TAUBMAN COLLEGE
ARCHITECTURE & URBAN PLANNING
UNIVERSITY OF MICHIGAN

Arch324
STRUCTURES II

Winter 2026
Recitation

FACULTY: Prof. Peter von Bülow
Mohsen Vatandoost

Welcome to the Recitation session 03/13

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Office: Room 3128

hours:

Fri: 11:30 – 12:30

Mon, Wed: 11:00 - 12:00

walk-ins welcome!

Click here to make an appointment

Please feel free to ask questions.

Welcome to the Recitation session 03/13

Outline:

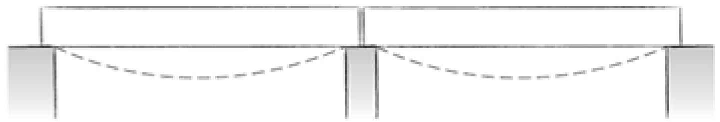
- Quick **Recap** of the week
- Provide the solution for the assignment **(Homework 7)**
- Answering students' questions
- Lab: **Continuous Beams**
- **Tower Project:** Test date: **March 23**

Please feel free to ask questions.

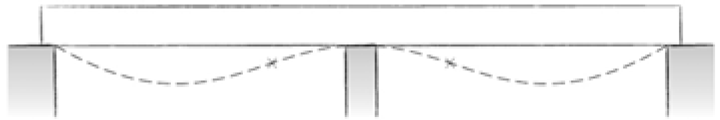
Continuous beams

Methods for solving internal forces in Continuous beams:

- Deflection Method
- Slope Method
- Three-Moment Theorem



two spans - simply supported



two spans - continuous

Statically indeterminate:

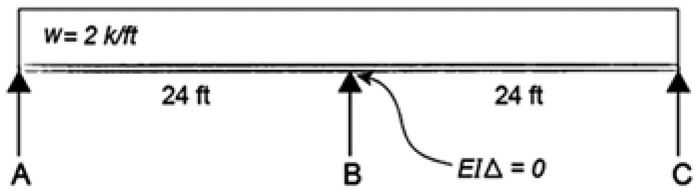
- Cannot be solved by the three equations of statics alone
- Internal forces (shear & moment) as well as reactions are affected by movement or settlement of the supports

Recap of the week

Continuous beams

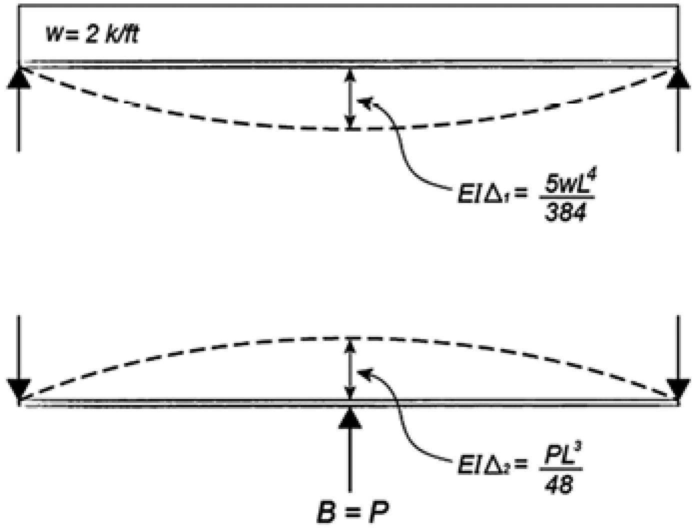
Deflection Method

- Two continuous, symmetric spans
- Symmetric Load



Procedure:

1. Remove the center support, and calculate the center deflection for each load case as a simple span.
2. Remove the applied loads and replace the center support. Set the deflection equation for this case (center point load) equal to the deflection from step 1.
3. Solve the resulting equation for the center reaction force. (upward point load)
4. Calculate the remaining two end reactions.
5. Draw shear and moment diagrams as usual.



