Architecture 324: Structures II

Three Moment Theorem — Guided Worksheet

Determine Support Moments and Reactions for a Continuous Beam

Given Data:

Span A (L_1): 19 FT	**Span B (L_2):** 28 FT	**Span C (L_3):** 10 FT
Load w_1 (Span A): 4 KLF	**Load P (Span B):** 66 K	**Dist. P from R2 (D):** 14 FT
Load w_2 (Span C): 5 KLF		

Part 1: Simple End Moments & Load Terms ($EI\theta$)

Q1 Moment at support R1, M1 (- if tension on top)

Since R1 is a simple exterior support (no fixity/overhang), its moment is known.

1. **Final M_1:** _____ K-FT

Q2 EI Theta on left side of R2

Rotation at right end of Span A due to uniform load.

1. $EI\theta_{2L} = \frac{w_1 L_1^3}{24} = \frac{\_\_\_(\_\_\_)^3}{24} =$ _____

Q3 EI Theta on right side of R2

Rotation at left end of Span B due to point load at distance D.

1. $EI\theta_{2R} = \frac{PL_2^2}{16} = \frac{\_\_\_(\_\_\_)^2}{16} =$ _____

Q4 Moment at support R4, M4 (- if tension on top)

Since R4 is a simple exterior support, its moment is known.

1. **Final M_4:** _____ K-FT

Q5 EI Theta on left side of R3

Rotation at right end of Span B due to centered point load.

1. $EI\theta_{3L} = \frac{PL_2^2}{16} = \frac{\_\_\_(\_\_\_)^2}{16} =$ _____

Q6 EI Theta on right side of R3

Rotation at left end of Span C due to uniform load.

1. $EI\theta_{3R} = \frac{w_2 L_3^3}{24} = \frac{\_\_\_(\_\_\_)^3}{24} =$ _____

Part 2: Internal Support Moments

Intermediate Step: Set up 3-Moment Equations

Eq 1 (Spans A & B):

$M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 = -6(EI\theta_{2L} + EI\theta_{2R})$

Eq 2 (Spans B & C):

$M_2L_2 + 2M_3(L_2 + L_3) + M_4L_3 = -6(EI\theta_{3L} + EI\theta_{3R})$

Solve the two equations simultaneously for M_2 and M_3:

Q7 Moment at support R2, M2 (- if tension on top)

1. **Final M_2:** _____ K-FT

Q8 Moment at support R3, M3 (- if tension on top)

1. **Final M_3:** _____ K-FT

Part 3: Support Reactions

Superimpose the simple beam reactions with the moment adjustment ($\Delta M/L$).

Q9 Support reaction, R1 (- if downward)

1. *(Simple reaction Span A) - (Moment adjustment)*
   $\frac{w_1 L_1}{2} - \frac{M_2}{L_1} =$ _____ K

Q10 Support reaction, R2 (- if downward)

1. *(Right Span A sum) + (Left Span B sum)*
   $\left(\frac{w_1 L_1}{2} + \frac{M_2}{L_1}\right) + \left(\frac{P(L_2 - D)}{L_2} + \frac{M_2 - M_3}{L_2}\right) =$ _____ K

Q11 Support reaction, R3 (- if downward)

1. *(Right Span B sum) + (Left Span C sum)*
   $\left(\frac{PD}{L_2} - \frac{M_2 - M_3}{L_2}\right) + \left(\frac{w_2 L_3}{2} + \frac{M_3}{L_3}\right) =$ _____ K

Q12 Support reaction, R4 (- if downward)

1. *(Simple reaction Span C) - (Moment adjustment)*
   $\frac{w_2 L_3}{2} - \frac{M_3}{L_3} =$ _____ K

Continuous Beam Diagram

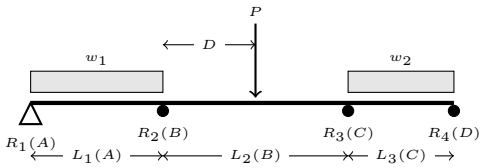

Figure 1: 3-Span continuous beam showing uniform and point loads.

