

Arch324

STRUCTURES II

Winter 2026
Recitation

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Recitation Guidelines

Homework Problem

No Lab

- Please complete the lab sheet during recitation and hand it in before leaving.
- Try to attend all sessions. Unexcused absences will **affect your grade** starting from the second missed class.

Analysis Example - HW9

9. Concrete Beam Design

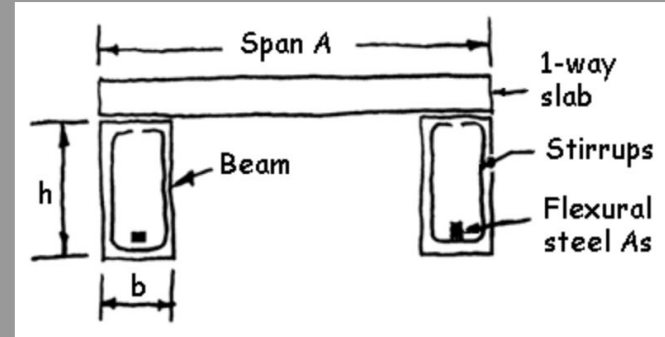
Using the Ultimate Strength Method, analyze the given section to determine its safe moment capacity, M_u , based on the given parameters. Check that the section is tension controlled ($\epsilon_t > 0.005$), and that the amount of steel, A_s is more than the minimum, A_{s_min} .

DATASET: 1

-2-

-3-

Span of slab	14 FT
Span of beam	23 FT
Thickness of slab	9 IN
section width, b	10 IN
section height, h	18 IN
max. aggregate size	0.75 IN
bar size number	8
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f_c	5500 PSI
steel yield strength, f_y	60000 PSI
Floor Live Load	55 PSF



Answer HW9

18 Questions

#	Question	Correct Answer
1	Unfactored dead load on beam from slab	787.5 PLF
2	Unfactored dead load on beam from the beam (beam selfweight)	187.5 PLF
3	Unfactored live load on beam, LL	385 PLF
4	Total factored beam load, wu	1786 PLF
5	Factored design moment from the loads, Mu	118.09925 FT-K
6	Distance from top beam edge to centroid of flexural steel, d	15.5 IN
7	The final calculated area of steel required, As,req	1.83215236 IN ²
8	Number of rebars used	3
9	Actual, final area of flexural steel used, As,used	2.37 IN ²
10	Minimum required area of steel, As,min (the greater of the 2 criteria)	0.574755383 IN ²
11	Depth of concrete stress block, a	3.04171123 IN
12	The factor beta_1	0.775
13	Distance to Neutral Axis from top of beam, c	3.924788684 IN
14	Strain in flexural steel, epsilon_t	0.008847772
15	Strength reduction factor, phi	0.9
16	Tensile force in the flexural steel, T	142.2 K
17	Nominal bending moment, Mn	1987.834332 K-IN
18	Factored bending resistance, phi Mn	149.0875749 K-FT

Analysis Example - HW9

Q1-5

take 1 feet of slab width

transfer to feet

reinforced concrete density = 150lb/ft³

Span of slab	14 FT
Span of beam	23 FT
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Floor Live Load	55 PSF

$$\frac{(1 \times \text{Span of slab} \times \frac{\text{Thickness of slab}}{12}) \times \text{density}}{2}$$

← half load on beam

$$= (1 \times 14 \times \frac{9}{12}) \times 150 / 2 = 787.5 \text{ PLF} \quad \text{Q1}$$

$$(1 \times \frac{h}{12} \times \frac{b}{12}) \times \text{density} = (1 \times \frac{18}{12} \times \frac{10}{12}) \times 150 = 187.5 \text{ PLF} \quad \text{Q2}$$

$$1 \times \text{Floor Live Load} \times \frac{\text{Span of slab}}{2} = 1 \times 55 \times \frac{14}{2} = 385 \text{ PLF} \quad \text{Q3}$$

$$W_u = 1.2D + 1.6L = 1.2(787.5 + 187.5) + 1.6(385) = 1786 \text{ PLF} \quad \text{Q4}$$

$$M_u = \frac{WL^2}{8} = \frac{W_u \cdot (\text{Span of beam})^2}{8} = \frac{1786 \times (23)^2}{8} = 118099.25 \text{ lb-ft} / 1000 = 118.09925 \text{ K-ft} \quad \text{Q5}$$

$$M_u = \frac{(Y_{DL}W_{DL} + Y_{LL}W_{LL})l^2}{8}$$

Analysis Example - HW9

Q6-7

section width, b 10 IN
section height, h 18 IN

bar size number 8
stirrup bar size number 4
concrete cover 1.5 IN

1. Calculate the factored load and find factored required moment, M_u
2. Find $d = h - \text{cover} - \text{stirrup} - d_b/2$
3. Estimate moment arm $z = jd$, for beams $j \approx 0.9$ for slabs $j \approx 0.95$
4. Estimate A_s based on estimate of jd .
5. Use A_s to find a
6. Use a to find A_s (repeat...until 2% accuracy)
7. Choose bars for A_s and check A_s max & min
8. Check that $\epsilon_t \geq 0.005$
9. Check $M_u \leq \phi M_n$ (final condition)

