# Arch 324
# Structures II

Winter 2026 Recitation 004

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# Recitation 004

Welcome to session 7!

- Quick Lecture Recap
- Homework #7 Three Moment Theorem
- Lab: Continuous Beams
- Tower Testing in a little over a week!! :0

*Feel free to ask questions anytime*

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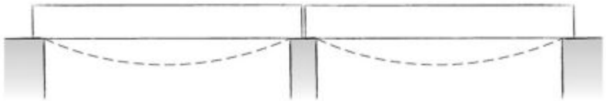
# Lecture: Continuous Beams (2/18)

## Continuous Beams

- Continuous over one or more supports
  - Most common in monolithic concrete
  - Steel: continuous or with moment connections
  - Wood: as continuous beams, e.g. long Glulam spans
- Statically indeterminate
  - Cannot be solved by the three equations of statics alone
  - Internal forces (shear & moment) as well as reactions are affected by movement or settlement of the supports



two spans - simply supported

two spans - continuous

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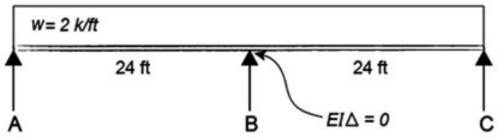
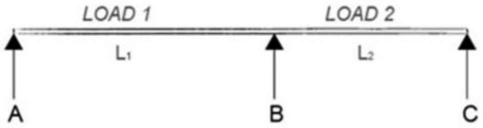
# Lecture: Continuous Beams (2/18)

## Continuous Beam Analysis

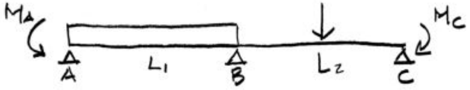
Deflection Method
- Two continuous, symmetric spans
- Symmetric Load




Slope Method
- Two continuous spans
- Non-symmetric loads and spans




3-Moment Theorem
- Any number of continuous spans
- Non-Symmetric Load and Spans




$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

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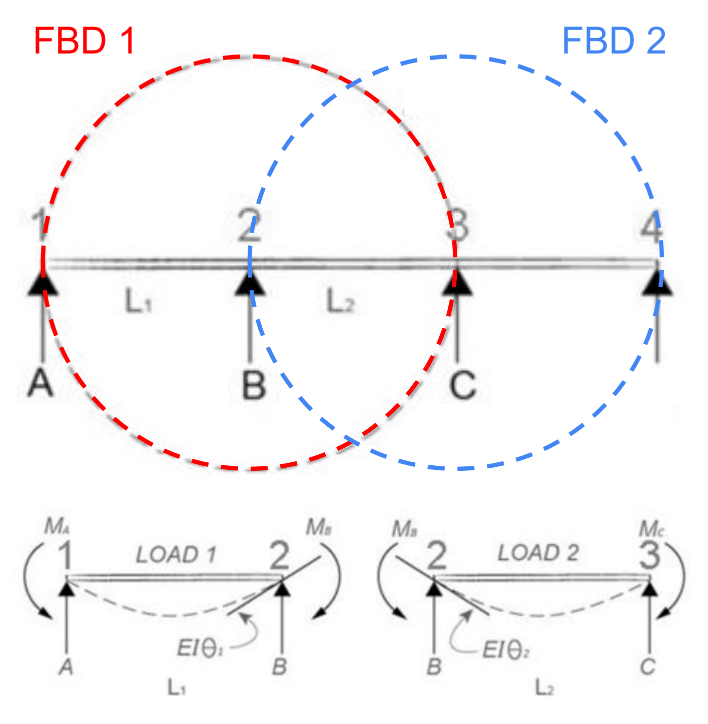
# Lecture: Three-Moment Theorem (2/23)