$$EI\Delta_1 + EI\Delta_2 = 0$$

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Recap of the week

Continuous beams

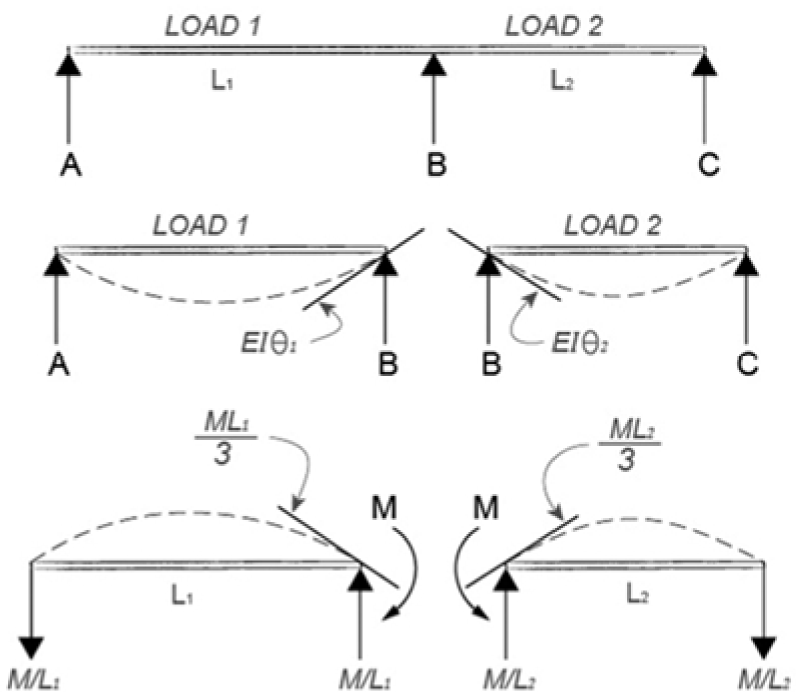
Slope Method

- Two continuous spans
- Non-symmetric loads and spans

Procedure:

1. Break the beam into two halves at the interior support, and calculate the interior slopes of the two simple spans.
2. Use the Slope Equation to solve for the negative interior moment.
3. Find the reactions of each of the simple spans plus the M/L reactions caused by the interior moment.
4. Add all the reactions by superposition.
5. Draw the shear and moment diagrams as usual.

$$M = \frac{3}{L_1 + L_2}\left[EI\Theta_1 + EI\Theta_2\right]$$



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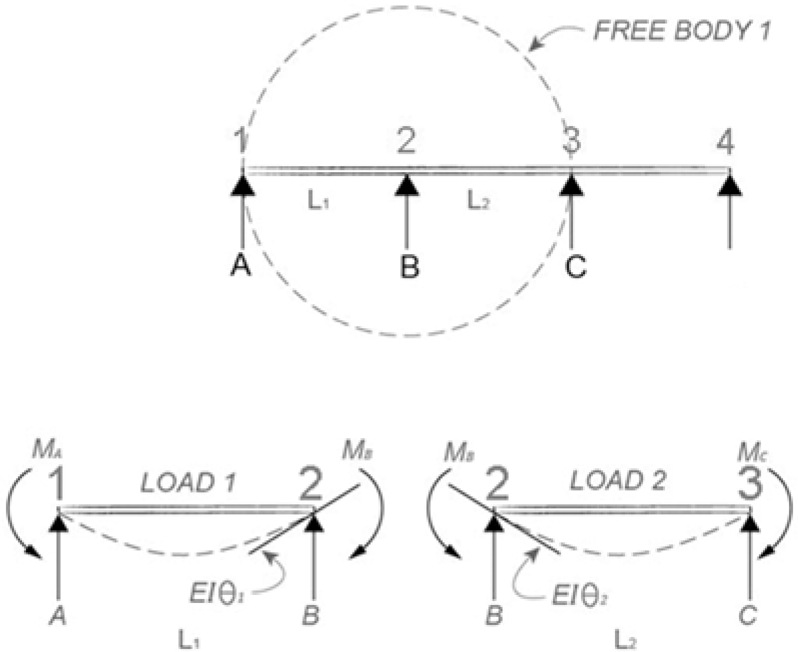
Recap of the week

Continuous beams

Three-Moment Theorem

- Any number of spans
- Symmetric or non-symmetric



$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

Recap of the week

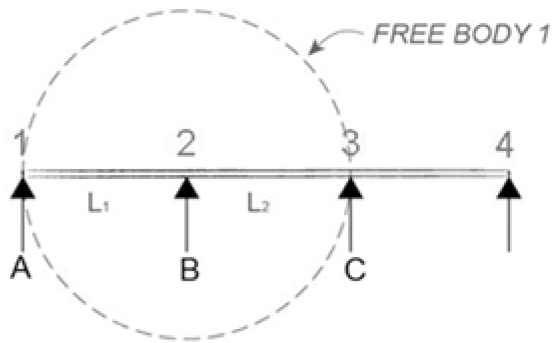
Continuous beams

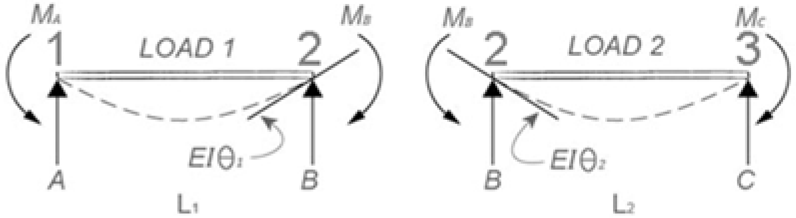
Three-Moment Theorem

- Any number of spans
- Symmetric or non-symmetric

Procedure:

1. Draw a free body diagram of the first two spans.
2. Label the spans L1 and L2 and the supports (or free end) A, B and C as show.
3. Use the Three-Moment equation to solve for each unknown moment, either as a value or as an equation.





