

# Architecture 324: Structures II

## LRFD Steel Beam Analysis – SOLUTIONS

Problem 4: Beam Capacity Analysis — Dataset 1

**Given Data:** W-Section: W12×35  $F_y$ : 50 KSI  $E$ : 29,000 KSI  
**Span A:** 25 FT (beam span) **Span B:** 11 FT (tributary width) **Floor DL:** 18 PSF  
**Assumptions:** Fully braced ( $L_b < L_p$ )  $\phi_b = 0.90$   $\phi_v = 1.0$  Deflection Limit:  $L/180$   
**LRFD Load Combo:**  $w_u = 1.2 w_{DL} + 1.6 w_{LL}$

### Part 1: Section Properties (AISC Table 1-1)

#### Q1 Plastic Section Modulus ( $Z_x$ )

Lookup: AISC Table 1-1 → W12×35

- Action:** Open AISC Table 1-1 (W-Shapes).
- Locate:** Row for W12×35.
- Extract:**  $Z_x = 51.2 \text{ IN}^3$
- Also note:**  $I_x = 285 \text{ IN}^4$ ,  $d = 12.5 \text{ IN}$ ,  $t_w = 0.300 \text{ IN}$ ,  $r_y = 1.54 \text{ IN}$

Final  $Z_x$ : **51.2 IN<sup>3</sup>**

### Part 2: Flexural Capacity (Zone 1: $L_b < L_p$ )

Procedure per Lecture: Verify Zone 1, then compute  $M_n$  and  $\phi_b M_n$ .

#### Q2 Nominal Bending Moment ( $M_n$ )

$$M_n = M_p = F_y \times Z_x$$

- Zone Check:**  $L_p = 1.76 r_y \sqrt{E/F_y} = 1.76(1.54) \sqrt{29,000/50} = 65.3 \text{ IN} = 5.44 \text{ FT}$ . Fully braced ( $L_b < L_p$ ) ⇒ **Zone 1 OK ✓**
- Calculate:**  $M_n = 50 \times 51.2 = 2,560 \text{ K-IN}$

Final  $M_n$ : **2,560 K-IN**

#### Q3 Factored Bending Resistance ( $\phi_b M_n$ )

$$\phi_b M_n = 0.90 \times M_n$$

- Calculate:**  $0.90 \times 2,560 = 2,304 \text{ K-IN}$

Final  $\phi_b M_n$ : **2,304 K-IN**

#### Q4 Factored Design Moment ( $M_u$ )

$$M_u = \phi_b M_n \quad (\text{set design at full capacity})$$

- Convert to K-FT:**  $M_u = 2,304 \div 12 = 192.0 \text{ K-FT}$

Final  $M_u$ : **192.0 K-FT**

### Part 3: Load Capacity

From  $M_u = w_u L^2 / 8$ , solve for  $w_u$ . Then:  $w_u = 1.2 w_{DL} + 1.6 w_{LL}$ , solve for  $w_{LL}$ .

#### Q5 Total Factored Design Load ( $w_u$ )

$$w_u = \frac{8 M_u}{L^2}$$

- Numerator:**  $8 \times 192.0 = 1,536$
- Denominator:**  $L^2 = 25^2 = 625$
- Divide:**  $1,536/625 = 2.458 \text{ KLF}$

Final  $w_u$ : **2.46 KLF**

#### Q6 Total Unfactored Dead Load ( $w_{DL}$ )

$$w_{DL} = w_{\text{beam}} + w_{\text{floor}}$$

- Beam Self-Wt:** W12×35 ⇒ 35 PLF
- Floor DL:**  $18 \times 11 = 198 \text{ PLF}$
- Sum & Convert:**  $(35 + 198)/1,000 = 0.233 \text{ KLF}$