**Three-Moment Theorem**

- Any number of spans
- Symmetric or non-symmetric

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

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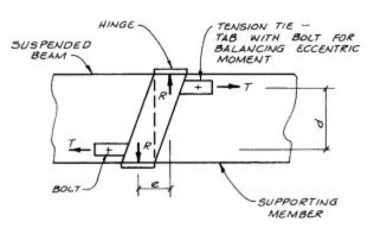
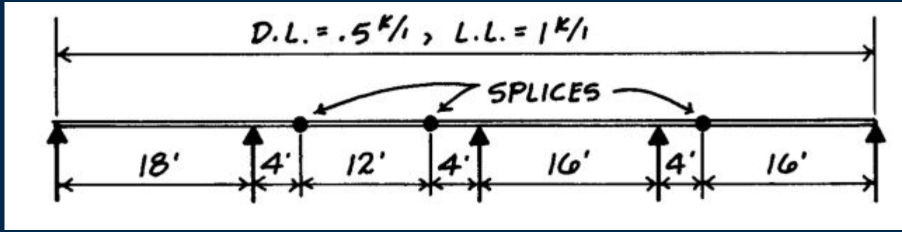
# Lecture: Gerber Beams (3/09)

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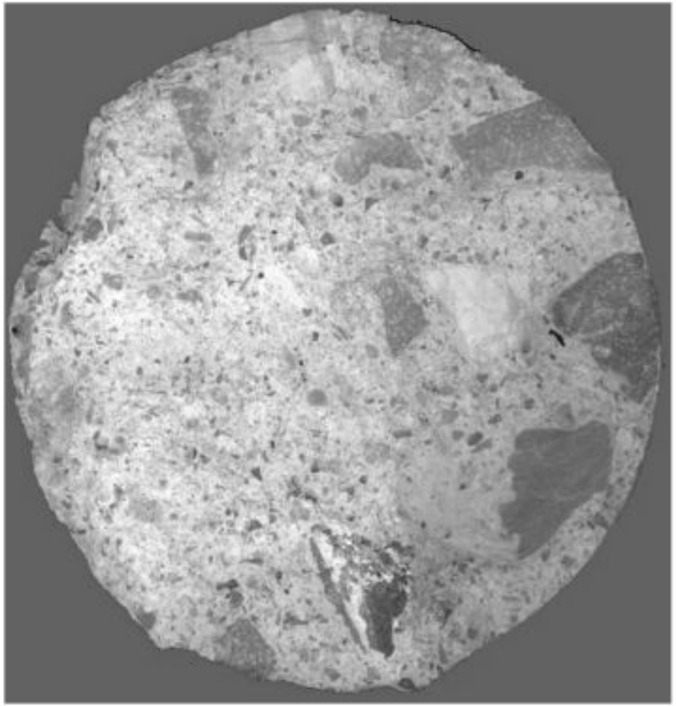
# Lecture: Intro to Concrete (3/11)

## Reinforced Concrete

- Material Properties
  - Aggregate
  - Cement
  - Water
  - Reinforcement
- Strength
  - Compression, f'c
  - Tension, f't
- PCA – Concrete Fundamentals

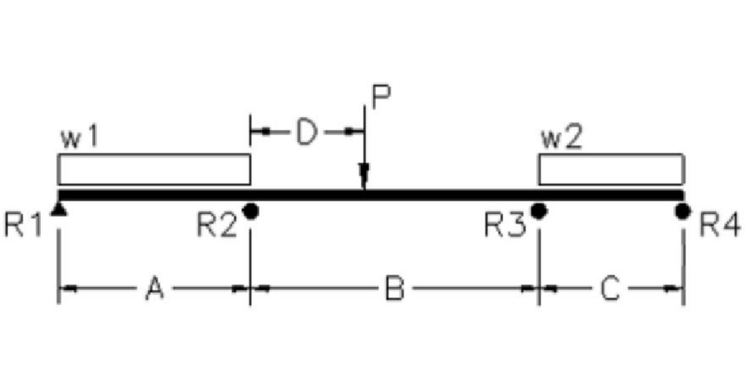


# HW #7: Three Moment Theorem

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

DATASET: 1 -2- -3-

| | |
|---|---|
| Span A | 21 FT |
| Span B | 16 FT |
| Span C | 20 FT |
| Uniform load on span A, w1 | 4 KLF |
| Uniform load on span C, w2 | 4 KLF |
| Point load on span b, P | 62 K |
| Distance to point load P from R2, D | 4 FT |