$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

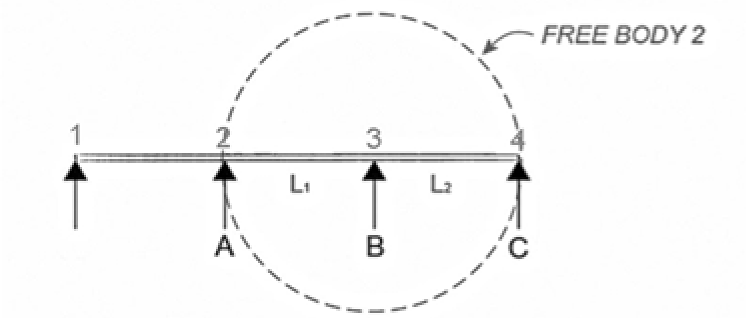
Recap of the week

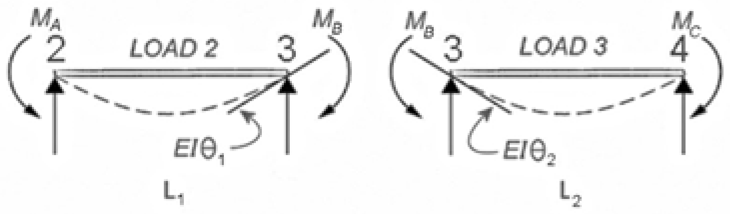
Continuous beams

Three-Moment Theorem

Procedure (continued):

4. Move one span further and repeat the procedure.
5. In a 3 span beam, the mid-moment from step 3 above (B), can now be solved using the two equations from step 4 and 3 together, by writing 2 equations with 2 unknowns.
6. Repeat as needed, always moving one span to the right and writing a new set of moment equations.
7. Solve 2 simultaneous equations for 3 spans, or 3 equations for more than 3 spans, to get the interior moments.
8. Once all interior moments are known, solve for reactions using free body diagrams of individual spans.
9. Draw shear and moment diagrams as usual. This will also serve as a check for the moment values.





$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

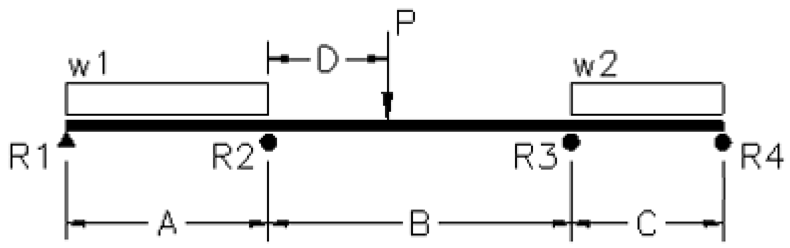
Provide the solution for the assignment – **HW7**

7. Three Moment Theorem

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

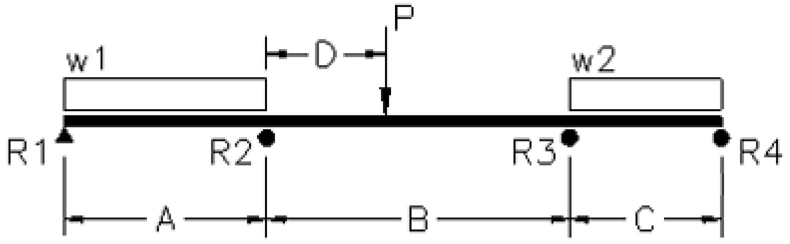
DATASET: 1 -2- -3-

Span A	28 FT
Span B	21 FT
Span C	24 FT
Uniform load on span A, w1	4 KLF
Uniform load on span C, w2	5 KLF
Point load on span b, P	60 K
Distance to point load P from R2, D	14 FT

