor β_1

$$\beta_1 = 0.85 - 0.05\left(\frac{6500 - 4000}{1000}\right) = 0.85 - 0.05 \times 2.5 = 0.85 - 0.125$$

Final β_1: 0.725

Q13 Neutral Axis Depth (c)

$$c = \frac{a}{\beta_1} = \frac{1.974}{0.725}$$

Final c: 2.723 IN

Q14 Strain in Flexural Steel (ε_t)

$$\varepsilon_t = \frac{d - c}{c} \times 0.003 = \frac{14.4375 - 2.723}{2.723} \times 0.003$$

1. Numerator: $14.4375 - 2.723 = 11.714$
2. Ratio: $11.714/2.723 = 4.302$
3. 4.302×0.003

Final ε_t: 0.01290 ($\gg 0.005 \Rightarrow$ **Tension Controlled**)

Q15 Strength Reduction Factor (ϕ)

Since $\varepsilon_t = 0.01290 \geq 0.005$, the section is tension-controlled:

Final ϕ: 0.90

Part 5: Moment Capacity

Q16 Tensile Force in Steel (T)

$$T = A_{s,\text{used}} \times f_y = 2.00 \times 60$$

Final T: 120.0 K

Q17 Nominal Bending Moment (M_n)

$$M_n = T\left(d - \frac{a}{2}\right)$$

1. Moment arm: $14.4375 - 1.974/2 = 14.4375 - 0.987 = 13.450$ IN
2. 120.0×13.450

Final M_n: 1,614.0 K-IN

Q18 Factored Bending Resistance (ϕM_n)

$$\phi M_n = \frac{\phi \times M_n}{12} = \frac{0.90 \times 1{,}614.0}{12} = \frac{1{,}452.6}{12}$$

1. $\phi M_n = 121.1$ K-FT
2. Check: $\phi M_n = 121.1$ K-FT $\geq$ $M_u = 89.65$ K-FT ✓ **OK**

Final ϕM_n: 121.1 K-FT

Summary of Answers

#	Question	Answer	Units
1	Unfactored DL from slab	**600**	PLF
2	Beam self-weight	**194.79**	PLF
3	Unfactored LL	**330**	PLF
4	Factored load, w_u	**1,481.75**	PLF
5	Factored moment from loads, M_u	**89.65**	FT-K
6	Effective depth, d	**14.4375**	IN
7	Required steel area, $A_{s,\text{req}}$	**1.45**	IN^2
8	Number of rebars	**2**	—
9	Actual steel area, $A_{s,\text{used}}$	**2.00**	IN^2
10	Minimum steel area, $A_{s,\text{min}}$	**0.640**	IN^2
11	Stress block depth, a	**1.974**	IN
12	Factor β_1	**0.725**	—
13	Neutral axis depth, c	**2.723**	IN
14	Steel strain, ε_t	**0.01290**	—
15	Strength reduction factor, ϕ	**0.90**	—
16	Tensile force, T	**120.0**	K
17	Nominal moment, M_n	**1,614.0**	K-IN
18	Factored resistance, ϕM_n	**121.1**	K-FT
Final Check: $\phi M_n = 121.1$ *K-FT* $\geq M_u = 89.65$ *K-FT* ✓ *Beam is adequate.*			