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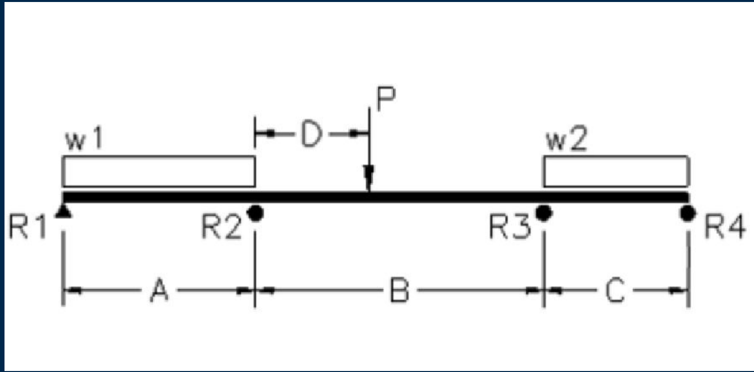
# HW #7 Three Moment Theorem

M1 = 0




M4 = 0

Q1 + 4 Simply supported beam - moment at end supports = 0K-FT

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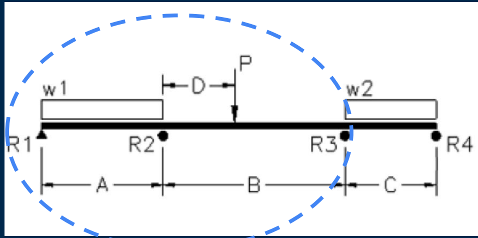
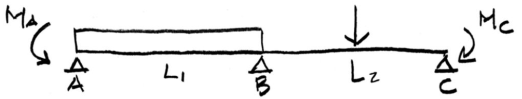
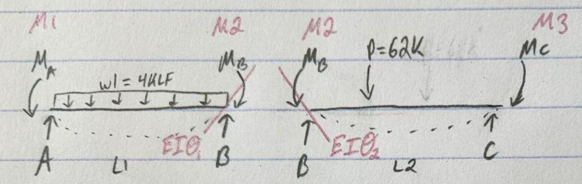
# HW #7: Three Moment Theorem




$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$




$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\theta_1 + EI\theta_2]$$

$M_A = 0$ $\quad L_1 = 21'$ (= Span A) $\quad EI\theta_1 = \frac{w\ell^2}{24} = \frac{(4\times21)(21^2)}{24} = \boxed{1{,}543.5}$ Q#2

$M_B = M_2$ $\quad L_2 = 16'$ (= Span B)

$M_C = M_3$ $\quad L_1 + L_2 = 37'$ $\quad EI\theta_2 = \frac{7P\ell^2}{128} = \frac{7(62)(16^2)}{128} = \boxed{868}$ Q#3

*Reference lecture table for proper EIθ formulas*

See next slide for lecture table*

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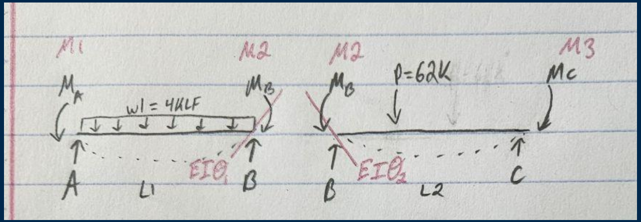
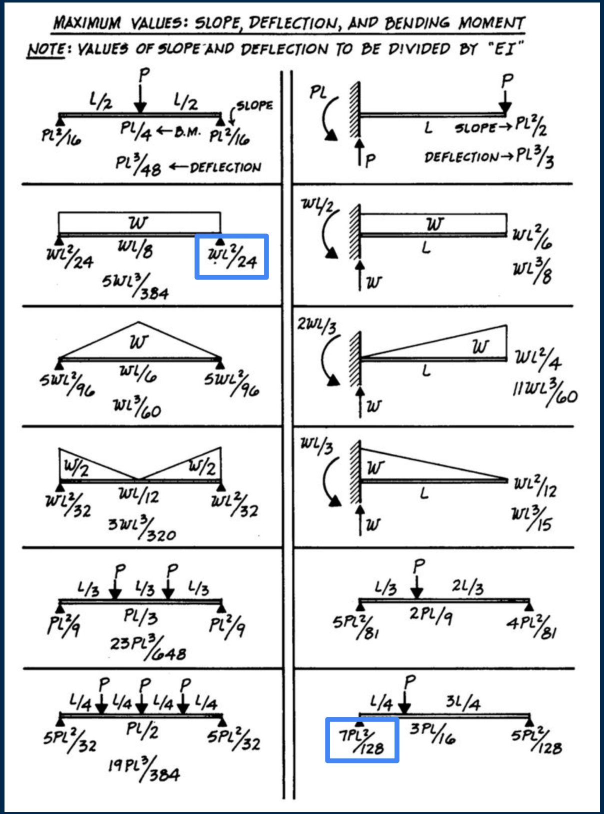
# HW #7: Three Moment Theorem