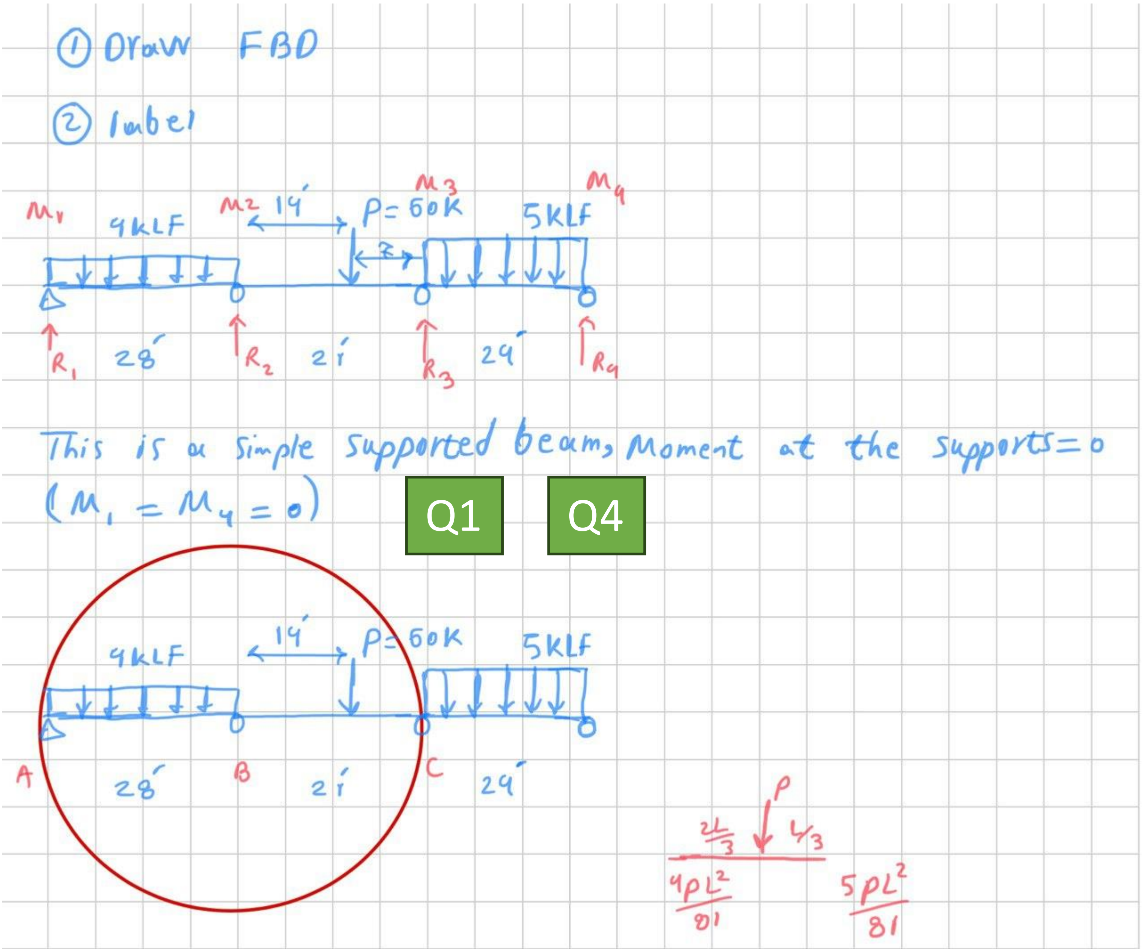


Provide the solution for the assignment – **HW7**

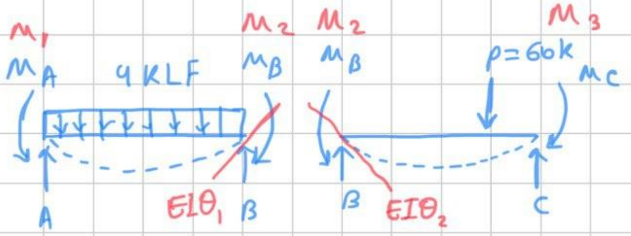
#	Question	Your Response	
1	Moment at support R1, M1 (- if tension on top)		K-FT
2	EI Theta on left side of R2		
3	EI Theta on right side of R2		
4	Moment at support R4, M4 (- if tension on top)		K-FT
5	EI Theta on left side of R3		
6	EI Theta on right side of R3		
7	Moment at support R2, M2 (- if tension on top)		K-FT
8	Moment at support R3, M3 (- if tension on top)		K-FT
9	Support reaction, R1 (- if downward)		K
10	Support reaction, R2 (- if downward)		K
11	Support reaction, R3 (- if downward)		K
12	Support reaction, R4 (- if downward)		K



Provide the solution for the assignment – **HW7**



Provide the solution for the assignment – **HW7**



$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6\left[EI\theta_1 + EI\theta_2\right]$$

$M_A = 0 \qquad L_1 = 28 \qquad EI\theta_1 = \dfrac{wl^2}{24} = \dfrac{(4\times 28)(28)^2}{24} = 3658.66$ **Q2**