Three-Moment Theorem Procedure

1. Draw a free body diagram of the first two spans.
2. Label the spans L_1 and L_2 and the supports (or free end) A, B, and C as shown.
3. Use the Three-Moment equation to solve for each unknown moment, either as a value or as an equation.
4. Move one span further and repeat the procedure.
5. In a 3-span beam, the mid-moment from step 3 above (B) can now be solved using the two equations from step 4 and 3 together, by writing 2 equations with 2 unknowns.
6. Repeat as needed, always moving one span to the right and writing a new set of moment equations.
7. Solve 2 simultaneous equations for 3 spans (or 3 equations for more than 3 spans) to get the interior moments.
8. Once all interior moments are known, solve for reactions using free body diagrams of individual spans.
9. Draw shear and moment diagrams as usual. This will also serve as a check for the moment values.

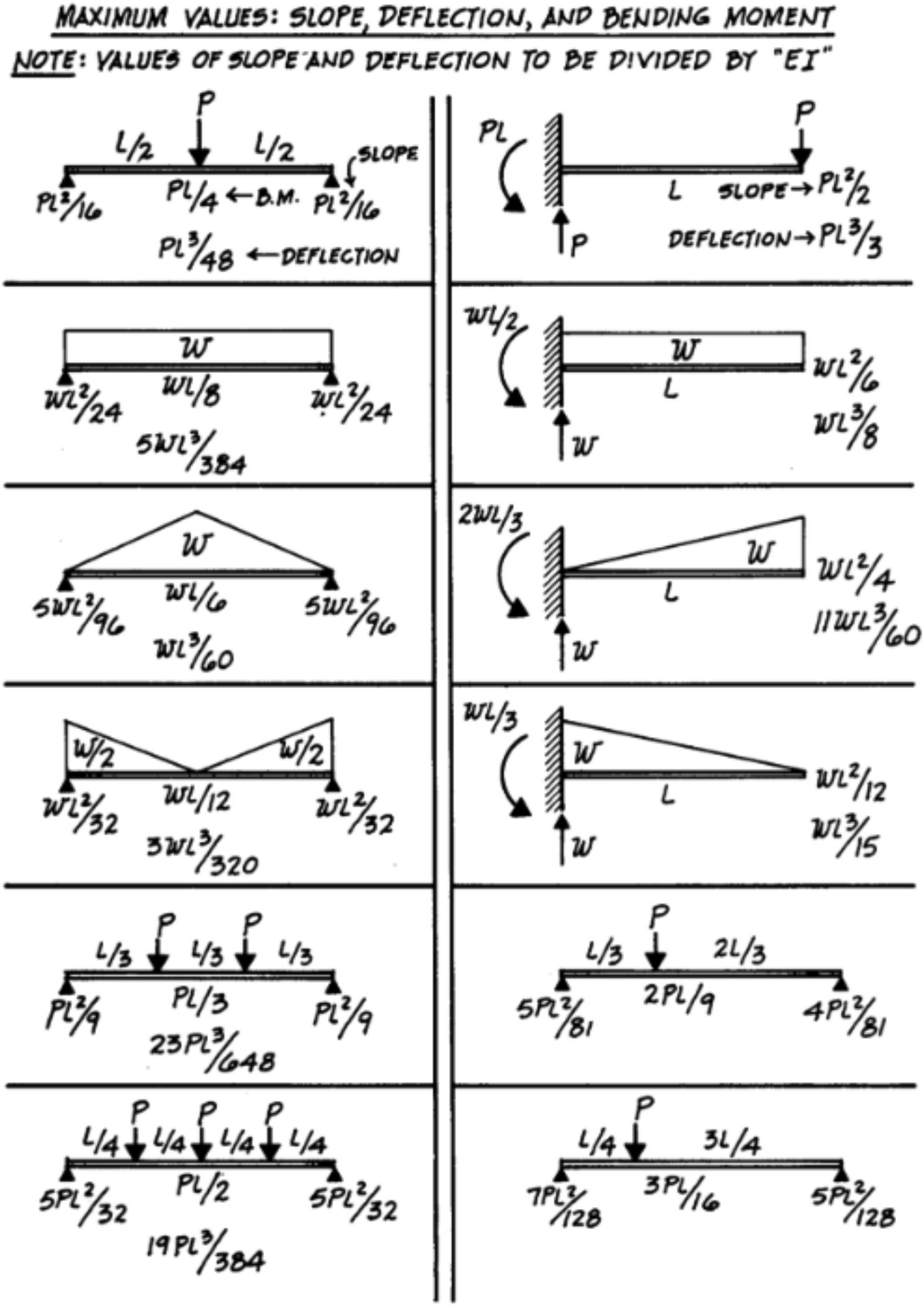

Figure 2: Caption