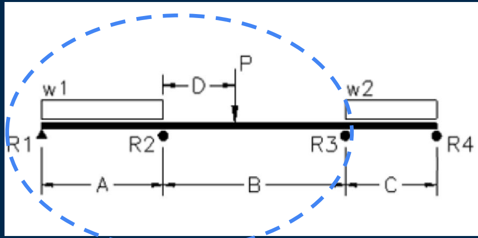


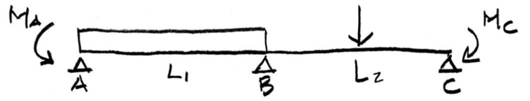

¼ L = 4’ ¾ L = 12’

L = 16’

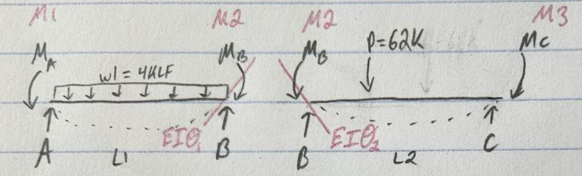


# HW #7: Three Moment Theorem




$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$




$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\theta_1 + EI\theta_2]$$

$M_A = 0$ $\quad L_1 = 21'$ (= Span A) $\quad EI\theta_1 = \frac{wl^2}{24} = \frac{(4\times21)(21^2)}{24} = \boxed{1{,}543.5}$ Q#2

$M_B = M_2$ $\quad L_2 = 16'$ (= Span B)

$M_C = M_3$ $\quad L_1 + L_2 = 37'$ $\quad EI\theta_2 = \frac{7Pl^2}{128} = \frac{7(62)(16^2)}{128} = \boxed{868}$ Q#3

*Reference lecture table for proper EIθ formulas*

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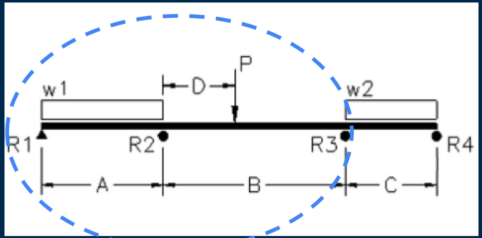
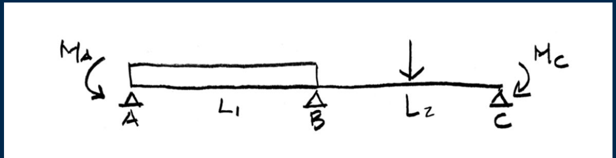
# HW #7: Three Moment Theorem




$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

$M_A = 0$ $L_1$ (= Span A) $= 21'$ $EI\theta_1 = \frac{wl^2}{24} = \frac{(4\times21)(21^2)}{24} = \boxed{1,543.5}$ Q#2

$M_B = M_2$ $L_2$ (= Span B) $= 16'$

$M_C = M_3$ $L_1 + L_2 = 37'$ $EI\theta_2 = \frac{7Pl^2}{128} = \frac{7(62)(16^2)}{128} = \boxed{868}$ Q#3