$M_B = M_2 \qquad L_2 = 21$

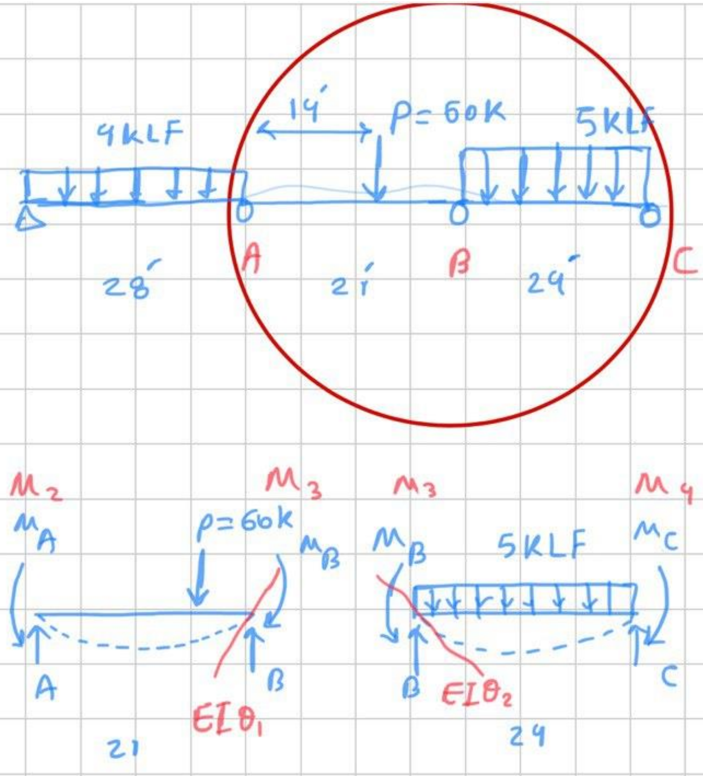
$M_C = M_3 \qquad L_1 + L_2 = 49 \qquad EI\theta_2 = \dfrac{4Pl^2}{81} = \dfrac{4(60)(21)^2}{81} = 1306.67$ **Q3**

$$0 + 2M_2(28+21) + M_3(21) = 6\left[3658.66 + 1306.67\right]$$

$$2M_2(49) + M_3(21) = 29791.98 \quad (1)$$

$$M_2 = \frac{29791.98 - 21M_3}{98}$$

Provide the solution for the assignment – **HW7**



$M_A = M_2$ $\qquad L_1 = 21$

$M_B = M_3$ $\qquad L_2 = 24$

$M_C = M_4 = 0$ $\qquad L_1 + L_2 = 45$

$$EI\theta_1 = \frac{5Pl^2}{81} = \frac{5(60)(21)^2}{81} = 1633.34$$ **Q5**

$$EI\theta_2 = \frac{wl^2}{24} = \frac{(5\times24)(24)^2}{24} = 2880$$ **Q6**

Provide the solution for the assignment – **HW7**

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6\,[EI\theta_1 + EI\theta_2]$$

$$M_2(21) + 2M_3(45) + \cdot = 6\,[1633.34 + 2880]$$

$$M_2(21) + 2M_3(45) = 27080.04 \quad (2)$$

Replace M_2 From (1)

$$\rightarrow \left(\frac{29791.98 - 21M_3}{98}\right)(21) + 2M_3(45) = 27080.04$$