$$M_u = \frac{(\gamma_{DL}W_{DL} + \gamma_{LL}W_{LL})l^2}{8}$$

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

Table A.2 Designations, Areas, Perimeters, and Weights of Standard Bars

Bar No.	Customary Units			SI Units		
	Diameter (in.)	Cross-sectional Area (in. ²)	Unit Weight (lb/ft)	Diameter (mm)	Cross-sectional Area (mm ²)	Unit Weight (kg/m)
3	0.375	0.11	0.376	9.52	71	0.560
4	0.500	0.20	0.668	12.70	129	0.994
5	0.625	0.31	1.043	15.88	200	1.552
6	0.750	0.44	1.502	19.05	284	2.235
7	0.875	0.60	2.044	22.22	387	3.042
8	1.000	0.79	2.670	25.40	510	3.973
9	1.128	1.00	3.400	28.65	645	5.060
10	1.270	1.27	4.303	32.26	819	6.404
11	1.410	1.56	5.313	35.81	1006	7.907
14	1.693	2.25	7.650	43.00	1452	11.384
18	2.257	4.00	13.600	57.33	2581	20.238

$$d_c = \text{cover} + \text{stirrup} + \frac{\text{Flex bar}}{2}$$

$$= 1.5 + 0.5 + \frac{1}{2} = 2.5 \text{ inch}$$

$$d = h - d_c = 18 - 2.5 = 15.5 \text{ inch} \quad \text{Q6}$$

$$A_s = \frac{M_u}{\phi \cdot f_y \cdot z} = \frac{M_u}{\phi \cdot f_y (jd)} = \frac{118099.25 \times 12}{0.9(60000)(0.9 \times 15.5)}$$

$$= 1.8813 \text{ in}^2$$

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b} = \frac{1.8813 \times 60000}{0.85 \times 5500 \times 10} = 2.4145 \text{ inch}$$

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)} = \frac{118099.25 \times 12}{0.9(60000)\left(15.5 - \frac{2.4145}{2}\right)} = 1.8362 \text{ in}^2 \quad \text{Q7}$$

check the accuracy

$$\frac{1.8813 - 1.8362}{1.8813} = 2.397\% \approx 2\% \rightarrow \text{✓}$$

Analysis Example - HW9

Q8-9

Table A.2 Designations, Areas, Perimeters, and Weights of Standard Bars

Bar No.	Customary Units			SI Units		
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round up to whole number

$$n = \frac{A_s}{\text{Cross-sectional Area}} = \frac{1.8362}{0.79} = 2.324 \rightarrow 3 = n \quad \text{Q8}$$

$$A_s = n \times (\text{Cross-sectional Area}) = 3 \times 0.79 = 2.37 \text{ in}^2 \quad \text{Q9}$$

Analysis Example - HW9

Q10-18

$$c = \frac{a}{\beta_1}$$

As,min:
greater of a and b

(a) $\frac{3\sqrt{f'_c}}{f_y} b_w d$
 (b) $\frac{200}{f_y} b_w d$

Beta1:

$$0.85 \geq 0.85 - 0.05 \left(\frac{f'_c - 4000}{1000} \right) \geq 0.65$$

$$\epsilon_t = \frac{d - c}{c} 0.003 \geq 0.005$$

Check that $\epsilon_t \geq 0.005$
 (for tension controlled section)
 $\phi = 0.9$

$$\frac{3\sqrt{f'_c}}{f_y} bd = \frac{3\sqrt{5500}}{60000} (10)(15.5) = 0.5748 \quad \checkmark \quad \text{Q10}$$

$$\frac{200}{f_y} bd = \frac{200}{60000} (10)(15.5) = 0.5167$$

Q9 As,used

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b} = \frac{2.37 \times 60000}{0.85(5500)(10)} = 3.0417 \text{ inch} \quad \text{Q11}$$

$$\beta_1 = 0.85 - 0.05 \left(\frac{f'_c - 4000}{1000} \right) = 0.85 - 0.05 \left(\frac{5500 - 4000}{1000} \right) = 0.775 \leq 0.85 \quad \text{Q12}$$

$$c = \frac{a}{\beta_1} = \frac{3.0417}{0.775} = 3.9248 \quad \text{Q13}$$

Q14

$$\epsilon_t = \frac{d - c}{c} (0.003) = \frac{15.5 - 3.9248}{3.9248} (0.003) = 0.00885 > 0.005$$

Tension is controlled

transfer to k $\rightarrow \phi = 0.9$ Q15

$$T = A_s \cdot f_y = 2.37 \times 60000 / 1000 = 142.2 \text{ K} \quad \text{Q16}$$

$$M_n = A_s \cdot f_y \left(d - \frac{a}{2} \right) = (2.37)(60000) \left(15.5 - \frac{3.0417}{2} \right) = 1989754.83 \text{ lb-IN} \quad \text{Q17} = 1989.75 \text{ K-IN}$$

$$\phi M_n = 0.9 \times 1989.75 / 12 = 149.23 \text{ K-FT} \quad \text{Q18}$$

transfer to inch feet