*Reference lecture table for proper EIθ formulas*

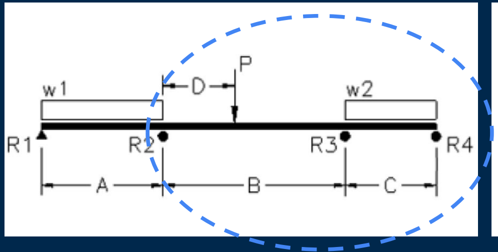
$$M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 = 6[EI\theta_1 + EI\theta_2]$$

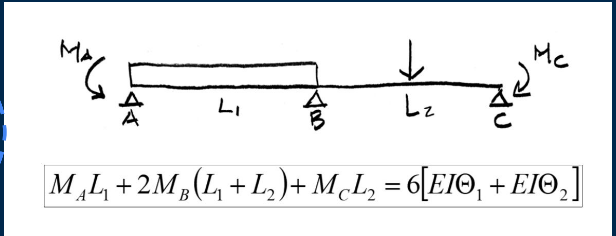
$$0 + 2M_2(37') + M_3(16') = 6(1,543.5 + 868)$$

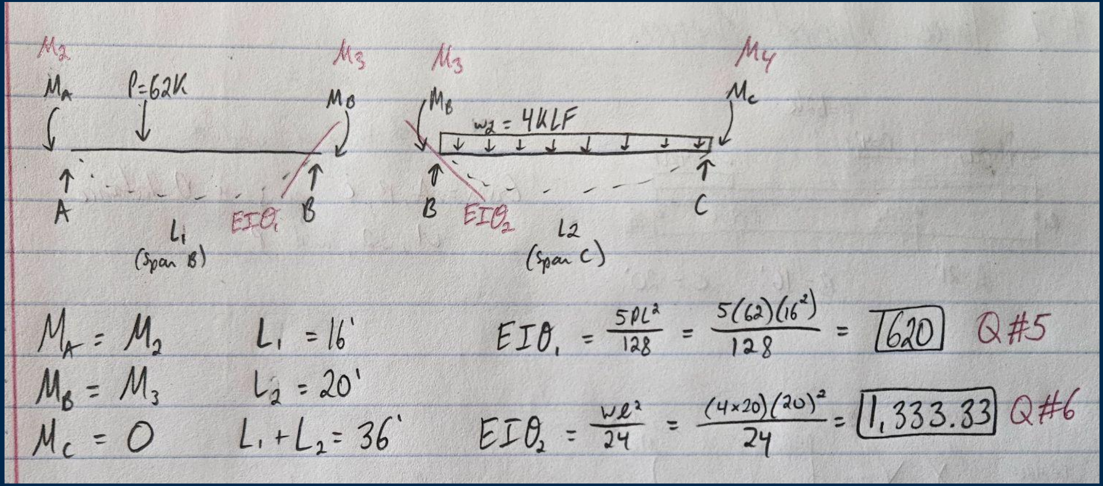
$$74M_2 + 16M_3 = 14,469$$

$$M_2 = \frac{14,469 - 16M_3}{74}$$ *Save for later*

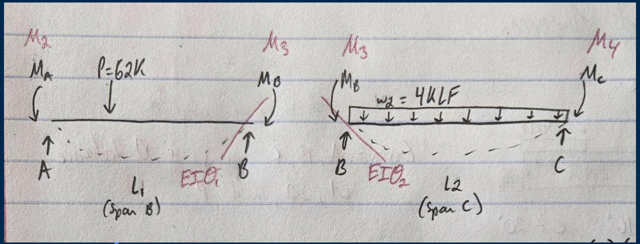
# HW #7: Three Moment Theorem



# HW #7: Three Moment Theorem

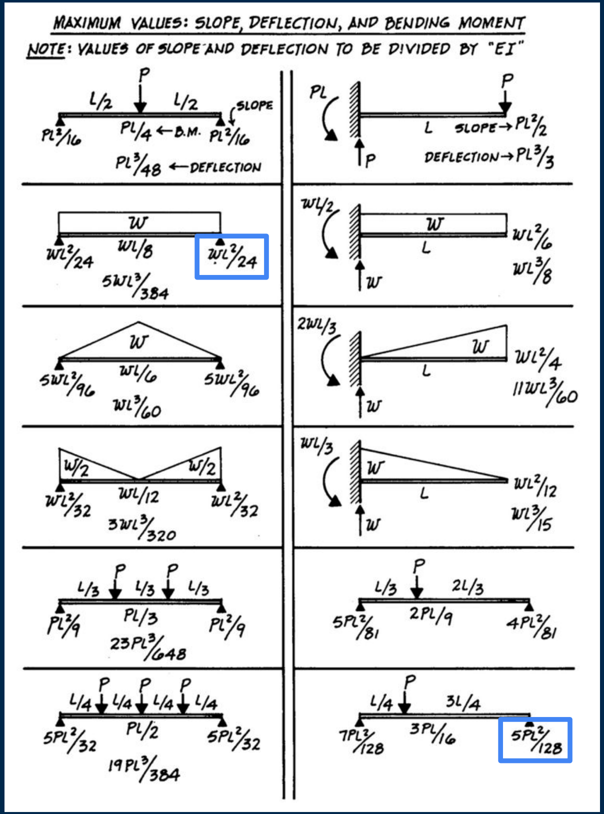



¼ L = 4'    ¾ L = 12'

L = 16'

$$EI\theta_1 = \frac{5PL^2}{128} = \frac{5(62)(16^2)}{128} = \boxed{620} \quad \text{Q\#5}$$

$$EI\theta_2 = \frac{wl^2}{24} = \frac{(4\times 20)(20)^2}{24} = \boxed{1{,}333.33} \quad \text{Q\#6}$$



# HW #7: Three Moment Theorem

$$M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 = 6\left[EI\theta_1 + EI\theta_2\right]$$