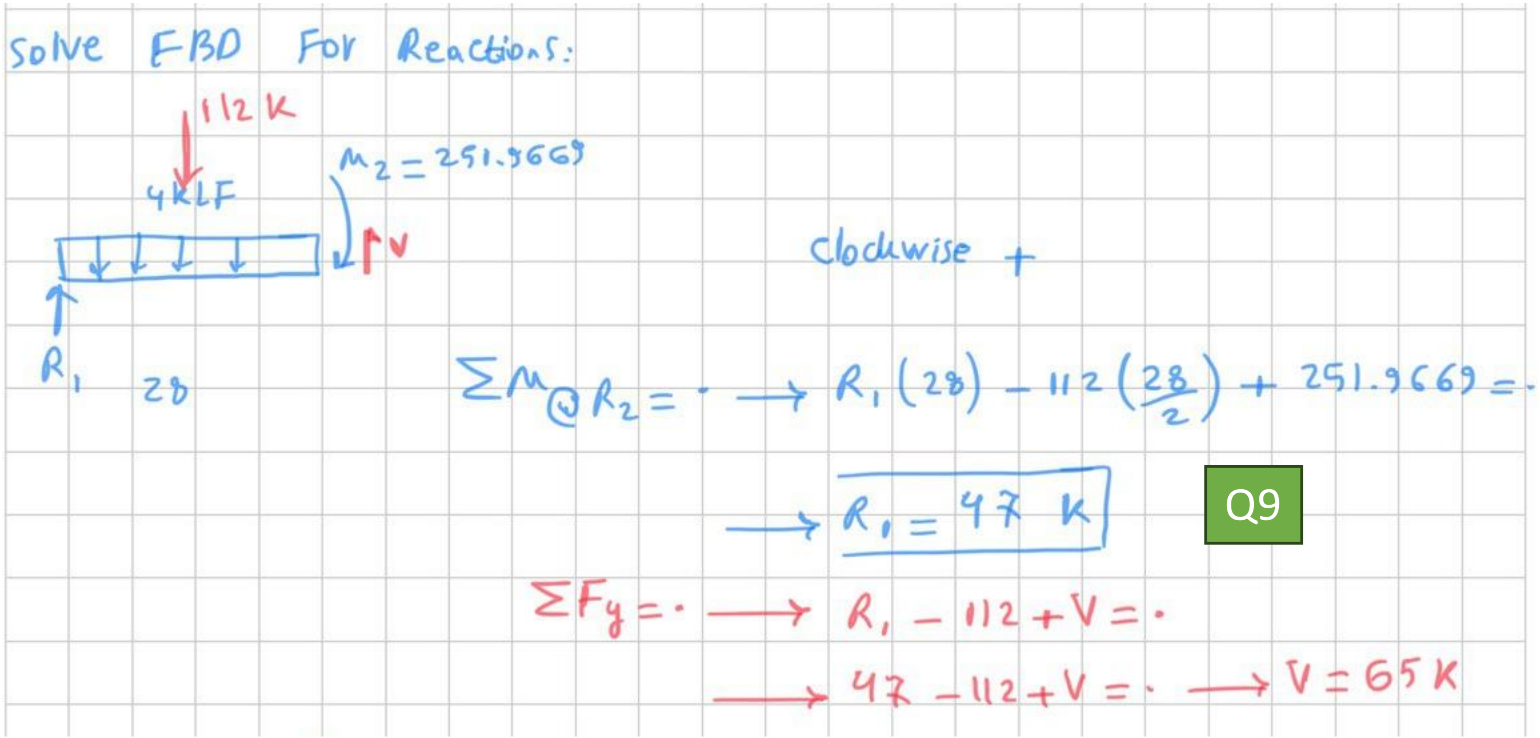
$$\rightarrow \begin{cases} M_3 = 242.82 \\ M_2 = 251.9669 \end{cases}$$