Final  $w_{DL}$ : **0.233 KLF**

#### Q7 Total Factored Dead Load ( $w_{u,DL}$ )

$$w_{u,DL} = 1.2 \times w_{DL}$$

- Calculate:**  $1.2 \times 0.233 = 0.280 \text{ KLF}$

Final  $w_{u,DL}$ : **0.280 KLF**

#### Q8 Factored Live Load ( $w_{u,LL}$ )

$$w_{u,LL} = w_u - w_{u,DL}$$

- Subtract:**  $2.458 - 0.280 = 2.178 \text{ KLF}$

Final  $w_{u,LL}$ : **2.18 KLF**

### Q9 Actual Beam Live Load Capacity ( $w_{LL}$ )

$$w_{LL} = \frac{w_{u,LL}}{1.6}$$

- Unfactor:**  $2.178/1.6 = 1.361 \text{ KLF}$

Final  $w_{LL}$ : **1.36 KLF**

### Q10 Floor Live Load Capacity ( $LL$ )

$$LL = \frac{w_{LL} \times 1,000}{\text{Span B}}$$

- Convert & Distribute:**  $1.361 \times 1,000/11 = 123.7 \text{ PSF}$

Final  $LL$ : **123.7 PSF**

### Part 4: Shear Check

Check shear zone via  $h/t_w$ , then verify  $V_u < \phi_v V_n$ .

#### Q11 Max. Factored Shear ( $V_{u,\text{max}}$ )

$$V_{u,\text{max}} = \frac{w_u \times L}{2}$$

- Calculate:**  $2.458 \times 25/2 = 30.72 \text{ K}$

Final  $V_{u,\text{max}}$ : **30.72 K**

#### Q12 Web Area ( $A_w$ )

$$A_w = d \times t_w$$

- From Table 1-1:**  $d = 12.5 \text{ IN}$ ,  $t_w = 0.300 \text{ IN}$
- Multiply:**  $12.5 \times 0.300 = 3.75 \text{ IN}^2$

Final  $A_w$ : **3.75 IN<sup>2</sup>**

#### Q13 Factored Shear Resistance ( $\phi_v V_n$ )

$$\phi_v V_n = \phi_v \times 0.6 \times F_y \times A_w \times C_v$$

- Zone Check:**  $h/t_w = 36.2 \leq 2.45 \sqrt{E/F_y} = 59.0$   
⇒ Zone 1 ( $C_v = 1.0$ ,  $\phi_v = 1.0$ )
- Factor:**  $\phi_v V_n = 1.0 \times 0.6 \times 50 \times 3.75 \times 1.0 = 112.5 \text{ K}$

Final  $\phi_v V_n$ : **112.5 K**

#### Q14 Shear Adequacy Check

- Compare:**  $30.72 \text{ K}$  ( $V_u$ ) vs.  $112.5 \text{ K}$  ( $\phi_v V_n$ )
- Check:**  $30.72 < 112.5$  ⇒ **YES**

Safe for Shear? **1 (Yes)**

### Part 5: Deflection Check

Unfactored total D+L load; limit  $L/180$ .

#### Q15 Actual Deflection ( $\Delta_{\text{actual}}$ )

$$\Delta = \frac{5 w L^4}{384 E I_x} \quad (\text{all units: K, IN})$$

- Load ( $w$ ):**  $0.233 + 1.361 = 1.594 \text{ KLF} \rightarrow 0.13285 \text{ K/IN}$
- Span ( $L$ ):**  $25' \times 12 = 300 \text{ IN}$   $I_x: 285 \text{ IN}^4$
- Calc:**  $\Delta = \frac{5(0.13285)(300^4)}{384(29,000)(285)} = \frac{5.381 \times 10^9}{3.174 \times 10^9} = 1.70 \text{ IN}$

Final  $\Delta_{\text{actual}}$ : **1.70 IN**

#### Q16 Deflection Limit ( $L/180$ )

$$\Delta_{\text{limit}} = \frac{L \text{ (in)}}{180}$$

- Calculate:**  $300/180 = 1.67 \text{ IN}$

Final  $\Delta_{\text{limit}}$ : **1.67 IN**

#### Q17 Deflection Adequacy Check

- Check:**  $1.70 \text{ IN}$  ( $\Delta_{\text{act}}$ )  $>$   $1.67 \text{ IN}$  ( $\Delta_{\text{lim}}$ ) ⇒ **NO**

Deflection OK? **0 (No)**