$$M_2(16') + 2M_3(36') + 0 = 6(620 + 1,333.33)$$

$$16M_2 + 72M_3 = 11,719.98$$

*plug in $M_2$ from earlier*

$$16\left(\frac{14,469 - 16M_3}{74}\right) + 72M_3 = 11,719.98$$

$$\frac{14,469 - 16M_3}{74} + 4.5M_3 = 732.49875$$

$$14,469 - 16M_3 + 333M_3 = 54,204.9075$$

$$317M_3 = 39,735.9075$$

$$M_3 = 125.35$$

$$16M_2 + 72M_3 = 11,719.98$$

$$16M_2 + 72(125.35) = 11,719.98$$

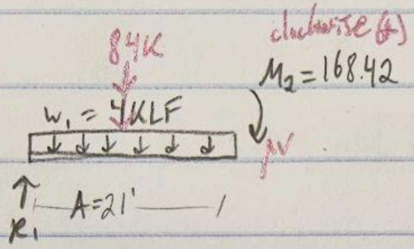
$$16M_2 = 2,694.78$$

$$M_2 = 168.42$$

*$M_2$ and $M_3$ were assumed to cause tension on top so add (-) sign to both!*

| | |
|---|---|
| $M_3 = -125.35$ | $M_2 = -168.42$ |
| Q#8 | Q#7 |

# HW #7: Three Moment Theorem




conc. load $w_1$

$$\Sigma M_{@R_2} = R_1(A) - 84\left(\tfrac{A}{2}\right) + M_2$$

$$= 21R_1 - 84\left(\tfrac{21}{2}\right) + 168.42$$

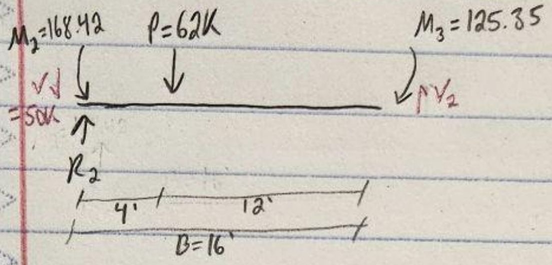
$$\boxed{R_1 = 33.98K}$$ Q #9

$$\Sigma F_y = R_1 - 84 + V$$

$$= 33.98 - 84 + V$$

$$\underline{V = 50K}$$

# HW #7: Three Moment Theorem




$$\Sigma M_{@R_3} = B(R_2) - B(V) - 12(P) + M_3 - M_2$$