Q8

Q7

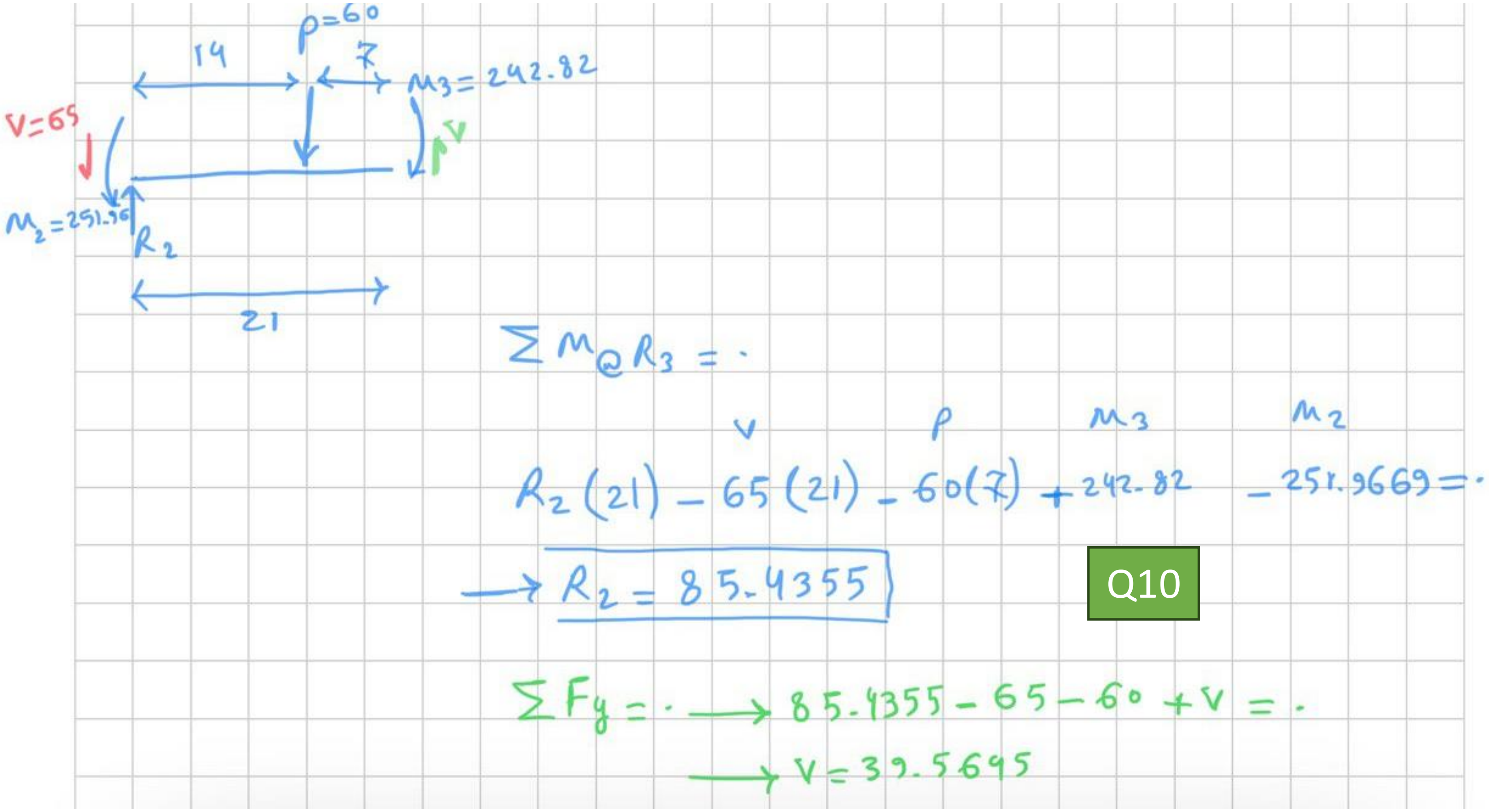
Remember M_2, M_3 → were assumed to cause tension on top, so don't forget to use (−) sign!