$$= 16R_2 - 16(50) - 12(62) + 125.35 - 168.42$$

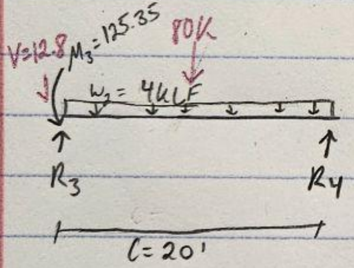
$$\boxed{R_2 = 99.2K}$$ Q#10

$$\Sigma F_y = R_2 - V - P + V_2$$

$$= 99.2 - 50 - 62 + V_2$$

$$V_2 = 12.8$$

# HW #7: Three Moment Theorem




$$\Sigma M_{@R_4} = -M_3 + R_3(C) - P\left(\frac{C}{2}\right) - V(C)$$

$$= -125.35 + 20R_3 - 80\left(\frac{20}{2}\right) - 12.8(20')$$

$$\boxed{R_3 = 59.1K}$$ Q#11

$$\Sigma F_y = R_3 - V - P + R_4$$

$$= 59.1 - 12.8 - 80 + R_4$$

$$\boxed{R_4 = 33.7K}$$ Q#12

# HW #7: Three Moment Theorem

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

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|---|---|
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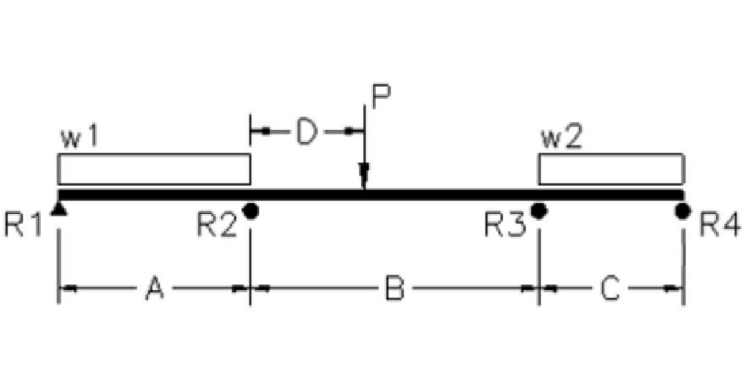
| Point load on span b, P | 62 K |
| Distance to point load P from R2, D | 4 FT |

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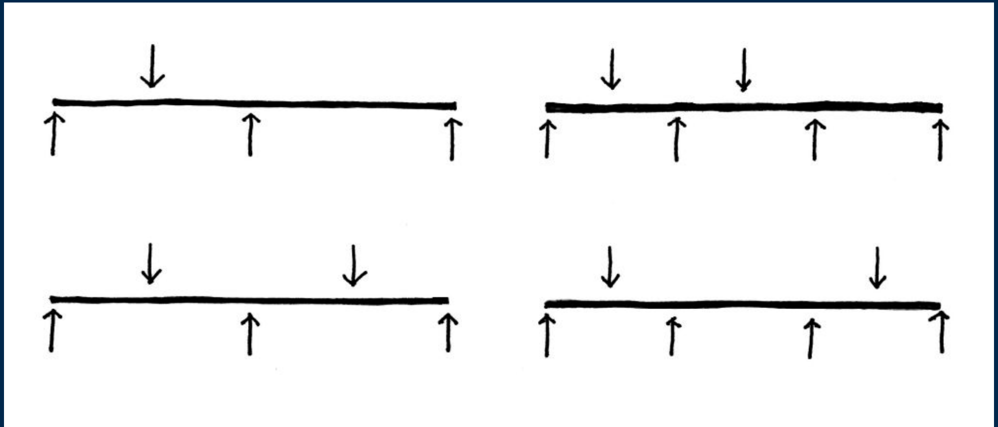

# Lab: Continuous Beams

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