Provide the solution for the assignment – **HW7**



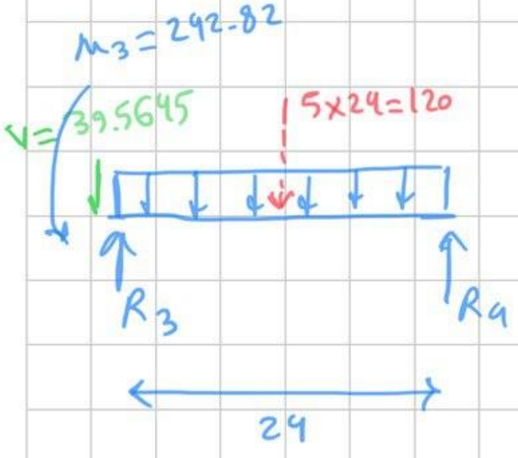
Q9

Provide the solution for the assignment – **HW7**



Q10

Provide the solution for the assignment – **HW7**



$$\sum M_{@R_4} = 0$$

$$-242.82 + R_3(24) - 120\left(\frac{24}{2}\right) - 39.5645(24) = 0$$

$$\rightarrow R_3 = 109.682$$

Q11

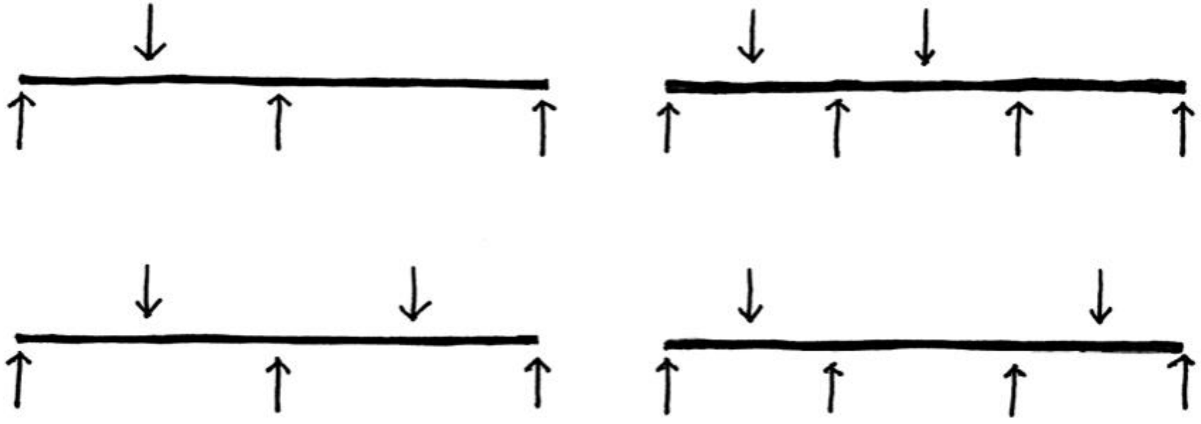
$$\sum F_y = 0 \rightarrow$$

$$109.682 - 39.5645 - 120 + R_4 = 0$$

$$\rightarrow R_4 = 49.8825$$

Q12

Lab : Continuous Beams



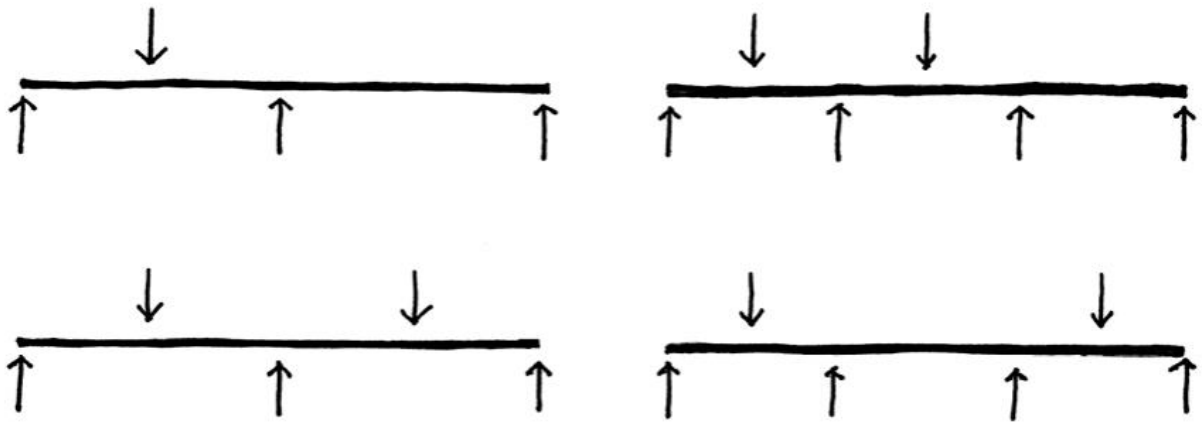
Description

This project uses observation to understand the behavior of beams continuous over multiple supports.

Goals

To observe the behavior of continuous beams under different loadings
To estimate locations of contraflexure and effective lengths
To determine areas of positive and negative moment based on curvature

Lab : Continuous Beams



Procedure

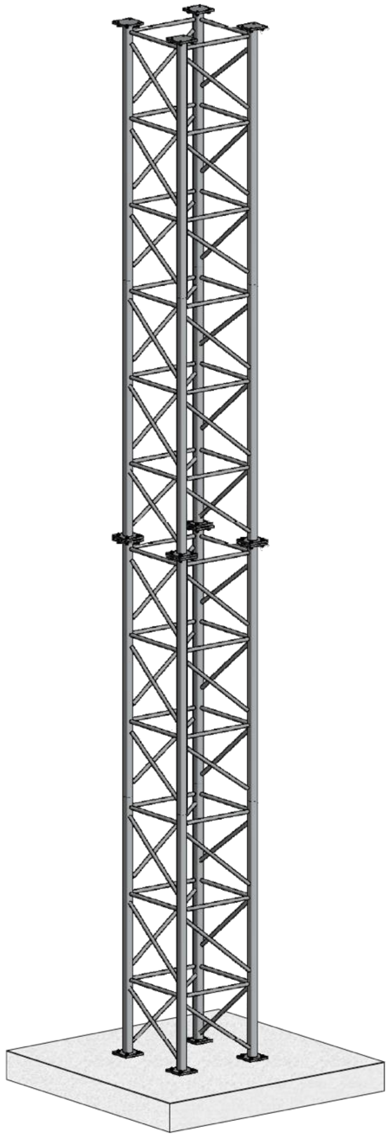
1. Using the 24 inch stick, position the supports and loads (with your finger) as shown in the diagrams below. Hold the beam down on the reactions if it lifts up.
2. For each case observe and draw the elastic curve.
3. Label + and – curvature (moment) and points of contraflexure.
4. Estimate the effective lengths, ℓ_e, across the beam. (between points of M=0)

Link to Prof. von Buelow's description of this lab.

Tower Project:

Due date for the submission of **the revision** of the Preliminary report (optional) is March 21

Tower Test: March 23



Thank you.

Any question?

Please feel free to ask questions.

Contact: Mohsenv@umich.edu